Complex Networks, CSYS/MATH 303
University of Vermont, Spring 2014
Assignment 9 • code name: Luft Balons

Dispersed: Thursday, April 18, 2014.
Due: By start of lecture, 2:30 pm, Thursday, April 25, 2014.
Some useful reminders:
Instructor: Peter Dodds
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Office hours: 3:45 pm to 4:15 pm, Tuesday, and 12:45 pm to 2:15 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-01UVM-303

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. (12 pts) Consider a family of undirected random networks with degree distribution

$$P_k = c\delta_{k1} + (1-c)\delta_{k3},$$

where δ_{ij} is the Kronecker delta function, and where c is a constant to be determined below. Also allow nodes to be correlated according to the following node-node mixing probabilities.

Conditional probability version, $P(k\,|k'\,)$:

$$\begin{aligned} P(1\,|\,1) &= \frac{1}{2}(1+r), \\ P(3\,|\,1) &= \frac{1}{2}(1-r), \\ P(1\,|\,3) &= \frac{1}{2}(1-r), \\ \text{and } P(3\,|\,3) &= \frac{1}{2}(1+r). \end{aligned}$$

where $-1 \le r \le 1$ is the family's tunable parameter.

Newman's correlation probability version:

$$E = [e_{ij}] = \begin{bmatrix} e_{00} & e_{02} \\ e_{20} & e_{22} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} (1+r) & (1-r) \\ (1-r) & (1+r) \end{bmatrix}$$

where e_{ij} is the probability that a randomly chosen edge connects a node of degree $i+1$ an a node of degree $j+1$, and only the non-zero values of E are shown.

For the following questions, you can use either formulation.

(a) Determine c so that purely disassortative networks are achievable if r is tuned to -1.

(b) Determine which networks in this family have a giant component. In other words, find the values of r for which a giant component exists.

Note which value (or values) of r mark a phase transition.

(c) Analytically determine the size of the giant component as a function of r.

(d) Determine the size of the largest component containing only degree 3 nodes as a function of r.

Hint: allow degree 3 nodes to be always vulnerable ($\beta_{3i} = 1$ for $i = 0$, 1, 2, and 3) and degree 1 nodes to be never vulnerable ($\beta_{1i} = 0$ for $i = 0$ and 1).

2. Spreading on assortative networks: Starting from

$$\theta_{j,t+1} = G_j(\vec{\theta}_t) = \phi_0 + (1 - \phi_0)\times$$

$$\sum_{k=1}^{\infty} \frac{e_{j-1,k-1}}{R_{j-1}} \sum_{i=0}^{k-1} \binom{k-1}{i} \theta_{k,t}^{i} (1 - \theta_{k,t})^{k-1-i} B_{ki}.$$

show the matrix for which we must have the largest eigenvalue greater than 1 for spreading to occur is

$$\frac{\partial G_j(\vec{0})}{\partial \theta_{k,t}} = \frac{e_{j-1,k-1}}{R_{j-1}}(k-1)(\beta_{k1} - \beta_{k0}).$$

3. Show that for uncorrelated networks, i.e, when $e_{jk} = R_j R_k$, the above condition collapses to the standard condition

$$\sum_{k=1}^{\infty}(k-1)\frac{kP_k}{\langle k \rangle}(\beta_{k1} - \beta_{k0}) > 1.$$