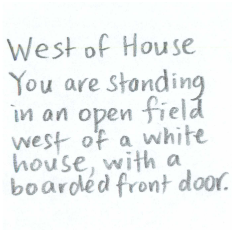
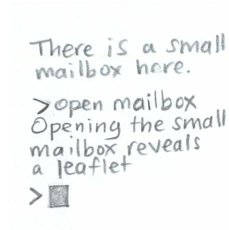


MATH 124: Matrixology (Linear Algebra)
Level Zork (1977), 2 of 10
University of Vermont, Spring 2015



Dispersed: Thursday, January 22, 2015.
Due: By start of lecture, Thursday, January 29, 2015.
Sections covered: 2.3, 2.4.

Some useful reminders:
Instructor: Prof. Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 2 to 2:45 pm, Mondays; 3 to 3:45 pm Tuesdays; and 1 to 2:30 pm Wednesdays
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2015-01UVM-124
Textbook: "Introduction to Linear Algebra" (3rd or 4th edition) by Gilbert Strang (published by Wellesley-Cambridge Press).

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

1. (similar to Q 24, Section 2.3) Apply elimination to the 2 by 3 augmented matrix $[\,A \;\; \vec{b}\,]$ for the equation given below. Do this using elimination matrix E_{21}. What is the triangular system $U\vec{x} = \vec{c}$? What is the solution $\vec{x}$?

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

2. Write down the 3 by 3 matrices that produce the following elimination or permutation steps:

   (a) E_{21} subtracts 4 times row 1 from row 2.

   (b) E_{32} subtracts -3 times row 2 from row 3.

   (c) P_{23} swaps rows 2 and 3.

3. (modified version of Q 3, Section 2.3) Which three matrices E_{21}, E_{31}, and E_{32} put A into triangular form U? What is U here? $A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 2 & 0 \\ 4 & 6 & 1 \end{bmatrix}$ and $E_{32}E_{31}E_{21}A = U$.

4. (Q 6, Section 2.4) Show that $(A+B)^2$ is different from $A^2+2AB+B^2$, when

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

Write down the correct rule for $(A+B)(A+B)$.

5. (Q 14, Section 2.4) True or false (briefly explain why):

   (a) If A^2 is defined then A is necessarily square.

   (b) if AB and BA are defined then A and B are square.

   (c) if AB and BA are defined then AB and BA are square.

   (d) if $AB = B$ then $A = I$ (where I is the identity matrix and the matrix B is not filled with zilches (0s)).

6. (Q 26, Section 2.4) Multiply AB using columns times rows:

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix}.$$

(You are calculating 'outer products' instead of inner products as we did for an example in Episode 4.)

7. Find all the powers A^2, A^3, ... and AB, $(AB)^2$, ... for

$$A = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

8. (Q 36, Section 2.4) Find all matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that satisfy

$$A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} A.$$

9. (2 pts)

   Matlab action: Compute the following

$$\mathbf{A} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

Incredibly, the above product of three matrices will be one very useful way to view A. More later.

10. (4 pts)

    Matlab action:

    For the A you found in the previous question, compute (a) A^2, (b) A^5, (c) A^{10}, and (d) A^{100}.

    What appears to be happening? We'll fully understand what's going on in about 10 more weeks.

11. (Bonus, 1 point)

    List five of the numerous peculiarities of the very curious species ornithorhynchus anatinus.