The Small-World Phenomenon

Complex Networks
CSYS/MATH 303, Spring, 2011

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University of Vermont

The Small-World Phenomenon

History

An online experiment

Previous theoretical work

An improved model

References

Outline

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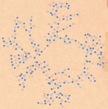
Some problems for people thinking about people?:

How are social networks structured?

- How do we define connections?
- How do we measure connections?
- (remote sensing, self-reporting)

What about the dynamics of social networks?

- How do social networks evolve?
- How do social movements begin?
- How does collective problem solving work?
- How is information transmitted through social networks?



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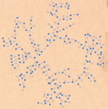


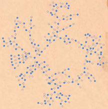
Social Search

A small slice of the pie:

- **Q.** Can people pass messages between distant individuals using only their existing social connections?
- **A.** Apparently yes...

Handles:

- The Small World Phenomenon
- or “Six Degrees of Separation.”



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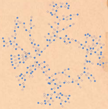
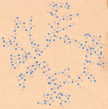


The problem

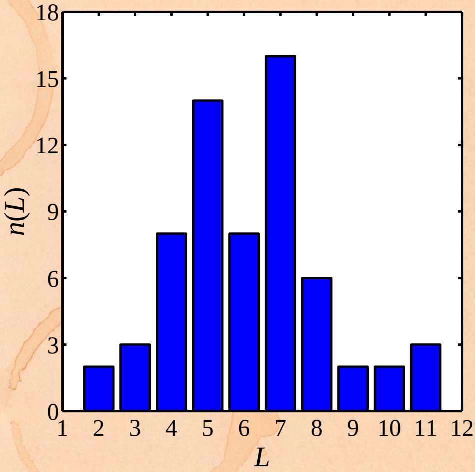
Stanley Milgram et al., late 1960's:

- Target person worked in Boston as a stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- average chain length $\simeq 6.5$.

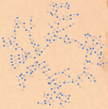


The problem

Lengths of successful chains:



From Travers and Milgram (1969) in Sociometry: [4]
"An Experimental Study of the Small World Problem."



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Two features characterize a social ‘Small World’:

1. Short paths exist

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2. People are good at finding them.

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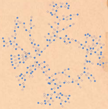

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Social Search

Milgram's small world experiment with e-mail [2]

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Social search—the Columbia experiment

- 60,000+ participants in 166 countries
- 18 targets in 13 countries including
  - a professor at an Ivy League university,
  - an archival inspector in Estonia,
  - a technology consultant in India,
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- 24,000+ chains

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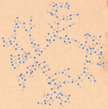

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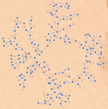
- Milgram's participation rate was roughly 75%
- Email version: Approximately 37% participation rate.
- Probability of a chain of length 10 getting through:

$$.37^{10} \simeq 5 \times 10^{-5}$$

- $\Rightarrow$ 384 completed chains (1.6% of all chains).



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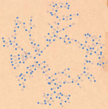

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- If target *seems* reachable
  ⇒ participation more likely.
- Small changes in attrition rates
  ⇒ large changes in completion rates
- e.g., ↘ 15% in attrition rate
  ⇒ ↗ 800% in completion rate

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Successful chains disproportionately used

- weak ties (Granovetter)
- professional ties (34% vs. 13%)
- ties originating at work/college
- target's work (65% vs. 40%)

. . . and disproportionately avoided

- hubs (8% vs. 1%) (+ no evidence of funnels)
- family/friendship ties (60% vs. 83%)

Geography → Work

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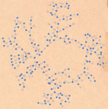

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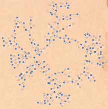

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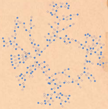

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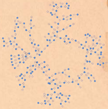

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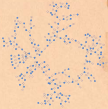

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Senders of successful messages showed little absolute dependency on

- age, gender
- country of residence
- income
- religion
- relationship to recipient

Range of completion rates for subpopulations:

30% to 40%

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Nevertheless, some weak discrepencies do exist...

An above average connector:

Norwegian, secular male, aged 30-39, earning over $100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

A below average connector:

Italian, Islamic or Christian female earning less than $2K, with elementary school education and retired, who uses strong ties to family members.

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Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain."

Why:

- Specificity important
- Successful links used relevant information. (e.g. connecting to someone who shares same profession as target.)

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Social search—the Columbia experiment

Basic results:

- $\langle L \rangle = 4.05$ for all completed chains
- $L_* =$ Estimated 'true' median chain length (zero attrition)
- Intra-country chains: $L_* = 5$
- Inter-country chains: $L_* = 7$
- All chains: $L_* = 7$
- Milgram: $L_* \simeq 9$

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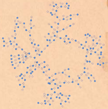

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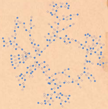

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- Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

N = population size,

d_{AB} = distance between nodes A and B.

- But: social networks aren't random...

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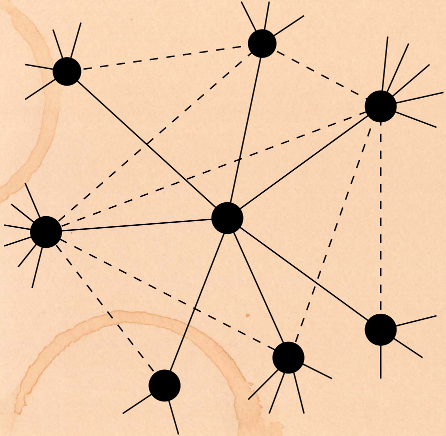

Need "clustering" (your friends are likely to know each other):

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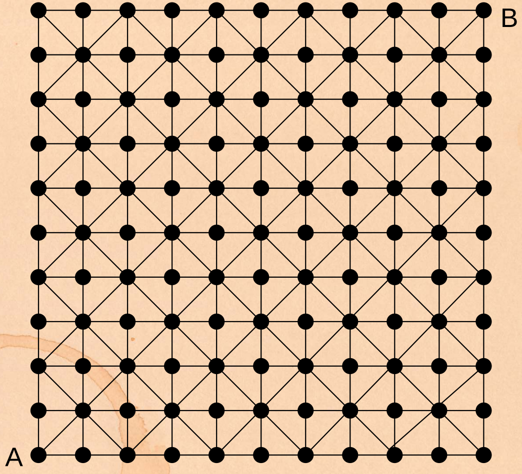
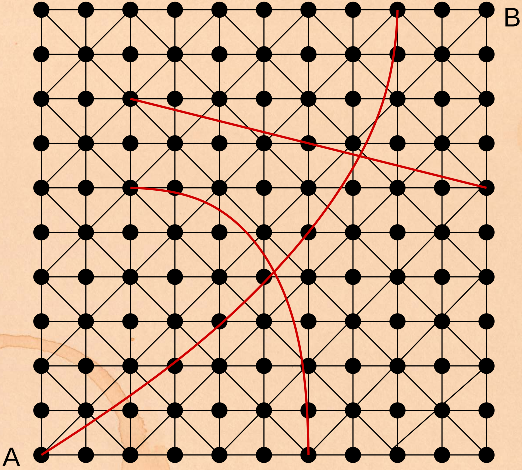
Non-randomness gives clustering



$d_{AB} = 10 \rightarrow$ too many long paths.

Randomness + regularity



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall

Small-world networks

Introduced by
Watts and Strogatz (Nature, 1998) [6]
“Collective dynamics of ‘small-world’ networks.”

Small-world networks were found everywhere:

- neural network of C. elegans,
- semantic networks of languages,
- actor collaboration graph,
- food webs,
- social networks of comic book characters,...

Very weak requirements:

- local regularity + random short cuts

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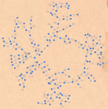

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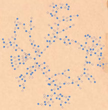

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Small-world networks

Introduced by
Watts and Strogatz (Nature, 1998) [6]
"Collective dynamics of 'small-world' networks."

Small-world networks were found everywhere:

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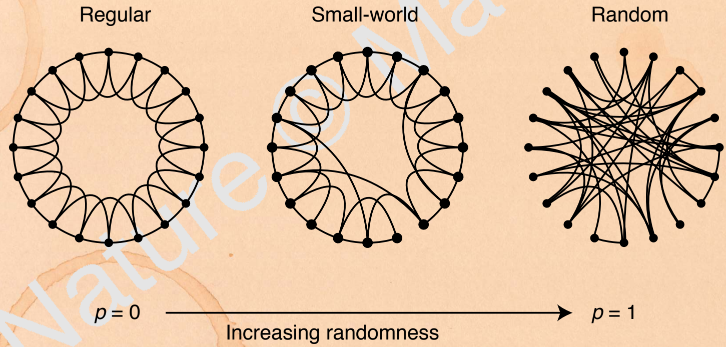
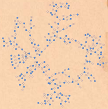
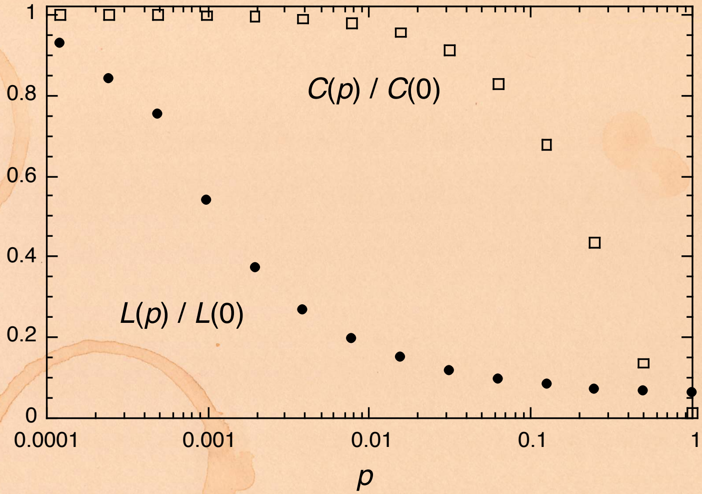
Toy model





The structural small-world property



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Previous work—finding short paths

But are these short cuts findable?

No.

Nodes cannot find each other quickly
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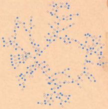

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- What can a local search method reasonably use?
- How to find things without a map?
- Need some measure of distance between friends and the target.

Some possible knowledge:

- Target's identity
- Friends' popularity
- Friends' identities
- Where message has been

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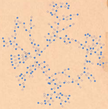

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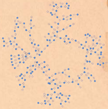

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Kleinberg's Network:

1. Start with regular d-dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q.
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto d_{ij}^{\;-\alpha}.$$

- $\alpha = 0$: random connections.
- α large: reinforce local connections.
- $\alpha = d$: same number of connections at all scales.

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Search time grows slowly with system size (like $\log^2 N$).

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Theoretical optimal search:

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Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.

Previous work—finding short paths

- If networks have hubs can also search well: Adamic et al. (2001) [1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

- Basic idea: get to hubs first (airline networks).
- But: hubs in social networks are limited.

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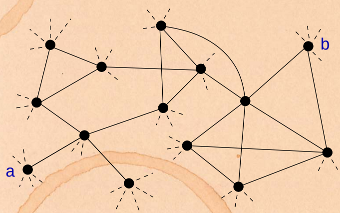
The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of a is closest to the target b?

What does 'closest' mean?

What is 'social distance'?

The model

One approach: incorporate identity.
(See “Identity and Search in Social Networks.” Science, 2002, Watts, Dodds, and Newman [5])

Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Religious beliefs
- Recreational activities.

Groups are formed by people with at least one similar attribute.

Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks.

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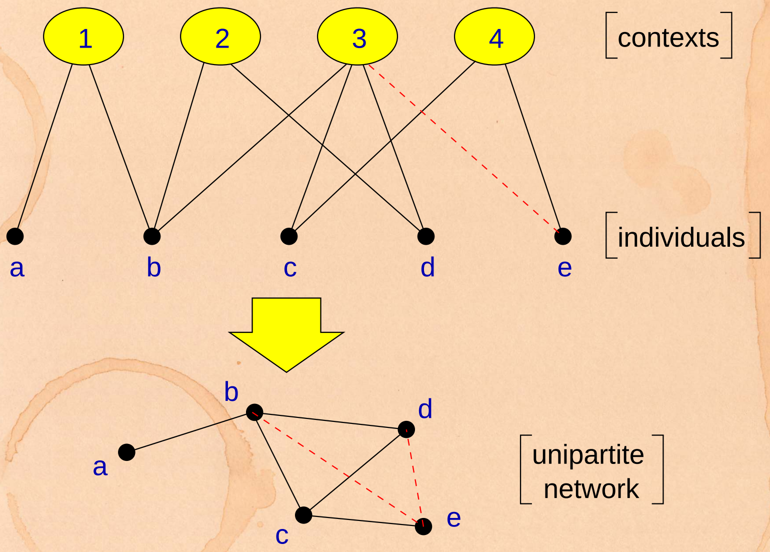
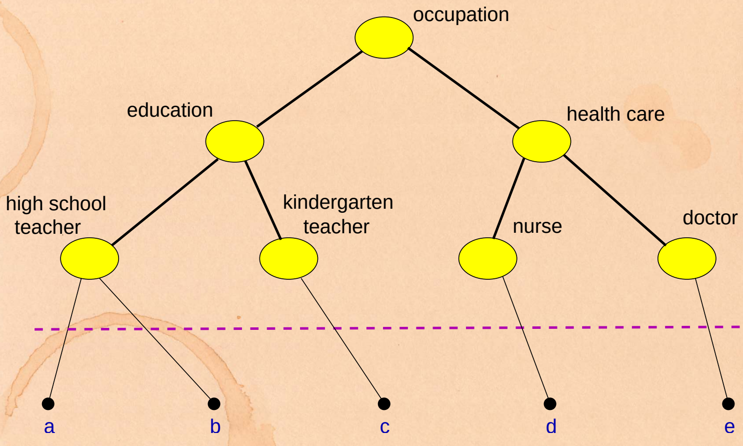
Social distance—Bipartite affiliation networks



Social distance—Context distance



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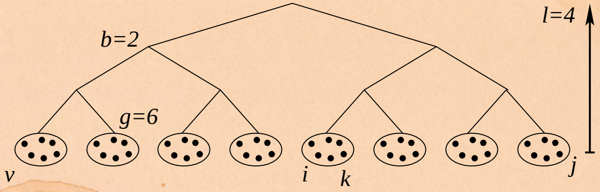
The model

Distance between two individuals x_{ij} is the height of lowest common ancestor.



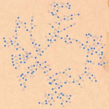
$x_{ij} = 3$, $x_{ik} = 1$, $x_{iv} = 4$.

The model

- Individuals are more likely to know each other the closer they are within a hierarchy.
- Construct z connections for each node using

$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$

- $\alpha = 0$: random connections.
- α large: local connections.



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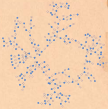
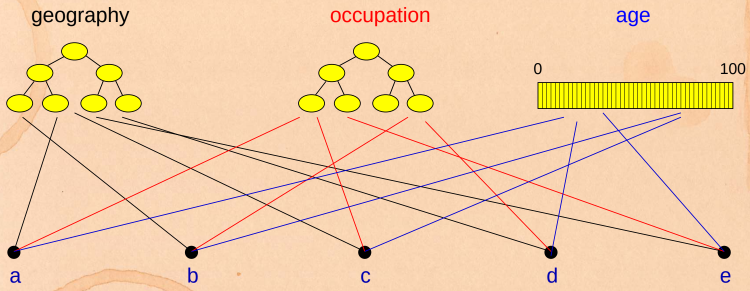
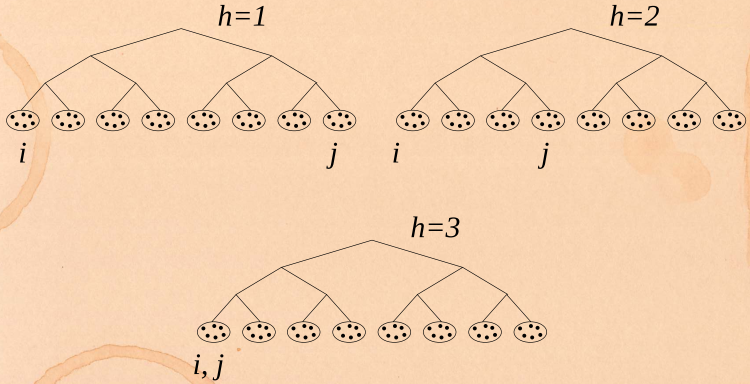


Social distance—Generalized context space



(Blau & Schwartz, Simmel, Breiger)

The model



$\vec{v}_i = [1\ 1\ 1]^T$, $\vec{v}_j = [8\ 4\ 1]^T$

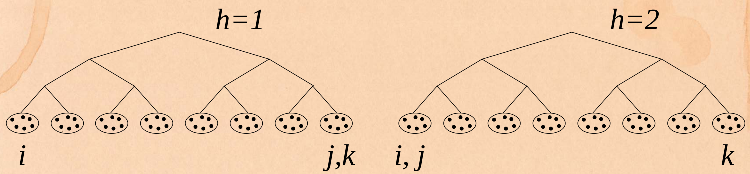
$x_{ij}^1 = 4,\ \ x_{ij}^2 = 3,\ \ x_{ij}^3 = 1.$

Social distance:

$$y_{ij} = \min_h x_{ij}^h.$$

The model

Triangle inequality doesn't hold:



$$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$$

The model

- Individuals know the identity vectors of
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  2. their friends,
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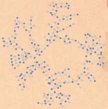
- Individuals can estimate the social distance between their friends and the target.
- Use a greedy algorithm + allow searches to fail randomly.



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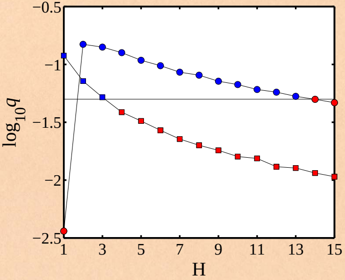
The model-results—searchable networks

$\alpha = 0$ versus $\alpha = 2$ for $N \simeq 10^5$:



$q \geq r$

$q < r$

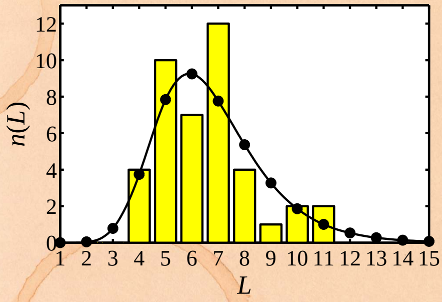
$r = 0.05$

q = probability an arbitrary message chain reaches a target.

- A few dimensions help.
- Searchability decreases as population increases.
- Precise form of hierarchy largely doesn't matter.

The model-results

Milgram's Nebraska-Boston data:



Model parameters:

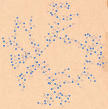
- $N = 10^8$,
- $z = 300$, $g = 100$,
- $b = 10$,
- $\alpha = 1$, $H = 2$;

- $\langle L_{\mathrm{model}} \rangle \simeq 6.7$
- $L_{\mathrm{data}} \simeq 6.5$

Social search—Data

Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Probability of connection as function of real distance $\propto 1/r$.



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Social Search—Real world uses

- Tags create identities for objects
- Website tagging: `http://www.del.icio.us`
- (e.g., Wikipedia)
- Photo tagging: `http://www.flickr.com`
- Dynamic creation of metadata plus links between information objects.
- Folksonomy: collaborative creation of metadata

Social Search—Real world uses

Recommender systems:

- Amazon uses people's actions to build effective connections between books.
- Conflict between 'expert judgments' and tagging of the hoi polloi.

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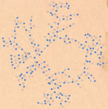
Conclusions

- Bare networks are typically unsearchable.
- Paths are findable if nodes understand how network is formed.
- Importance of identity (interaction contexts).
- Improved social network models.
- Construction of peer-to-peer networks.
- Construction of searchable information databases.



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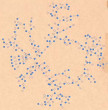

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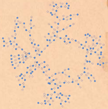

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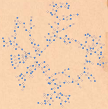

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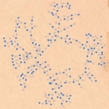


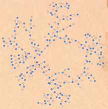
References I

[1] [1] L. Adamic, R. Lukose, A. Puniyani, and B. Huberman.
Search in power-law networks.
Phys. Rev. E, 64:046135, 2001. pdf (⊞)

[2] [2] P. S. Dodds, R. Muhamad, and D. J. Watts.
An experimental study of search in global social networks.
Science, 301:827–829, 2003. pdf (⊞)

[3] [3] J. Kleinberg.
Navigation in a small world.
Nature, 406:845, 2000. pdf (⊞)

[4] [4] J. Travers and S. Milgram.
An experimental study of the small world problem.
Sociometry, 32:425–443, 1969. pdf (⊞)



References II

[5] [5] D. J. Watts, P. S. Dodds, and M. E. J. Newman.
Identity and search in social networks.
Science, 296:1302–1305, 2002. pdf (⊞)

[6] [6] D. J. Watts and S. J. Strogatz.
Collective dynamics of 'small-world' networks.
Nature, 393:440–442, 1998. pdf (⊞)