Overview of Complex Networks

Principles of Complex Systems
CSYS/MATH 300, Spring, 2013 | #SpringPoCS2013

Prof. Peter Dodds
@peterdodds

Department of Mathematics & Statistics | Center for Complex Systems |
Vermont Advanced Computing Center | University of Vermont

Outline

net•work |ˈnetˌwərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

• a complex system of roads, railroads, or other transportation routes *: a network of railroads.*

2 a group or system of interconnected people or things *: a trade network.*

• a group of people who exchange information, contacts, and experience for professional or social purposes *: a support network.*

• a group of broadcasting stations that connect for the simultaneous broadcast of a program *: the introduction of a second TV network* | [as adj.] *network television.*

• a number of interconnected computers, machines, or operations *: specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*

• a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network *: the stock exchanges have proven to be resourceful in networking these deals.*

• link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*

• [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career *: the skills of networking, bargaining, and negotiation.*

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

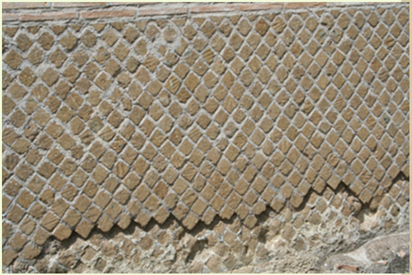
2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation: (⊞)

- Opus reticulatum:
- A Latin origin?



[http://serialconsign.com/2007/11/we-put-net-network]

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

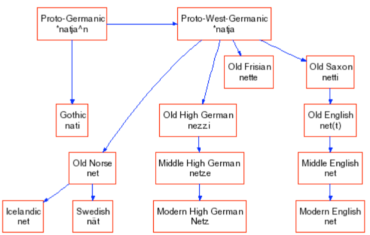
From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

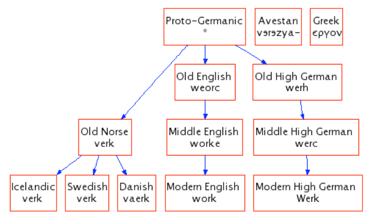
Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



The network of Germanic `net' words

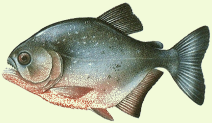


The network of `work' words

- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

Key Observation:

- Many complex systems
  can be viewed as complex networks
  of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
- ... largely due to your typical theoretical physicist:



- *Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

Popularity (according to Google Scholar)

"Collective dynamics of 'small-world' networks" [18]

- Watts and Strogatz
  Nature, 1998
- Cited $\approx$ 18,450 times (as of March 18, 2013)

"Emergence of scaling in random networks" [2]

- Barabási and Albert
  Science, 1999
- Cited $\approx$ 16,050 times (as of March 18, 2013)

Popularity (according to Google Scholar)

Review articles:

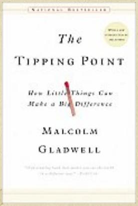
- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [3]
  Times cited: 3,500 (as of March 18, 2013)
- M. Newman
  "The structure and function of complex networks" [13]
  Times cited: 9,100 (as of March 18, 2013)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 11,600 (as of March 18, 2013)

Popularity according to textbooks:

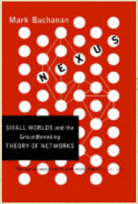
Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction" (⊞)
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" (⊞)

Popularity according to books:

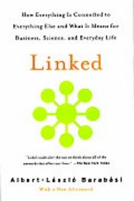


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]

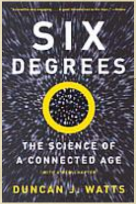


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [17]

Numerous others . . .

- Complex Social Networks—F. Vega-Redondo [16]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [14]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [7]

More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish mechanistic explanations.

** If this is upsetting, maybe string theory is for you...*

The UNIVERSITY of VERMONT

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)
- "The Unreasonable Effectiveness of Data," Halevy et al. [9].
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [19]

But:

- For scientists, description is only part of the battle.
- We still need to understand.

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

Super Basic definitions

Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- k_i = 0,1,2,. . . .
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)
- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

Super Basic definitions

Adjacency matrix:

- We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- (n.b., for numerical work, we always use sparse matrices.)

Examples

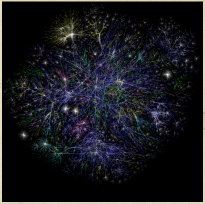
So what passes for a complex network?

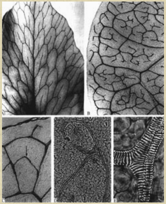
- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids





- Distribution (branching) versus redistribution (cyclical)

Examples

Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media

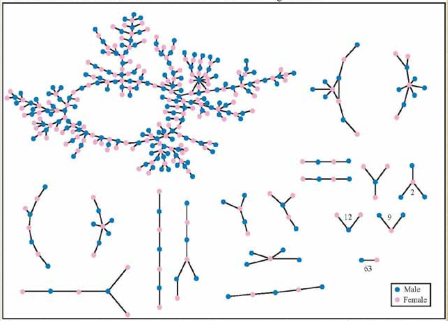
datamining.typepad.com (⊞)

Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook (⊞) twitter (⊞),

The Structure of Romantic and Sexual Relations at "Jefferson High School"



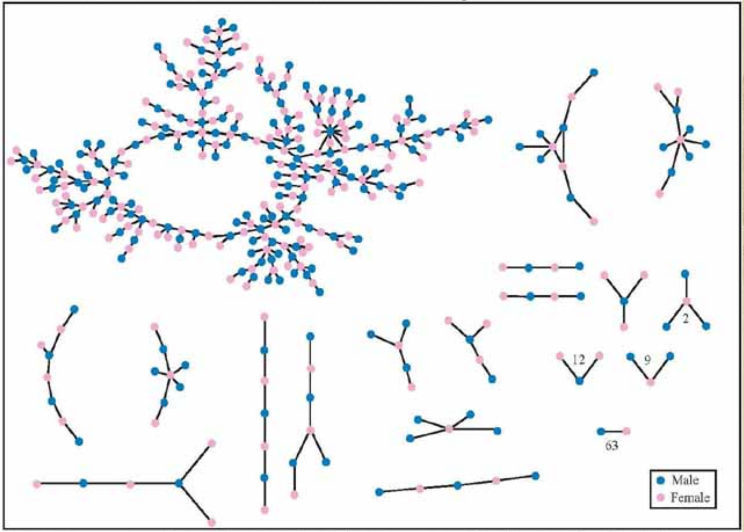
Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

- 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



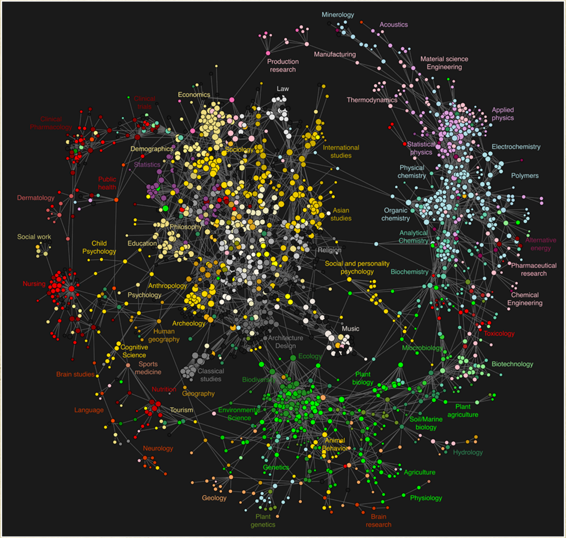
Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

Examples

Relational networks

- Consumer purchases
  (Wal-Mart: $\approx$ 1 petabyte $= 10^{15}$ bytes)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: bit.ly (⊞) flickr (⊞)

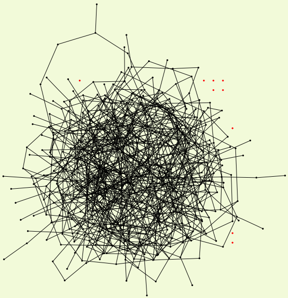
Clickworthy Science:



Bollen et al. [4]; a higher resolution figure is here (⊞)

A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.



$\Leftarrow$ Typical hairball

- number of nodes $N = 500$
- number of edges $m = 1000$
- average degree $\langle k \rangle = 4$

- And even when renderings somehow look good:
  "That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way"
  said Ponder [Stibbons] —*Making Money*, T. Pratchett.
- We need to extract digestible, meaningful aspects.

Properties

Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

- Plus coevolution of network structure and processes on networks.

* Degree distribution is the elephant in the room that we are now all very aware of...

Properties

1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k.
- k = node degree = number of connections.
- ex 1: Erdős-Rényi random networks have Poisson degree distributions:
  Insert question from assignment 5 (⊞)

$$P_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'.
- link cost controls skew.
- hubs may facilitate or impede contagion.

Properties

Note:

- Erdős-Rényi random networks are a *mathematical construct*.
- 'Scale-free' networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.

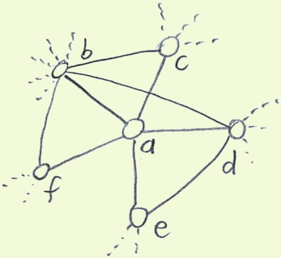
Properties

2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[12] similar degree nodes connecting to each other.
  Often social: company directors, coauthors, actors.
- Disassortative network: high degree nodes connecting to low degree nodes.
  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

Local socialness:

4. Clustering:



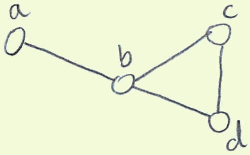
- Your friends tend to know each other.
- Two measures (explained on following slides):
  1. Watts & Strogatz [18]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

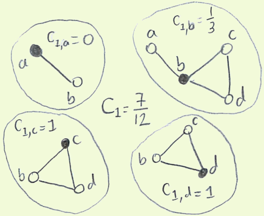
  2. Newman [13]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

Example network:



Calculation of C_1:



- C_1 is the average fraction of pairs of neighbors who are connected.
- Fraction of pairs of neighbors who are connected is

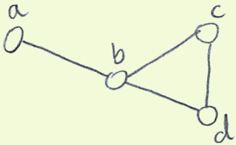
$$\frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

  where k_i is node i's degree, and $\mathcal{N}_i$ is the set of i's neighbors.
- Averaging over all nodes, we have:

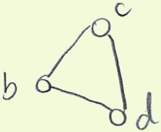
$$C_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i$$

Triples and triangles

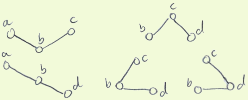
Example network:



Triangles:



Triples:



- Nodes i_1, i_2, and i_3 form a triple around i_1 if i_1 is connected to i_2 and i_3.
- Nodes i_1, i_2, and i_3 form a triangle if each pair of nodes is connected
- The definition $C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$ measures the fraction of closed triples
- The '3' appears because for each triangle, we have 3 closed triples.
- Social Network Analysis (SNA): fraction of transitive triples.

Clustering:

Sneaky counting for undirected, unweighted networks:

- If the path i–j–ℓ exists then $a_{ij}a_{j\ell} = 1$.
- Otherwise, $a_{ij}a_{j\ell} = 0$.
- We want $i \neq \ell$ for good triples.
- In general, a path of n edges between nodes i_1 and i_n travelling through nodes $i_2, i_3, \ldots i_{n-1}$ exists $\iff$ $a_{i_1 i_2} a_{i_2 i_3} a_{i_3 i_4} \cdots a_{i_{n-2} i_{n-1}} a_{i_{n-1} i_n} = 1$.
- $$\#\text{triples} = \frac{1}{2}\left(\sum_{i=1}^{N}\sum_{\ell=1}^{N}\left[A^2\right]_{i\ell} - \mathrm{Tr}A^2\right)$$
- $$\#\text{triangles} = \frac{1}{6}\mathrm{Tr}A^3$$

Properties

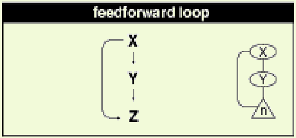
- For sparse networks, C_1 tends to discount highly connected nodes.
- C_2 is a useful and often preferred variant
- In general, $C_1 \neq C_2$.
- C_1 is a global average of a local ratio.
- C_2 is a ratio of two global quantities.

Properties

5. motifs:

- small, recurring functional subnetworks
- e.g., Feed Forward Loop:

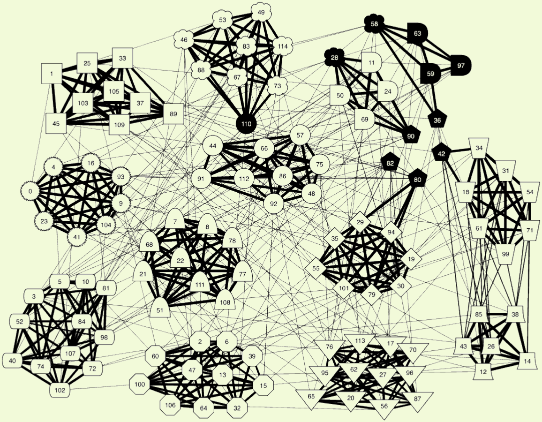
a



Shen-Orr, Uri Alon, *et al.* [15]

Properties

6. modularity and structure/community detection:



Clauset *et al.*, 2006 [6]: NCAA football

Properties

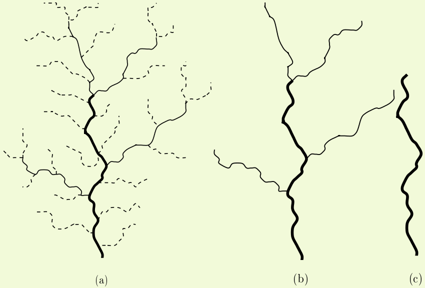
7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [11]

Properties

8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
  - Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_\omega \rangle$



Properties

9. network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
- Good algorithms exist for calculation.
- Weighted links can be accommodated.

Properties

9. network distances:
   - network diameter $d_{\max}$:
     Maximum shortest path length between any two nodes.
   - closeness $d_{\mathrm{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:
     Average ‘distance’ between any two nodes.
   - Closeness handles disconnected networks ($d_{ij} = \infty$)
   - $d_{\mathrm{cl}} = \infty$ only when all nodes are isolated.
   - Closeness perhaps compresses too much into one number

Properties

10. centrality:

- Many such measures of a node's 'importance.'
- ex 1: Degree centrality: k_i.
- ex 2: Node i's betweenness
  = fraction of shortest paths that pass through i.
- ex 3: Edge ℓ's betweenness
  = fraction of shortest paths that travel along ℓ.
- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg [10])

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

References I

[1] R. Albert and A.-L. Barabási.
Statistical mechanics of complex networks.
Rev. Mod. Phys., 74:47–97, 2002. pdf (⊞)

[2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. pdf (⊞)

[3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang.
Complex networks: Structure and dynamics.
Physics Reports, 424:175–308, 2006. pdf (⊞)

References II

[4] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila.
Clickstream data yields high-resolution maps of science.
PLoS ONE, 4:e4803, 2009. pdf (⊞)

[5] S. Bornholdt and H. G. Schuster, editors.
Handbook of Graphs and Networks.
Wiley-VCH, Berlin, 2003.

[6] A. Clauset, C. Moore, and M. E. J. Newman.
Structural inference of hierarchies in networks, 2006.
pdf (⊞)

[7] S. N. Dorogovtsev and J. F. F. Mendes.
Evolution of Networks.
Oxford University Press, Oxford, UK, 2003.

References III

[8] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.

[9] A. Halevy, P. Norvig, and F. Pereira.
The unreasonable effectiveness of data.
IEEE Intelligent Systems, 24:8–12, 2009. pdf (⊞)

[10] J. M. Kleinberg.
Authoritative sources in a hyperlinked environment.
Proc. 9th ACM-SIAM Symposium on Discrete Algorithms, 1998. pdf (⊞)

[11] M. Kretzschmar and M. Morris.
Measures of concurrency in networks and the spread of infectious disease.
Math. Biosci., 133:165–95, 1996. pdf (⊞)

References IV

[12] M. Newman.
Assortative mixing in networks.
Phys. Rev. Lett., 89:208701, 2002. pdf (⊞)

[13] M. E. J. Newman.
The structure and function of complex networks.
SIAM Review, 45(2):167–256, 2003. pdf (⊞)

[14] I. Rodríguez-Iturbe and A. Rinaldo.
Fractal River Basins: Chance and Self-Organization.
Cambridge University Press, Cambrigde, UK, 1997.

[15] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon.
Network motifs in the transcriptional regulation network of *Escherichia coli*.
Nature Genetics, 31:64–68, 2002. pdf (⊞)

References V

[16] F. Vega-Redondo.
Complex Social Networks.
Cambridge University Press, 2007.

[17] D. J. Watts.
Six Degrees.
Norton, New York, 2003.

[18] D. J. Watts and S. J. Strogatz.
Collective dynamics of ‘small-world’ networks.
Nature, 393:440–442, 1998. pdf (⊞)

[19] E. Wigner.
The unreasonable effectivenss of mathematics in the natural sciences.
Communications on Pure and Applied Mathematics, 13:1–14, 1960. pdf (⊞)