Overview of Complex Networks

Principles of Complex Systems
CSYS/MATH 300, Spring, 2013 | #SpringPoCS2013

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Outline

net•work |ˈnetˌwərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes *: a network of railroads.*

2 a group or system of interconnected people or things *: a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes *: a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program *: the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations *: specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network *: the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career *: the skills of networking, bargaining, and negotiation.*

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation: (⊞)

- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
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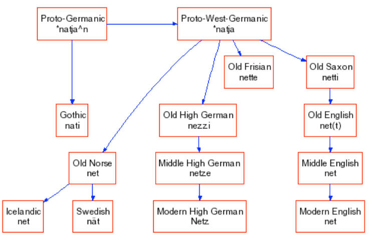
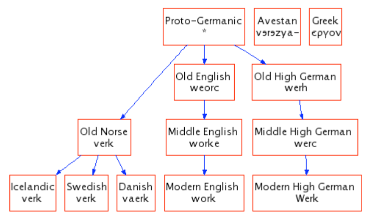
Net and Work are venerable old words:

- ‘Net’ first used to mean spider web (King Ælfréd, 888).
- ‘Work’ appear to have long meant purposeful action.



The network of Germanic `net' words



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- c.f., ironwork, stonework, fretwork.

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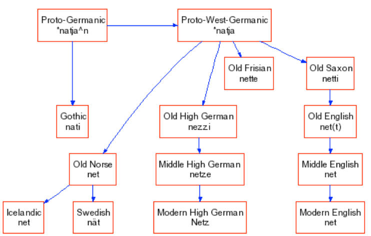

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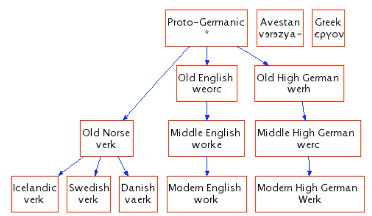

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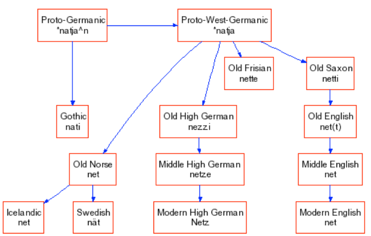

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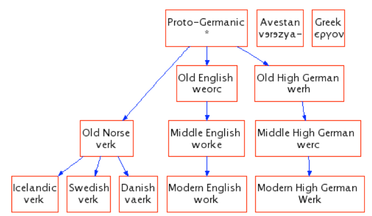

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- Many complex systems
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- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
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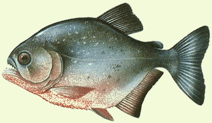

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- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

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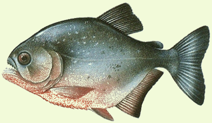

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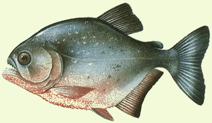

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Popularity (according to Google Scholar)

"Collective dynamics of 'small-world' networks"[18]

- Watts and Strogatz
  Nature, 1998
- Cited $\approx$ 18,450 times (as of March 18, 2013)

"Emergence of scaling in random networks"[2]

- Barabási and Albert
  Science, 1999
- Cited $\approx$ 16,050 times (as of March 18, 2013)

Popularity (according to Google Scholar)

Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [3]
  Times cited: 3,500 (as of March 18, 2013)
- M. Newman
  "The structure and function of complex networks" [13]
  Times cited: 9,100 (as of March 18, 2013)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 11,600 (as of March 18, 2013)

Popularity according to textbooks:

Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction" (⊞)
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" (⊞)

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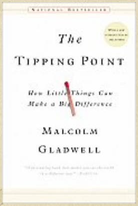
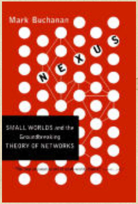


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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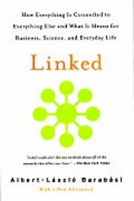
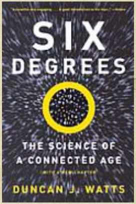


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [17]

Numerous others . . .

- Complex Social Networks—F. Vega-Redondo [16]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [14]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [7]

More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories
  with a much more measurable reality.*
- A worthy goal: establish mechanistic explanations.

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** If this is upsetting, maybe string theory is for you...*

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)
- “The Unreasonable Effectiveness of Data,” Halevy et al.[9].
- c.f. Wigner’s “The Unreasonable Effectiveness of Mathematics in the Natural Sciences”[19]

But:

- For scientists, description is only part of the battle.
- We still need to understand.

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Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

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Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- k_i = 0,1,2,. . . .
- Notation: the average degree of a network = $\langle k \rangle$
- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

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Adjacency matrix:

- We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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Examples

So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

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- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids

- Distribution (branching) versus redistribution (cyclical)

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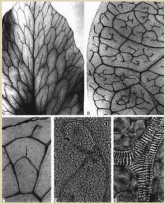

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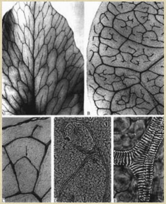

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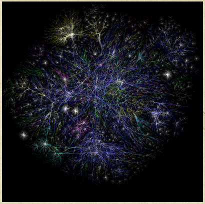

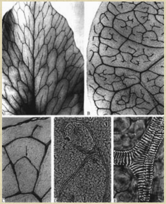

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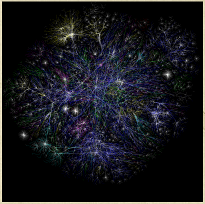

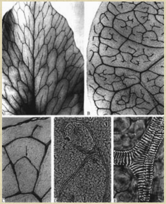

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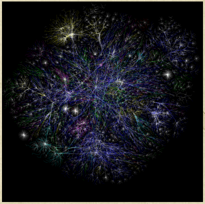

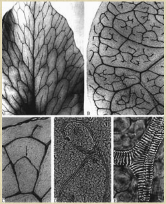

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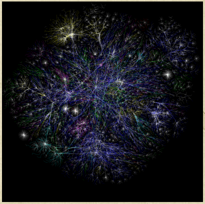

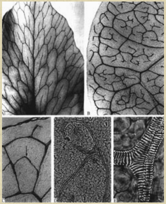

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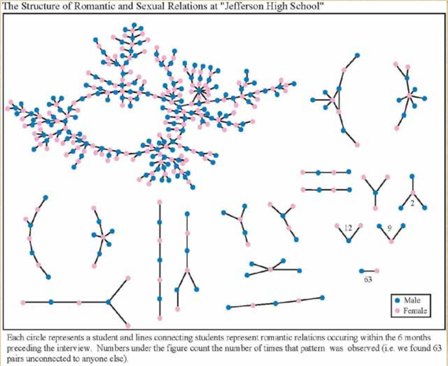

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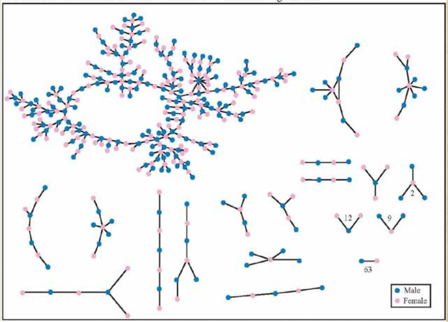

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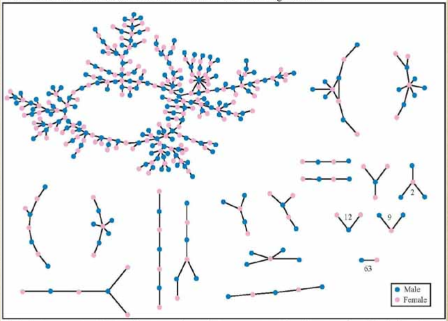

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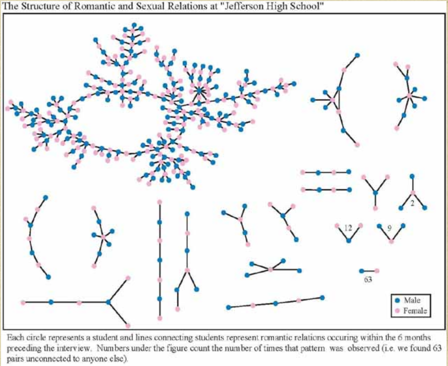

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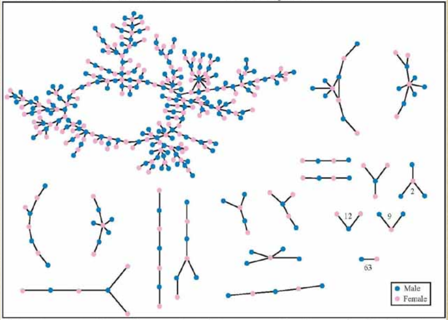

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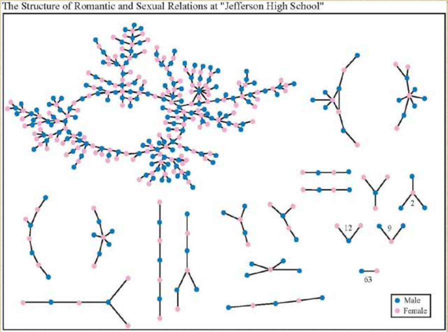

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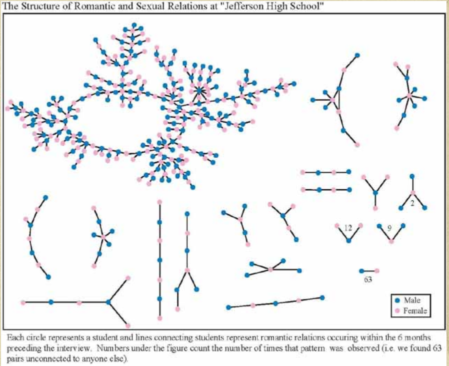

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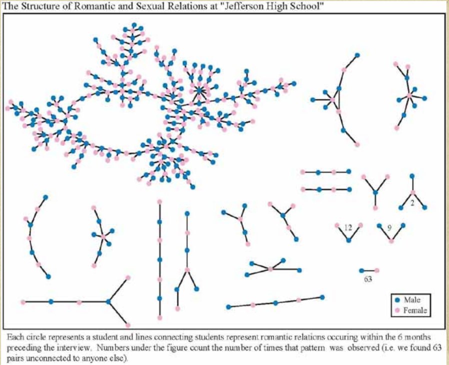

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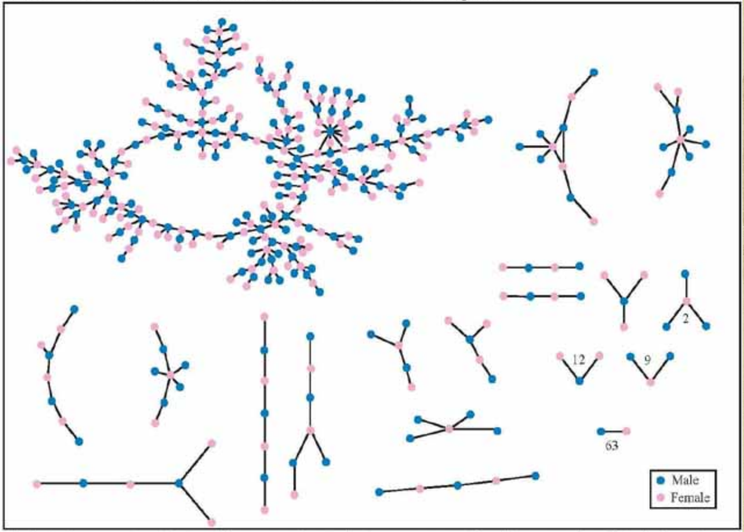

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Relational networks

- Consumer purchases
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: bit.ly (⊞) flickr (⊞)

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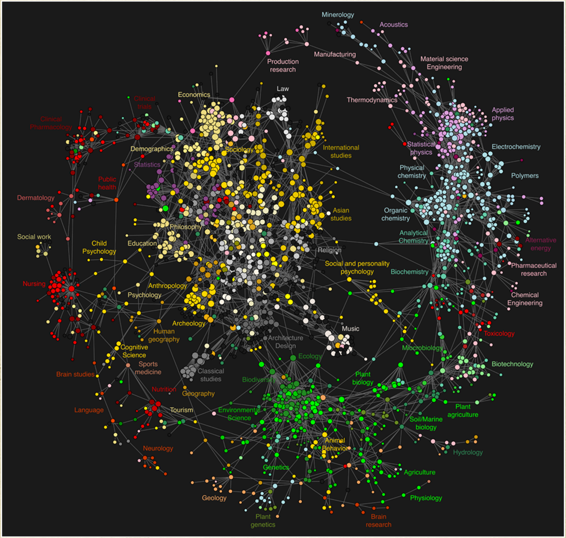
common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

Clickworthy Science:



Bollen et al. [4]; a higher resolution figure is here (⊞)

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- Graphical renderings are often just a big mess.
- And even when renderings somehow look good:
- We need to extract digestible, meaningful aspects.

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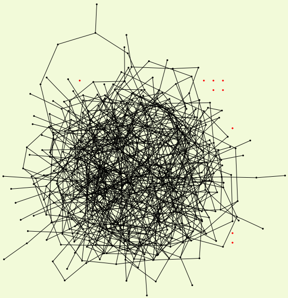

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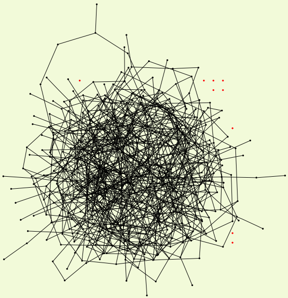

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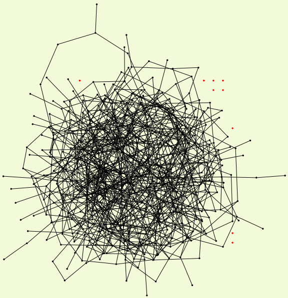

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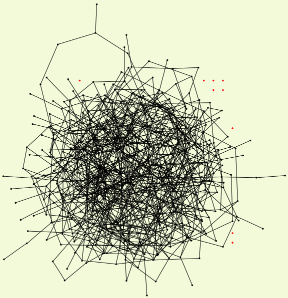

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Properties

Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

- Plus coevolution of network structure and processes on networks.

* Degree distribution is the elephant in the room that we are now all very aware of...

Properties

1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k.
- k = node degree = number of connections.
- ex 1: Erdős-Rényi random networks have Poisson degree distributions:
  Insert question from assignment 5 (⊞)

$$P_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'.
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- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[12] similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.

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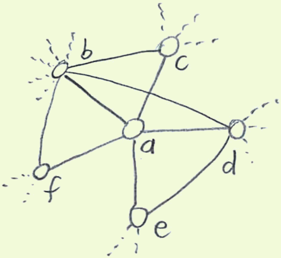
Local socialness:

4. Clustering:



- Your friends tend to know each other.
- Two measures (explained on following slides):
  1. Watts & Strogatz [18]
  $$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$
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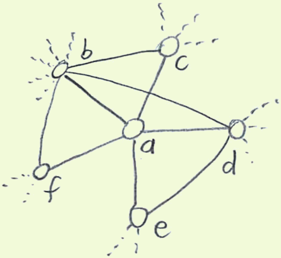

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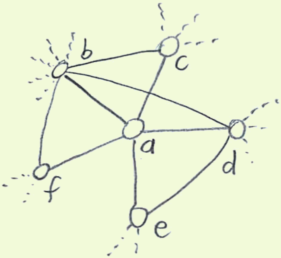

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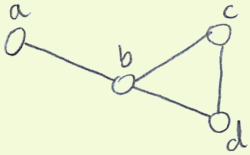
  1. Watts & Strogatz [18]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

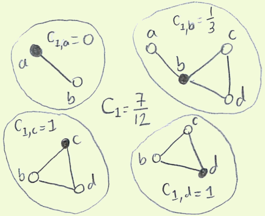
  2. Newman [13]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

Example network:



Calculation of C_1:



- C_1 is the average fraction of pairs of neighbors who are connected.
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$$\frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

where k_i is node i's degree, and $\mathcal{N}_i$ is the set of i's neighbors.

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$C_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2}$

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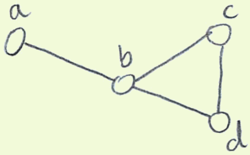

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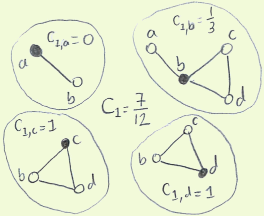

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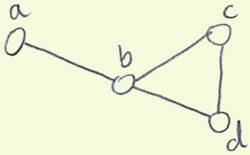

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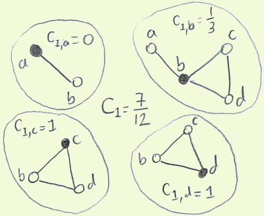

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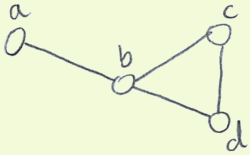

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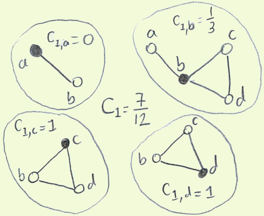

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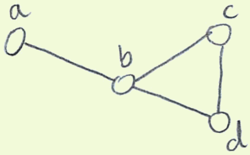

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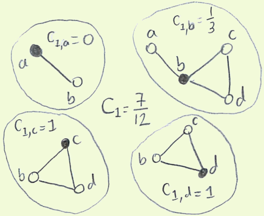

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Triples and triangles

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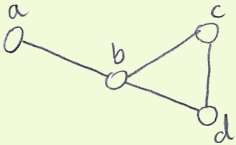

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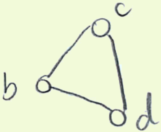

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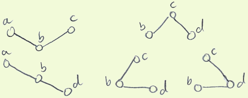

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- Nodes i_1, i_2, and i_3 form a triangle if each pair of nodes is connected
- The definition $C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$ measures the fraction of closed triples
- The '3' appears because for each triangle, we have 3 closed triples.
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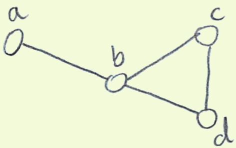

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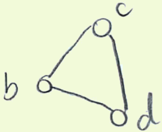

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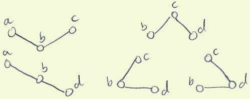

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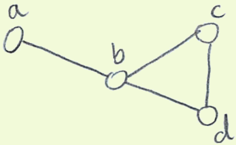

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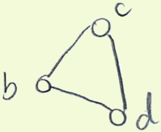

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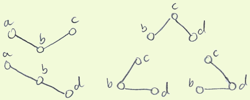

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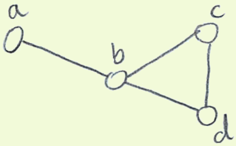

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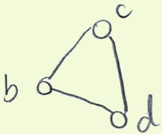

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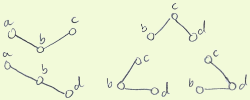

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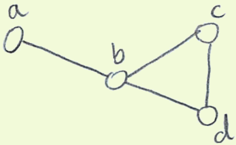

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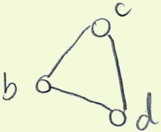

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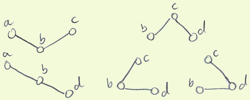

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Clustering:

Sneaky counting for undirected, unweighted networks:

- If the path i–j–ℓ exists then $a_{ij}a_{j\ell} = 1$.
- Otherwise, $a_{ij}a_{j\ell} = 0$.
- We want $i \neq \ell$ for good triples.
- In general, a path of n edges between nodes i_1 and i_n travelling through nodes $i_2, i_3, \ldots i_{n-1}$ exists $\iff$ $a_{i_1 i_2} a_{i_2 i_3} a_{i_3 i_4} \cdots a_{i_{n-2} i_{n-1}} a_{i_{n-1} i_n} = 1$.
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Properties

- For sparse networks, C_1 tends to discount highly connected nodes.
- C_2 is a useful and often preferred variant
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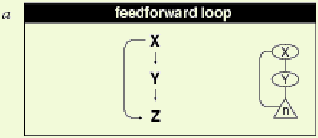

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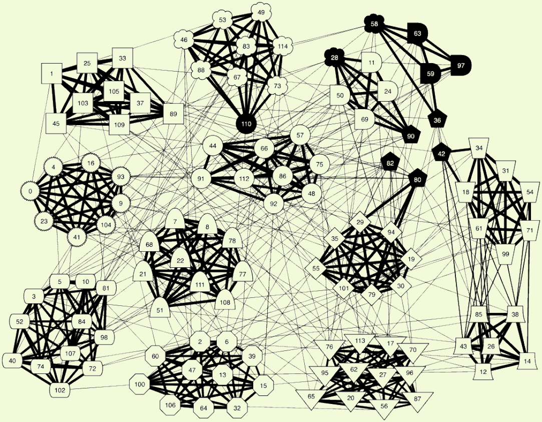
Properties

6. modularity and structure/community detection:



Clauset *et al.*, 2006 [6]: NCAA football

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7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
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  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
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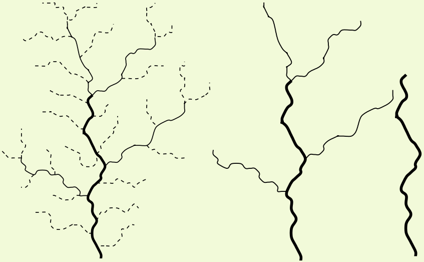

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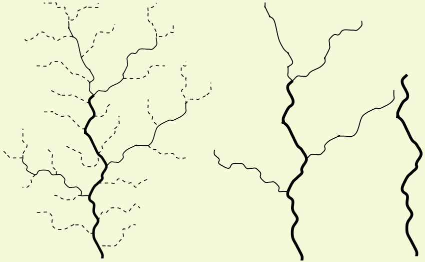

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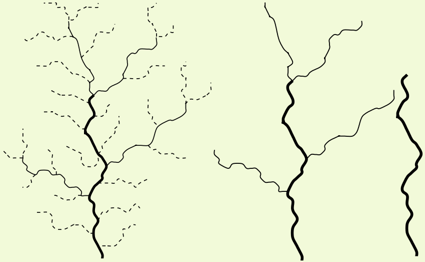

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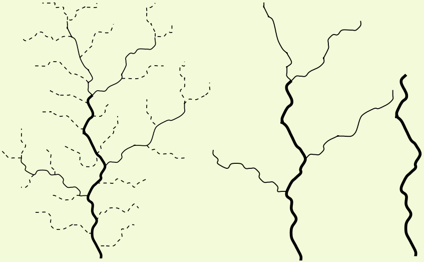
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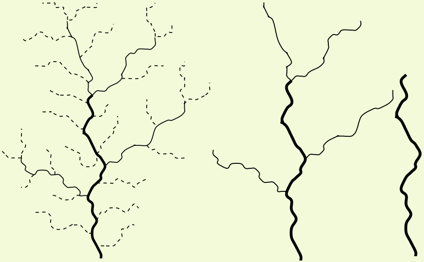

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9. network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
- Good algorithms exist for calculation.
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- closeness $d_{\mathrm{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:
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10. centrality:

- Many such measures of a node's 'importance.'
- ex 1: Degree centrality: k_i.
- ex 2: Node i's betweenness
  = fraction of shortest paths that pass through i.
- ex 3: Edge ℓ's betweenness
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Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
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