Review

Matrixology (Linear Algebra)—Lecture 7/25
MATH 124, Fall, 2011

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Outline

Review for Exam 1

Basics:

Sections covered on first midterm:

- Chapter 1 and Chapter 2 (Sections 2.1–2.7)
- Chapter 2 is our focus
- Knowledge of Chapter 1 as needed for Chapter 2 = solving $A\vec{x} = \vec{b}$.
- Want 'understanding' and 'doing' abilities.

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Stuff to know:

Row, Column, & Matrix Pictures of Linear Systems ($A\vec{x} = \vec{b}$)

- What dimensions of A mean:
  - m = number of equations
  - n = number of unknowns (x_1, x_2, ...)
- How to draw the row and column pictures.
- Be able to identify row picture
  (e.g., as representing 2 planes in 3-d).
- How to convert between the three pictures.

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Solving $A\vec{x} = \vec{b}$ by elimination

Solve four equivalent ways:

1. Simultaneous equations (snore)
2. Row operations on augmented matrix
   - Systematically transform $A\vec{x} = \vec{b}$ into $U\vec{x} = \vec{c}$
   - Solve by back subsitution
3. Row operations with E_{ij} and P_{ij} matrices
4. Factor A as $A = LU$
   - Solve two triangular systems by forward and back substitution
   - First $L\vec{c} = \vec{b}$ then $U\vec{x} = \vec{c}$.
   - More generally, $PA = LU$.

Understand number of solutions business:

- 0, 1, or ∞: why, when, ...

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Stuff to know:

More on $A = LU$:

- Be able to find the pivots of A (they live in U)
- Understand how elimination matrices (E_{ij}'s) are constructed from multipliers (l_{ij}'s)
- Understand how L is made up of inverses of elimination matrices
  - e.g.: $L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}A$.
- Understand how L is made up of the l_{ij} multipliers.
- Understand how inverses of elimination matrices are simply related to elimination matrices.

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Stuff to know:

Matrix algebra

- Understand basic matrix algebra
- Understand matrix multiplication
- Understand multiplication order matters
- Understand $AB = BA$ is rarely true

Inverses

- Understand identity matrix I
- Understand $A\,A^{-1} = A^{-1}A = I$
- Find A^{-1} with Gauss-Jordan elimination
- Perform row reduction on augmented matrix $[\,A\,|\,I\,]$.
- Understand that that finding A^{-1} solves $A\vec{x} = \vec{b}$ but is often prohibitively expensive to do.
- $(AB)^{-1} = B^{-1}A^{-1}$

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Stuff to know:

Transposes

- Definition: flip entries across main diagonal
- $A = A^{\mathrm{T}}$: A is symmetric
- Important property: $(AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}}$

Extra pieces:

- If $A\vec{x} = \vec{0}$ has a non-zero solution, A has no inverse
- If $A\vec{x} = \vec{0}$ has a non-zero solution, then $A\vec{x} = \vec{b}$ always has infinitely many solutions.
- $(A^{-1})^{\mathrm{T}} = (A^{\mathrm{T}})^{-1}$

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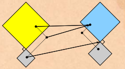

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