Applications of Random Networks

Complex Networks CSYS/MATH 303, Spring, 2011

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- Problem: How much of a real network's structure is non-random?
- Key elephant in the room. The degree distribution F
 First observe departure of P_k from a Poisson distribution.
- Next: measure the departure of a real network with a degree frequency N_k from a random network with the same degree frequency.
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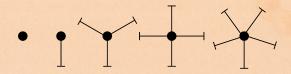
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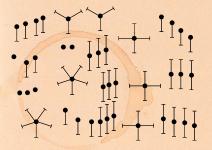


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Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.
- Initially allow self- and repeat connections.

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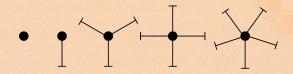
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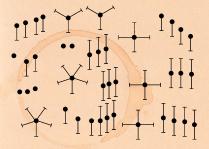


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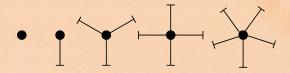
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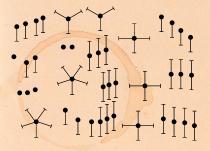


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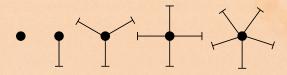
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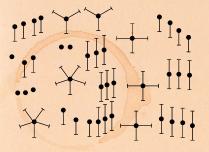
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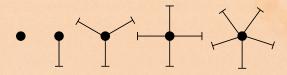
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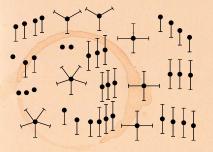
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Building random networks: First rewiring

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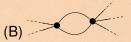
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Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





Being careful: we can't change the degree of any noterise we can't simply move links around.
 Simplest solution: randomly rewire two edges at a time.

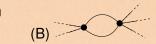


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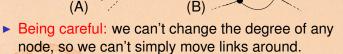


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Building random networks: First rewiring

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Randomly choose two edges. (Or choose problem edge and a random edge)

Rewire one end of each edge

- Node degrees do not change.
- Works if e₁ is a self-loop or repeated edge.
- Same as finding on/off/on/off 4-cycles. and rotating them.

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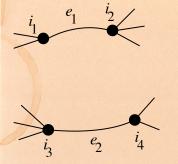
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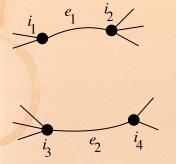
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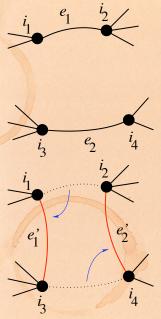
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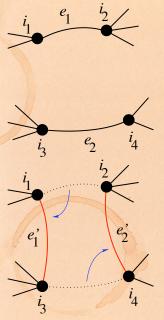
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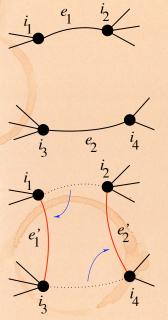
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 Use rewiring algorithm to remove all self and repeat loops.

Phase 3

 Randomize network wiring by applying rewiring algorithm liberally.

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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

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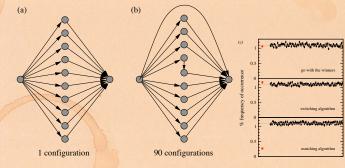


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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

Example from Milo et al. (2003)^[1]:



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What if we have P_k instead of N_k?

construction algorithm.

 Generate N nodes by sampling from degree distribution P.

Easy to do exactly numerically since k is discrete.
 Note: not all Pk will always give nodes that can be



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Network motifs

 Idea of motifs^[2] introduced by Shen-Orr, Alon et al. in 2002.

 Looked at gene expression within full context of transcriptional regulation networks.
 Specific example of Excharichia coli

 Directed network with 577 interactions (edges) and 424 operons (nodes).

Used network randomization to produce ensemble of alternate networks with same degree frequency N_k . Looked for certain subnetworks (motifs) that appeared more or less often than expected

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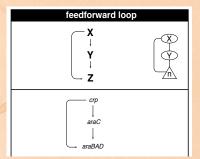


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a input input X € 0.5 10 12 14 16 18 20 6 8 € 0.5 AND 12 14 16 18 20 output Z E 0.5 output 2 6 14 16 18 20 4 10 12 time

Z only turns on in response to sustained activity in X.

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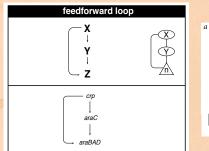
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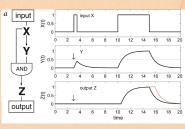
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Turning off X rapidly turns off Z.



Z only turns on in response to sustained activity in X.

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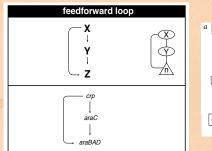
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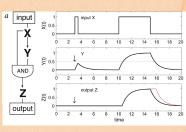
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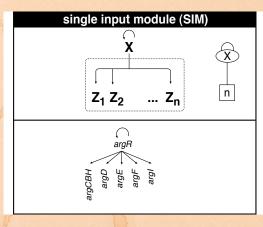
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Motifs

- Z only turns on in response to sustained activity in X.
- Furning off X rapidly turns off Z.
- Analogy to elevator doors.





Master switch.

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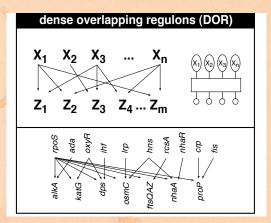
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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.



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Motifs

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- For more, see work carried out by Wiggins et al. at Columbia.



References I

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References

 [1] R. Milo, N. Kashtan, S. Itzkovitz, M. E. J. Newman, and U. Alon.
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[2] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon. Network motifs in the transcriptional regulation network of *Escherichia coli*. <u>Nature Genetics</u>, pages 64–68, 2002. pdf (⊞)



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