



**Complex Networks, CSYS/MATH 303**  
**University of Vermont, Spring 2014**  
**Assignment 6 • code name: Chief Vitalstatistix**

**Dispersed:** Friday, March 21, 2014.

**Due:** By start of lecture, 2:30 pm, Thursday, April 3, 2014.

*Some useful reminders:*

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**Office hours:** 3:45 pm to 4:15 pm, Tuesday, and 12:45 pm to 2:15 pm, Wednesday

**Course website:** <http://www.uvm.edu/~pdodds/teaching/courses/2014-01UVM-303>

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All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use  $\LaTeX$  (or related  $\TeX$  variant).

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1. Generating functions and giant components: In this question, you will use generating functions to obtain a number of results we found in class for standard random networks.
  - (a) For an infinite standard random network (Erdős-Rényi/ER network) with average degree  $\langle k \rangle$ , compute the generating function  $F_P$  for the degree distribution  $P_k$ .  
(Recall the degree distribution is Poisson:  $P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$ ,  $k \geq 0$ .)
  - (b) Show that  $F'_P(1) = \langle k \rangle$  (as it should).
  - (c) Using the joyous properties of generating functions, show that the second moment of the degree distribution is  $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$ .
2.
  - (a) Continuing on from Q1 for infinite standard random networks, find the generating function  $F_R(x)$  for the  $\{R_k\}$ , where  $R_k$  is the probability that a node arrived at by following a random direction on a randomly chosen edge has  $k$  outgoing edges.
  - (b) Now, using  $F_R(x)$  determine the average number of outgoing edges from a randomly-arrived-at-along-a-random-edge node.
  - (c) Given your findings above, what is the condition on  $\langle k \rangle$  for a standard random network to have a giant component?

3. (a) Find the generating function for the degree distribution  $P_k$  of a finite random network with  $N$  nodes and an edge probability of  $p$ .
- (b) Show that the generating function for the finite ER network tends to the generating function for the infinite one. Do this by taking the limit  $N \rightarrow \infty$  and  $p \rightarrow 0$  such that  $p(N - 1) = \langle k \rangle$  remains constant.
4. (a) Prove that if random variables  $U$  and  $V$  are distributed over the non-negative integers then the generating function for the random variable  $W = U + V$  is

$$F_W(x) = F_U(x)F_V(x).$$

Denote the specific distributions by  $\Pr(U = i) = U_i$ ,  $\Pr(V = i) = V_i$ , and  $\Pr(W = i) = W_i$ .

- (b) Using the your result in part (a), argue that if

$$W = \sum_{j=1}^U V^{(j)}$$

where  $V^{(j)} \stackrel{d}{=} V$  then

$$F_W(x) = F_U(F_V(x)).$$

Hint: write down the generating function of probability distribution of  $\sum_{j=1}^k V^{(j)}$  in terms of  $F_V(x)$ .

5. (a) Again, given

$$W = \sum_{i=1}^U V^{(i)} \text{ with each } V^{(i)} \stackrel{d}{=} V$$

where we know that

$$F_W(x) = F_U(F_V(x)),$$

determine the mean of  $W$  in terms of the means of  $U$  and  $V$ .

- (b) For  $W = U + V$ , similarly find the mean of  $W$  in terms of  $U$  and  $V$  via generating functions. Your answer should make sense.