# Social Contagion

Principles of Complex Systems
CSYS/MATH 300, Spring, 2013 | #SpringPoCS2013

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## Outline







## Things that spread well:

buzzfeed.com (⊞):

LOL win omg cute geeky trashy fail wtf

+ News ...



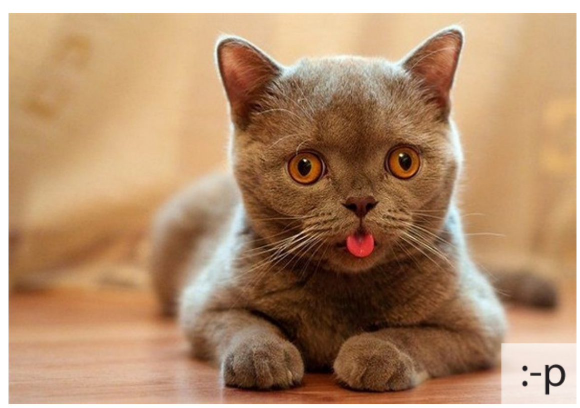


## LOL + cute + fail + wtf:






## The whole lolcats thing:



## Some things really stick:







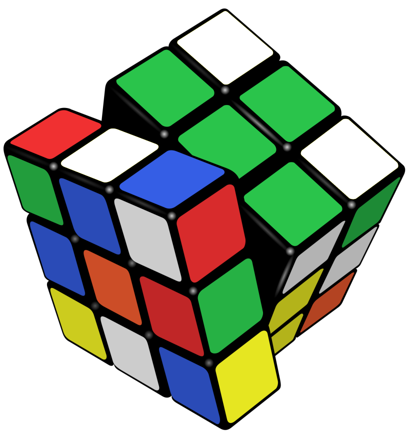
## Social Contagion







## wtf + geeky + omg:









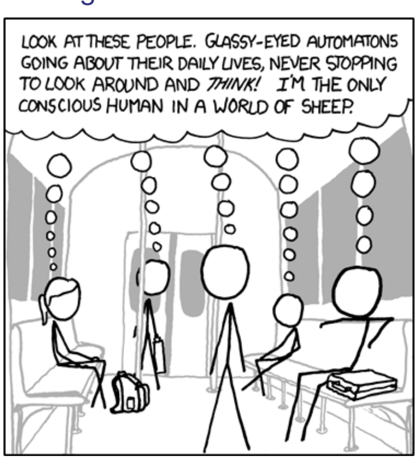
## Social Contagion







## Social Contagion




http://xkcd.com/610/ (⊞)







## Social Contagion

### Examples abound

- fashion
- striking
- smoking (⊞) [7]
- residential segregation [19]
- ipods
- obesity (⊞) [6]
- Harry Potter
- voting
- gossip
- Rubik's cube
- religious beliefs
- leaving lectures

### SIR and SIRS contagion possible

- Classes of behavior versus specific behavior: dieting

## Framingham heart study:

### Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking (⊞) [7]
- The spread of spreading (⊞) [6]
- Also: happiness (⊞) [9], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (⊞)

### Controversy:

- Are your friends making you fat? (⊞) (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious (⊞)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).







## Social Contagion

### Two focuses for us

- Widespread media influence
- Word-of-mouth influence

### We need to understand influence

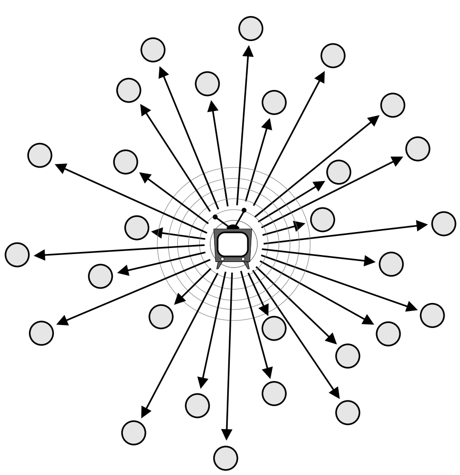
- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers? Highly popularized by Gladwell [10] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [16]







## The hypodermic model of influence

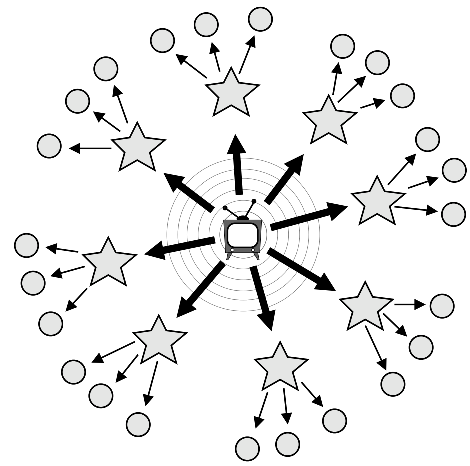








## The two step model of influence [16]

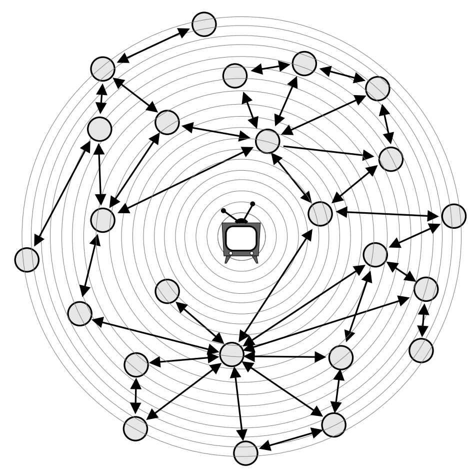








## The general model of influence









## Social Contagion

### Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

## The Mona Lisa

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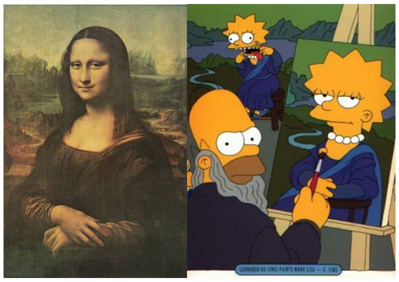


- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism, parody, ...





## The completely unpredicted fall of Eastern Europe

Timur Kuran:[17, 18] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"





## The dismal predictive powers of editors...






## Social Contagion

### Messing with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent (⊞)
- One of Facebook's early advertising attempts: Beacon (⊞)
- All of Facebook's advertising attempts.





## Getting others to do things for you

A very good book: 'Influence'[8] by Robert Cialdini (⊞)

### Six modes of influence:

1. Reciprocation: *The Old Give and Take... and Take*; e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*; e.g., Hazing.
3. Social Proof: *Truths Are Us*; e.g., Jonestown (⊞), Kitty Genovese (⊞) (contested).
4. Liking: *The Friendly Thief*; e.g., Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*; e.g., Milgram's obedience to authority experiment. (⊞)
6. Scarcity: *The Rule of the Few*; e.g., Prohibition.





## Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

### Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

# Social Contagion



## Some important models:

- Tipping models—Schelling (1971) [19, 20, 21]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) online implementation (⊞) [26]
- Threshold models—Granovetter (1978) [13]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
  - Social learning theory, Informational cascades,...





# Social contagion models



## Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (unrealistic).
- Assumption: level of influence per person is uniform (unrealistic).





# Social Contagion



## Some possible origins of thresholds:

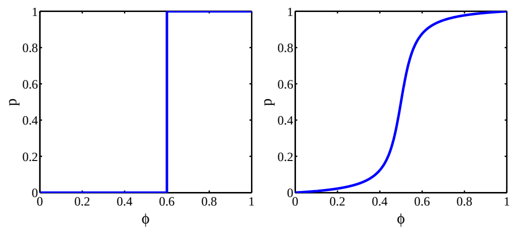
- Inherent, evolution-devised inclination to coordinate, to conform, to imitate. [1]
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
  - Externalities = Effects on others not directly involved in a transaction
  - Examples: telephones, fax machine, Facebook, operating systems
  - An individual's utility increases with the adoption level among peers and the population in general





# Threshold models—response functions






- Example threshold influence response functions: deterministic and stochastic
- $\phi$ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.





# Social Contagion



## Granovetter's Threshold model—definitions

- $\phi^*$ = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- $\phi_t$ = fraction of people 'rioting' at time step $t$.

- At time $t+1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.
- $$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*)\mathrm{d}\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$
- $\Rightarrow$ Iterative maps of the unit interval $[0, 1]$.

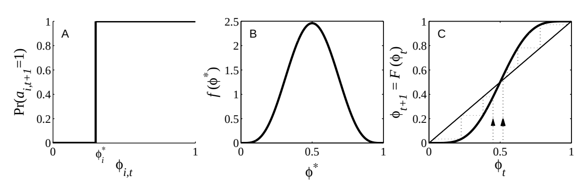




# Threshold models



## Action based on perceived behavior of others:

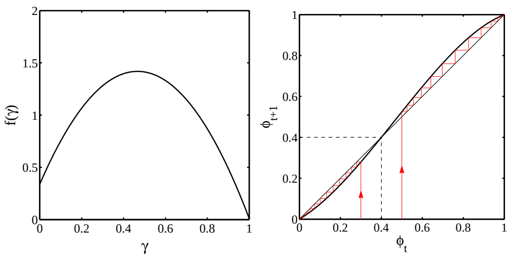



- Two states: S and I.
- $\phi$ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

# Threshold models



## Another example of critical mass model:



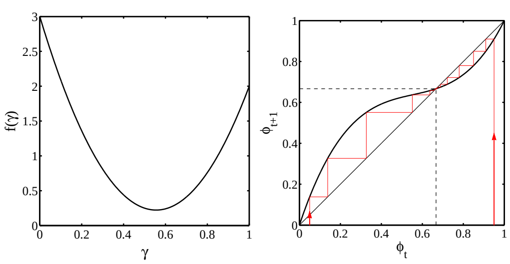





# Threshold models



## Example of single stable state model:



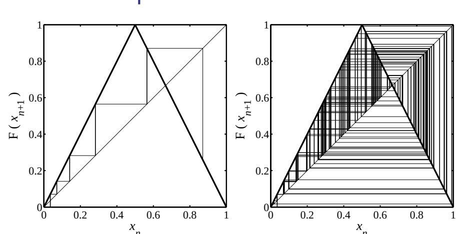





# Threshold models



## Chaotic behavior possible [15, 14]




- Period doubling arises as map amplitude $r$ is increased.
- Synchronous update assumption is crucial





# Threshold models—Nutshell



## Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes





## "A simple model of global cascades on random networks"

- Many years after Granovetter and Soong's work: D. J. Watts. Proc. Natl. Acad. Sci., 2002 [23]
  - Mean field model $\rightarrow$ network model
  - Individuals now have a limited view of the world



## We'll also explore:

- "Seed size strongly affects cascades on random networks" [12]
  Gleeson and Cahalane, Phys. Rev. E, 2007.
- "Influentials, Networks, and Public Opinion Formation" [24]
  Watts and Dodds, J. Cons. Res., 2007.
- "Threshold models of Social Influence" [25]
  Watts and Dodds, The Oxford Handbook of Analytical Sociology, 2009.



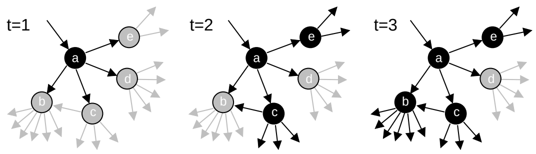


# Threshold model on a network



- Interactions between individuals now represented by a network
- Network is sparse
- Individual $i$ has $k_i$ contacts
- Influence on each link is reciprocal and of unit weight
- Each individual $i$ has a fixed threshold $\phi_i$
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual $i$ becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)

## Threshold model on a network




- All nodes have threshold $\phi = 0.2$.







## Snowballing

### First study random networks:

- Start with $N$ nodes with a degree distribution $p_k$
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

### The Cascade Condition:

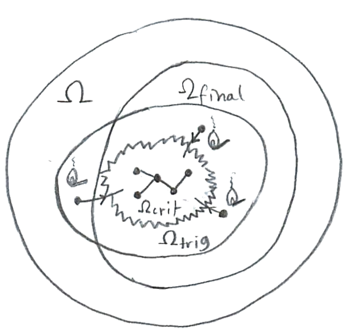
1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?







## Example random network structure:




- $\Omega_{\rm crit} = \Omega_{\rm vuln}$ = critical mass = global vulnerable component
- $\Omega_{\rm trig}$ = triggering component
- $\Omega_{\rm final}$ = potential extent of spread
- $\Omega$ = entire network

$\Omega_{\rm crit} \subset \Omega_{\rm trig}$; $\Omega_{\rm crit} \subset \Omega_{\rm final}$; and $\Omega_{\rm trig}, \Omega_{\rm final} \subset \Omega$.







## Snowballing

### Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.







## The most gullible

### Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node $i$:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [23]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.







## Cascade condition

### Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree $k$ node with probability $\propto kP_k$.
- Follows from there being $k$ ways to connect to a node with degree $k$.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

## Cascade condition



### Next: Vulnerability of linked node

- Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) \mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces $k-1$ new outgoing active links
- If linked node is not vulnerable, it produces no active links.





## Cascade condition



### Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \underbrace{\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1-\beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}}$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$





## Cascade condition



So... for random networks with fixed degree distributions, cacades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- $\beta_k$ = probability a degree $k$ node is vulnerable.
- $P_k$ = probability a node has degree $k$.





## Cascade condition



### Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- (2) Giant component exists: $\beta = 1$

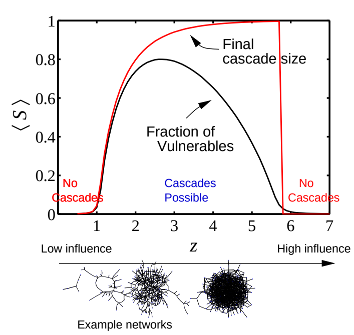
$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$





## Cascades on random networks





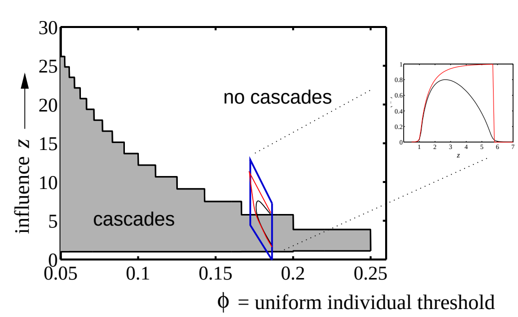

- Cascades occur only if size of max vulnerable cluster $> 0$.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.





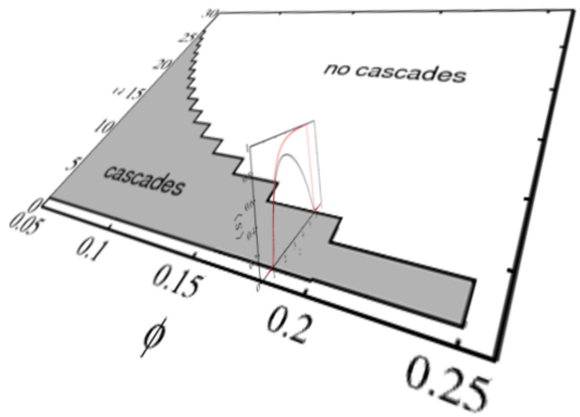
## Cascade window for random networks






- 'Cascade window' widens as threshold $\phi$ decreases.
- Lower thresholds enable spreading.

## Cascade window for random networks

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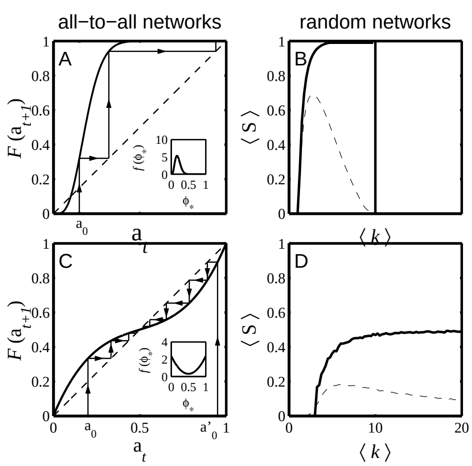
## All-to-all versus random networks

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## Cascade window—summary

For our simple model of a uniform threshold:

1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

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## Threshold contagion on random networks

- Next: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need $\geq 2$ hits switch on.
- Problem solved for infinite seed case by Gleeson and Cahalane:
  "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007.[12]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008.[11]

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## Expected size of spread

Idea:

- Randomly turn on a fraction $\phi_0$ of nodes at time $t = 0$
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node $i$ to become active at time $t$:
  - $t = 0$: $i$ is one of the seeds (prob = $\phi_0$)
  - $t = 1$: $i$ was not a seed but enough of $i$'s friends switched on at time $t = 0$ so that $i$'s threshold is now exceeded.
  - $t = 2$: enough of $i$'s friends and friends-of-friends switched on at time $t = 0$ so that $i$'s threshold is now exceeded.
  - $t = n$: enough nodes within $n$ hops of $i$ switched on at $t = 0$ and their effects have propagated to reach $i$.

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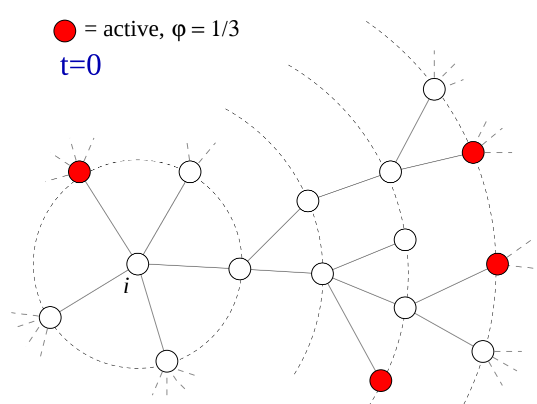
## Expected size of spread

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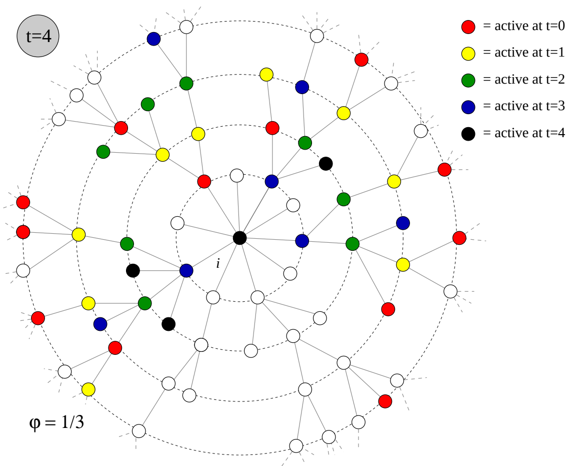
## Expected size of spread










## Expected size of spread

Notes:

- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine **Pr**(node of degree $k$ switching on at time $t$).
- Asynchronous updating can be handled too.



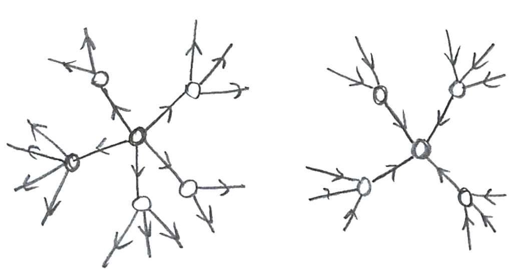




## Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.









## Expected size of spread

- Notation: $\phi_{k,t} = \mathbf{Pr}$(a degree $k$ node is active at time $t$).
- Notation: $B_{kj} = \mathbf{Pr}$ (a degree $k$ node becomes active if $j$ neighbors are active).
- Our starting point: $\phi_{k,0} = \phi_0$.
- $\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j}$ = **Pr** ($j$ of a degree $k$ node's neighbors were seeded at time $t=0$).
- Probability a degree $k$ node was a seed at $t=0$ is $\phi_0$ (as above).
- Probability a degree $k$ node was not a seed at $t=0$ is $(1-\phi_0)$.
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1-\phi_0)\sum_{j=0}^{k}\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j}B_{kj}.$$







## Expected size of spread

- For general $t$, we need to know the probability an edge coming into a degree $k$ node at time $t$ is active.
- Notation: call this probability $\theta_t$.
- We already know $\theta_0 = \phi_0$.
- Story analogous to $t=1$ case. For node $i$:

$$\phi_{i,t+1} = \phi_0 + (1-\phi_0)\sum_{j=0}^{k_i}\binom{k_i}{j}\theta_t^j(1-\theta_t)^{k_i-j}B_{k_i j}.$$

- Average over all nodes to obtain expression for $\phi_{t+1}$:

$$\phi_{t+1} = \phi_0 + (1-\phi_0)\sum_{k=0}^{\infty}P_k\sum_{j=0}^{k}\binom{k}{j}\theta_t^j(1-\theta_t)^{k-j}B_{kj}.$$

- So we need to compute $\theta_t$... massive excitement...







## Expected size of spread

First connect $\theta_0$ to $\theta_1$:

- $\theta_1 = \phi_0 +$

$$(1-\phi_0)\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_0^j(1-\theta_0)^{k-1-j}B_{kj}$$

- $\frac{kP_k}{\langle k\rangle} = R_k$ = **Pr** (edge connects to a degree $k$ node).
- $\sum_{j=0}^{k-1}$ piece gives **Pr**(degree node $k$ activates) of its neighbors $k-1$ incoming neighbors are active.
- $\phi_0$ and $(1-\phi_0)$ terms account for state of node at time $t=0$.
- See this all generalizes to give $\theta_{t+1}$ in terms of $\theta_t$...

## Expected size of spread

### Two pieces: edges first, and then nodes

1. $\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$

$$+(1-\phi_0)\underbrace{\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_t^{\,j}(1-\theta_t)^{k-1-j}B_{kj}}_{\text{social effects}}$$

with $\theta_0 = \phi_0$.

2. $\phi_{t+1} =$

$$\underbrace{\phi_0}_{\text{exogenous}} \quad +(1-\phi_0)\underbrace{\sum_{k=0}^{\infty}P_k\sum_{j=0}^{k}\binom{k}{j}\theta_t^{\,j}(1-\theta_t)^{k-j}B_{kj}}_{\text{social effects}}.$$







## Expected size of spread:

- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \to 0$.
- Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

$$G(0;\phi_0) = \sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.

- If $\theta = 0$ is a fixed point of $G$ (i.e., $G(0;\phi_0) = 0$) then spreading occurs if

$$G'(0;\phi_0) = \sum_{k=0}^{\infty}\frac{kP_k}{\langle k\rangle}\bullet(k-1)\bullet B_{k1} > 1.$$







## Expected size of spread:

### In words:

- If $G(0;\phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If $G$ has an unstable fixed point at $\theta = 0$, then cascades are also always possible.

### Non-vanishing seed case:

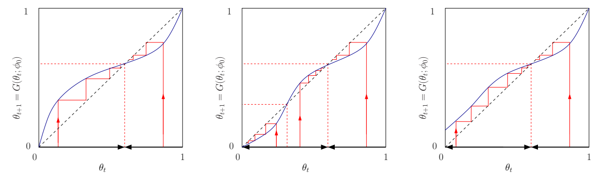
- Cascade condition is more complicated for $\phi_0 > 0$.
- If $G$ has a stable fixed point at $\theta = 0$, and an unstable fixed point for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- Tricky point: $G$ depends on $\phi_0$, so as we change $\phi_0$, we also change $G$.







## General fixed point story:



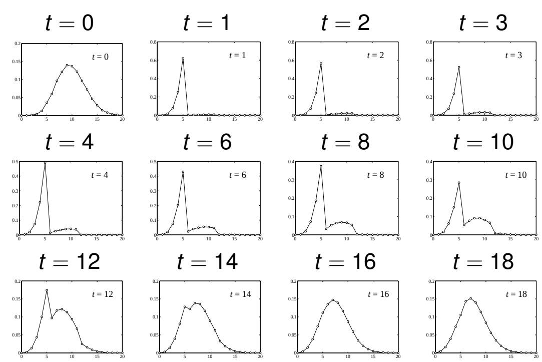
- Given $\theta_0 (= \phi_0)$, $\theta_\infty$ will be the nearest stable fixed point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.
- Important: Actual form of $G$ depends on $\phi_0$.
- So choice of $\phi_0$ dictates both $G$ and starting point—can't start anywhere for a given $G$.







## Early adopters—degree distributions



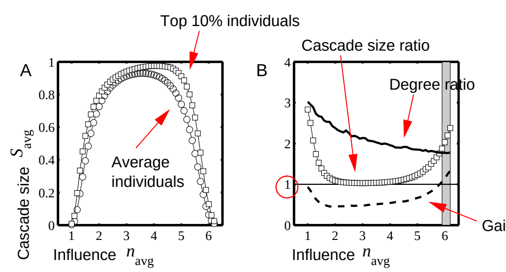

$P_{k,t}$ versus $k$





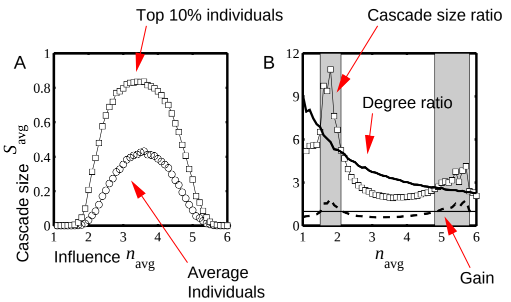


## The multiplier effect:




- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

## The multiplier effect:



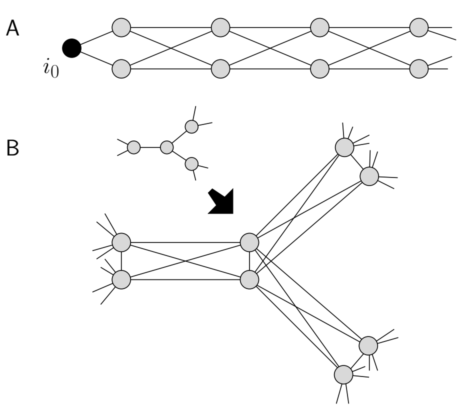

- Skewed influence distribution example.







## Special subnetworks can act as triggers



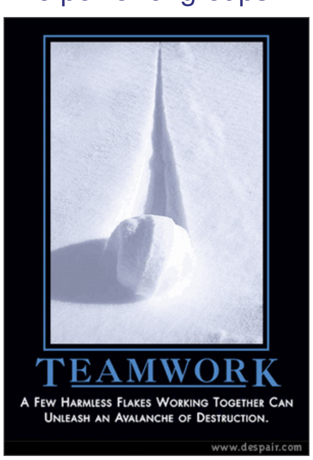

- $\phi = 1/3$ for all nodes







## The power of groups...




despair.com

"A few harmless flakes working together can unleash an avalanche of destruction."







## Extensions

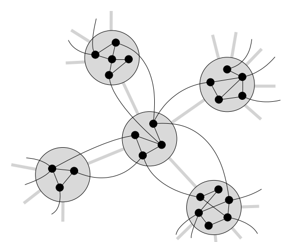
- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure







## Group structure—Ramified random networks



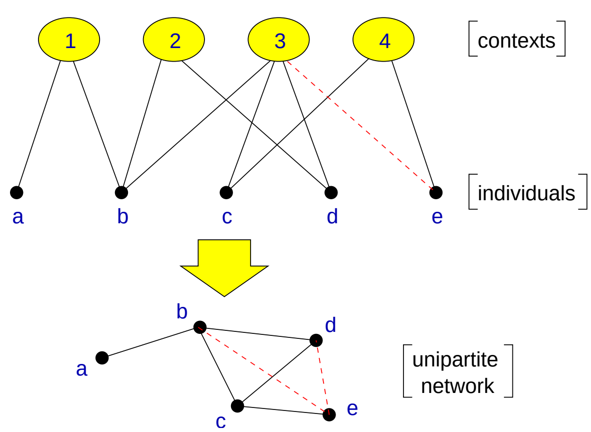
$p$ = intergroup connection probability
$q$ = intragroup connection probability.



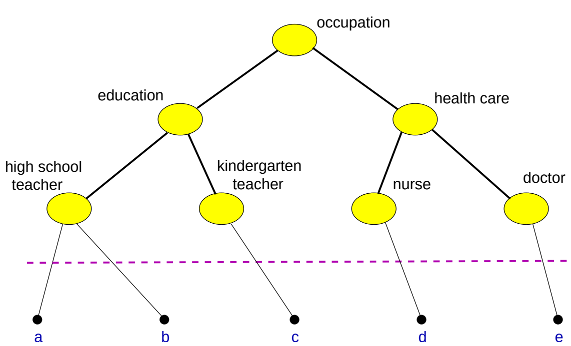




## Bipartite networks



## Context distance

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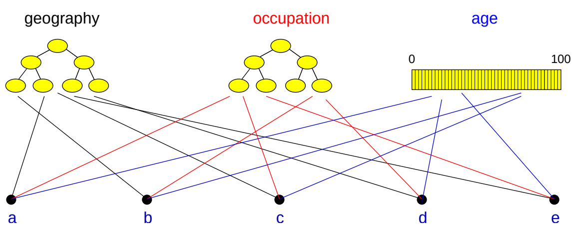
## Generalized affiliation model




(Blau & Schwartz, Simmel, Breiger)

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## Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto \exp^{-\alpha d}$ where
  $\alpha$ = homophily parameter
  and
  $d$ = distance between nodes (height of lowest common ancestor)
- $\tau_1$ = intergroup probability of friend-of-friend connection
- $\tau_2$ = intragroup probability of friend-of-friend connection

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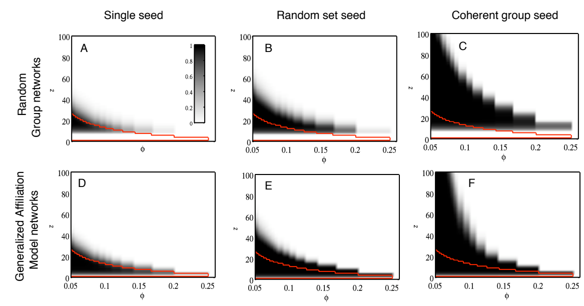
## Cascade windows for group-based networks

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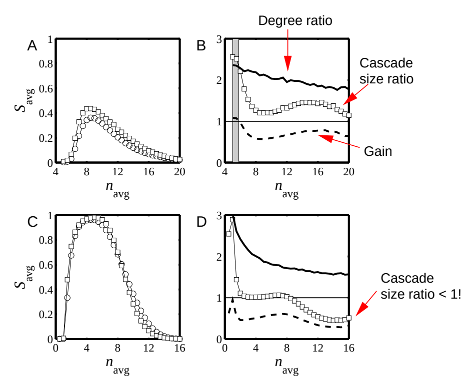
## Multiplier effect for group-based networks:




- Multiplier almost always below 1.

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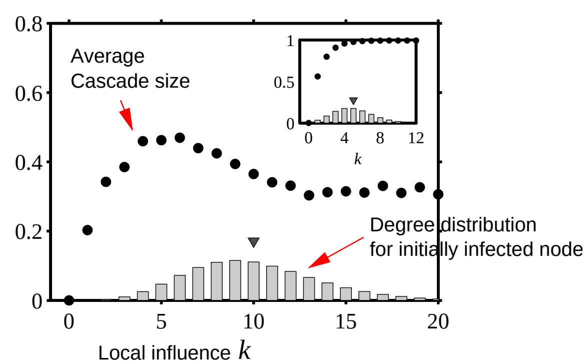
## Assortativity in group-based networks




- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

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# Social contagion





## Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.



# Social contagion





## Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.



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