Complex Networks CSYS/MATH 303, Spring, 2011

## Prof. Peter Dodds

Department of Mathematics & Statistics Center for Complex Systems Vermont Advanced Computing Center University of Vermont













Licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





200 1 of 65

# Outline

## **Basics**

Definitions How to build Some visual examples

## Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

## References

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





# Outline

## Basics Definitions

How to build Some visual examples

## Structure

Clustering Degree distributions Configuration model Barmon kiends are strange Largest component Simple, physically-motivated analysis

## References

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 3 of 65

## Pure, abstract random networks:

Consider set of all networks with *N* labelled noc and *m* edges.
Standard random network = one randomly chosen network from this set.
To be clear: each network is equally probable.
Sometimes equiprobability is a good assumptio it is always an assumption.
Known as Erdős-Rényi random networks or ER

### **Random Networks**

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Pure, abstract random networks:

 Consider set of all networks with N labelled nodes and m edges.

Standard random network =
one randomly chosen network from this set.
To be clear: each network is equally probable.
Sometimes equiprobability is a good assumption.
Known as Erdős-Rényi random networks or ER graphs.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- Sometimes equiprobability is a good assumption.
   it is always an assumption.
   Known as Ercos Renyi random networks or ER graphs

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- To be clear: each network is equally probable.

Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.

graphs

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





# Random network generator for N = 3:



- Get your own exciting generator here  $(\boxplus)$ .
- ► As N /, our polyhedral die rapidly becomes a ball...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





990 5 of 65

Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

Limit of m = 0, empty graph.
 Limit of m = (<sup>N</sup>/<sub>2</sub>): complete or fully-connected graph
 Number of possible networks with N labelled nodes

Given on ecces, mere are (v<sub>m</sub>) unterent possible networks.
 Orazy factorial explosion for 1 < m < (<sup>N</sup><sub>2</sub>)
 Real world: links are usually costly so real netwo are almost always sparse.

### Random Networks

#### Basics

Definitions How to build Some visual examples

> tructure Clustering

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 C 6 of 65

Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

Limit of m = 0: empty graph.

Given *m* edges, there are  $(\sqrt{n})$  different possible networks. Crazy factorial explosion for  $1 \ll m \ll {n \choose 2}$ Real world: links are usually costly so real networ

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 C 6 of 65

Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

- Limit of m = 0: empty graph.
- Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph.

Given *m* edges, there are (<sup>(n)</sup>/<sub>m</sub>) different possible networks.
 Crazy factorial explosion for 1 < m < (<sup>N</sup>/<sub>2</sub>).
 Real world: links are usually costly so teat networks are linkest always sparse.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 C 6 of 65

Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

- Limit of m = 0: empty graph.
- Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

Given *m* edges, there are (<sup>v</sup><sub>m</sub>) different possible networks.
 Crazy factorial explosion for 1 < *m* < (<sup>N</sup><sub>2</sub>)
 Real world: links are usually costly so real netwo are almost always sparse.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

- Limit of m = 0: empty graph.
- Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

Given *m* edges, there are  $\binom{\binom{N}{2}}{m}$  different possible networks.

Real world: links are usually costly so real netw

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

- Limit of m = 0: empty graph.
- Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

Given *m* edges, there are  $\binom{\binom{N}{2}}{m}$  different possible networks.

Crazy factorial explosion for 1 

 *m (<sup>N</sup><sub>2</sub>)* 

 Real world: 
 *m*

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

- Limit of m = 0: empty graph.
- Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

- Given *m* edges, there are  $\binom{\binom{N}{2}}{m}$  different possible networks.
- Crazy factorial explosion for  $1 \ll m \ll \binom{N}{2}$ .
- Real world: links are usually costly so real networks are almost always sparse.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





# Outline

Basics Definitions How to build Some visual examples

## Structure

Clustering Degree distributions Configuration model Barmon kiends are strange Largest component Simple, physically-motivated analysis

## References

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, ohveirally-motivated





## How to build standard random networks:

Given N and m.

(we'll see a third later on)

- probability n
  - Useful for theoretical work.
- Take N nodes and add exactly m links by selecting edges without replacement.
  - Algorithm: Randomly choose a pair of nodes i and j, i ≠ j, and connect if unconnected; repeat until all m edges are allocated.
  - Best for adding relatively small numbers of links (most cases).
  - 1 and 2 are effectively equivalent for large N

### Random Networks

#### Basics

Definition

How to build Some visual examples

### Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods well see a third later only

- Useful for theoretical work.
- Algorithm: Randomly choose a pair of nodes *i* and *j i* ≠ *j*, and connect if unconnected; repeat until all *m* edges are allocated.
- Best for adding relatively small numbers of links (most cases).
- 1 and 2 are effectively equivalent for large N

### Random Networks

#### Basics

Definition

How to build Some visual examples

### Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)

Useful for theoretical work.

- Algorithm: Randomly choose a pair of nodes *i* and *j i* ≠ *j*, and connect if unconnected; repeat until all *m* edges are allocated.
- Best for adding relatively small numbers of links (most cases).
- 1 and 2 are effectively equivalent for large N

### Random Networks

#### Basics

Definition

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
  - Useful for theoretical work.
  - lake /v hodes and add exactly m links by selecting edges without replacement
    - Algorithm: Randomly choose a pair of nodes i and j, i \neq j, and connect if unconnected; repeat until all m edges are allocated.
    - Best for adding relatively small numbers of links (most cases).
    - 1 and 2 are effectively equivalent for large N

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
- 2. Take *N* nodes and add exactly *m* links by selecting edges without replacement.

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
  - Useful for theoretical work.
- Take N nodes and add exactly m links by selecting edges without replacement.

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
  - Useful for theoretical work.
- 2. Take *N* nodes and add exactly *m* links by selecting edges without replacement.
  - ► Algorithm: Randomly choose a pair of nodes *i* and *j*, *i* ≠ *j*, and connect if unconnected; repeat until all *m* edges are allocated.

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
  - Useful for theoretical work.
- 2. Take *N* nodes and add exactly *m* links by selecting edges without replacement.
  - ► Algorithm: Randomly choose a pair of nodes *i* and *j*, *i* ≠ *j*, and connect if unconnected; repeat until all *m* edges are allocated.
  - Best for adding relatively small numbers of links (most cases).

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
  - Useful for theoretical work.
- 2. Take *N* nodes and add exactly *m* links by selecting edges without replacement.
  - ► Algorithm: Randomly choose a pair of nodes *i* and *j*, *i* ≠ *j*, and connect if unconnected; repeat until all *m* edges are allocated.
  - Best for adding relatively small numbers of links (most cases).
  - 1 and 2 are effectively equivalent for large N.

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2}$$

d or average degree is

$$=\frac{2}{N}\rho_{2}^{1}N(N-1)=\frac{2}{N}\rho_{2}^{1}N(N-1)=\rho(N-1)$$

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





9 9 0 9 of 65

## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

or average degree is

$$= \frac{2}{N} \rho \frac{1}{2} N(N-1) = \frac{2}{N} \rho \frac{1}{2} N(N-1) = \rho(N-1)$$

### Random Networks

#### Basics

Definition

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

### Random Networks

#### Basics

Definition

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)$$

### Random Networks

#### Basics

Definition

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





200 9 of 65

## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{2}{N}p\frac{1}{2}N(N-1)$$

### Random Networks

#### Basics

Definition

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{2}{N}p\frac{1}{2}N(N-1)=p(N-1).$$

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{2}{N}p\frac{1}{2}N(N-1)=p(N-1).$$

Which is what it should be...

### Random Networks

#### Basics

Definitions

How to build Some visual examples

Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{2}{N}p\frac{1}{2}N(N-1)=p(N-1).$$

- Which is what it should be...
- If we keep  $\langle k \rangle$  constant then  $p \propto 1/N \to 0$  as  $N \to \infty$ .

### Random Networks

#### Basics

Definition

How to build Some visual examples

tructure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





# Outline

## Basics Definitions How to build Some visual examples

## tructure

Clustering Degree distributions Configuration model Bandom kiends are strange Largest component Simple, physically-motivated analysis

## References

### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





200 10 of 65
Next slides: Example realizations of random networks

Vary *m*, the number of edges from 100 to 1000.
 Average degree (*k*) runs from 0.4 to 4.
 Lexit—a, full network plus the largest component.

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





na (~ 11 of 65

Next slides: Example realizations of random networks

- ► *N* = 500
- Nary *m*, the number of edges from 100 to 1000
  Average degree (k) runs from 0.4 to 4.
  - Leok at full network plus the largest componen

#### **Random Networks**

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





200 11 of 65

### Next slides: Example realizations of random networks

- ► *N* = 500
- Vary m, the number of edges from 100 to 1000.

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





### Next slides:

Example realizations of random networks

- ► *N* = 500
- Vary m, the number of edges from 100 to 1000.
- Average degree (k) runs from 0.4 to 4.

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





### Next slides:

Example realizations of random networks

- ► *N* = 500
- Vary m, the number of edges from 100 to 1000.
- Average degree (k) runs from 0.4 to 4.
- Look at full network plus the largest component.

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





### entire network:

### largest component:



N = 500, number of edges m = 100average degree  $\langle k \rangle = 0.4$ 

#### Random Networks

#### **Basics**

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





entire network:

largest component:

N = 500, number of edges m = 200average degree  $\langle k \rangle = 0.8$ 

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





entire network:

largest component:

Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References



N = 500, number of edges m = 230average degree  $\langle k \rangle = 0.92$ 



### entire network:



### largest component:



N = 500, number of edges m = 240average degree  $\langle k \rangle = 0.96$ 

#### Random Networks

#### Basics

Definitions

How to build

#### Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





entire network:

### largest component:



Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References

N = 500, number of edges m = 250average degree  $\langle k \rangle = 1$ 



N,m

entire network:



N = 500, number of edges m = 260average degree  $\langle k \rangle = 1.04$ 

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





entire network:



largest component:



N = 500, number of edges m = 280average degree  $\langle k \rangle = 1.12$ 

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





### entire network:

### largest component:



N = 500, number of edges m = 300average degree  $\langle k \rangle = 1.2$ 

### Random Networks

#### **Basics**

Definitions

How to build

#### Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





entire network:



largest component:



N = 500, number of edges m = 500average degree  $\langle k \rangle = 2$ 

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 of 65

entire network:

largest component:

N = 500, number of edges m = 1000average degree  $\langle k \rangle = 4$ 

#### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





### Random networks: examples for N=500











m = 100 $\langle k \rangle = 0.4$  m = 200 $\langle k \rangle = 0.8$ 



 $\begin{array}{l}m=240\\\langle k\rangle=0.96\end{array}$ 

 $\begin{array}{l}m=250\\\langle k\rangle=1\end{array}$ 







大

m = 260 $\langle k \rangle = 1.04$ 





m = 300 $\langle k \rangle = 1.2$ 

m = 500  $\langle k \rangle = 2$ 

 $\begin{array}{l} m = 1000 \\ \langle k \rangle = 4 \end{array}$ 

### Random Networks

#### Basics

Definitions

How to build

Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





### Random networks: largest components





m = 100 $\langle k \rangle = 0.4$ 

m = 200 $\langle k \rangle = 0.8$ 



m = 230 $\langle k \rangle = 0.92$ 





m = 250 $\langle k \rangle = 1$ 



Definitions

How to build

Some visual examples

Degree distributions Configuration model Random friends are strange Simple. physically-motivated





m = 260 $\langle k \rangle = 1.04$ 

m = 280

 $\langle k \rangle = 1.12$ 

m = 300 $\langle k \rangle = 1.2$ 



m = 500

 $\langle k \rangle = 2$ 

m = 1000 $\langle k \rangle = 4$ 





23 of 65

### Random networks: examples for N=500





m = 250 $\langle k \rangle = 1$ 

m = 250

 $\langle k \rangle = 1$ 



m = 250

 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 





m = 250 $\langle k \rangle = 1$ 





m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 

### **Bandom Networks**

Definitions

How to build

Some visual examples

Degree distributions Configuration model Random friends are strange Simple. physically-motivated





### Random networks: largest components





m = 250 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 

#### **Bandom Networks**

Definitions

How to build

Some visual examples

Degree distributions Random friends are strange physically-motivated

References





25 of 65





m = 250 $\langle k \rangle = 1$ 

# Outline

### Basics

Definitions How to build Some visual examples

### Structure Clustering

Degree distributions Configuration model Barateon kiends are strange Cargest component Simple, physically-motivated analysis

### References

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





26 of 65

# For method 1, what is the clustering coefficient for a finite network?

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





27 of 65

- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient:<sup>[1]</sup>

 $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ 

#### Random Networks

#### **Basics**

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient:<sup>[1]</sup>

 $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ 

Recall: C<sub>2</sub> = probability that two friends of a node are also friends.



#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient:<sup>[1]</sup>

 $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ 

- Recall: C<sub>2</sub> = probability that two friends of a node are also friends.
- Or: C<sub>2</sub> = probability that a triple is part of a triangle.

#### Random Networks

#### **Basics**

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient:<sup>[1]</sup>

 $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ 



- Recall: C<sub>2</sub> = probability that two friends of a node are also friends.
- Or: C<sub>2</sub> = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





References

Expected number of triples in entire network:

$$\frac{1}{2}N(N-1)(N-2)p^2$$

(Double counting dealt with by  $\frac{1}{2}$ .)

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Expected number of triples in entire network:

$$\frac{1}{2}N(N-1)(N-2)p^{2}$$

(Double counting dealt with by <sup>1</sup>/<sub>2</sub>.)
 Expected number of triangles in entire network:

$$\frac{1}{6}N(N-1)(N-2)p^{3}$$

(Over-counting dealt with by  $\frac{1}{6}$ .)

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Expected number of triples in entire network:

$$\frac{1}{2}N(N-1)(N-2)p^{2}$$

(Double counting dealt with by <sup>1</sup>/<sub>2</sub>.)
 Expected number of triangles in entire network:

$$\frac{1}{6}N(N-1)(N-2)p^{3}$$

(Over-counting dealt with by  $\frac{1}{6}$ .)

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}} = \frac{3 \times \frac{1}{6}N(N-1)(N-2)p^3}{\frac{1}{2}N(N-1)(N-2)p^2} = p$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





DQ @ 28 of 65

### Or: take any three nodes, call them a, b, and c.

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple. physically-motivated analysis

References





29 of 65

- Or: take any three nodes, call them a, b, and c.
  Triple a-b-c centered at b occurs with probability
  - $p^2 \times (1-p) + p^2 \times p = p^2$ .

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physical/-motivated analysis





- Or: take any three nodes, call them a, b, and c.
- Triple *a-b-c* centered at *b* occurs with probability  $p^2 \times (1-p) + p^2 \times p = p^2$ .

Triangle occurs with probability p<sup>3</sup>.

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





- Or: take any three nodes, call them a, b, and c.
- Triple *a-b-c* centered at *b* occurs with probability  $p^2 \times (1-p) + p^2 \times p = p^2$ .
- Triangle occurs with probability p<sup>3</sup>.
- Therefore,

$$C_2=\frac{p^3}{p^2}=p.$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





So for large random networks (N→∞), clustering drops to zero.

Key structural feature of random networks is that the locally look like pure branching networks

No small loops.

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 0 30 of 65



- So for large random networks (N → ∞), clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strangee Largest component Simple, physically-motivated analysis

References





20 0 30 of 65



- So for large random networks (N→∞), clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks
- No small loops.

#### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physical/-motivated analysis





# Outline

### Basics

Definitions How to build Some visual examples

### Structure

### Degree distributions

Configuration model Banagent kiends are strange Largest contronent Simple, physically-motivated analysis

### References

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, ohvsically-motivated

References





DQ @ 31 of 65
## Degree distribution:

- Recall P<sub>k</sub> = probability that a randomly selected node has degree k.
- networks: each possible link is realized with probability p.
  Now consider one node: there are 'N 1 choose ways the node can be connected to k of the othe N 1 nodes.
  - Each connection occurs with probability p, each non-connection with probability (1 - p) Therefore have a binomial distribution:



### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated

References





2 C 32 of 65

## Degree distribution:

- Recall P<sub>k</sub> = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.

Exercise section occurs with probability  $\rho$ , each non-connection with probability  $(1 - \rho)$ .

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, ohysically-motivated

References





2 C 32 of 65

## Degree distribution:

- Recall P<sub>k</sub> = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N 1 choose k' ways the node can be connected to k of the other N 1 nodes.

Federation occurs with probability p, each non-connection with probability (1 - p). Therefore have a binomial distribution: (N = 1) to a solution

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Degree distribution:

- Recall P<sub>k</sub> = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N 1 choose k' ways the node can be connected to k of the other N 1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1 p).

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Degree distribution:

- Recall P<sub>k</sub> = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N 1 choose k' ways the node can be connected to k of the other N 1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1 p).
- Therefore have a binomial distribution:

$$P(k; \boldsymbol{p}, \boldsymbol{N}) = \binom{N-1}{k} \boldsymbol{p}^{k} (1-\boldsymbol{p})^{N-1-k}.$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strang Largest component Simple, physically-motivated analysis





## Limiting form of P(k; p, N):

P(k; p, N) = (<sup>\*</sup><sub>k</sub>) p<sup>n</sup> (1 − p)<sup>n</sup>
What happens as N → ∞?
We must end up with the normal distribution right
If p is fixed, then we would end up with a Gaussia with average degree (k) ≃ pN → ∞.
But we want to keep (k) fixed...
So examine limit of P(k; p, N) when p → 0 and N → ∞. with (k) = p(N − 1) = constant.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 33 of 65

## Limiting form of P(k; p, N):

• Our degree distribution:  $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}.$ 

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





2 C 33 of 65

## Limiting form of P(k; p, N):

- Our degree distribution:  $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}.$
- What happens as  $N \to \infty$ ?

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 C 33 of 65

## Limiting form of P(k; p, N):

- Our degree distribution:  $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}.$
- What happens as  $N \to \infty$ ?
- We must end up with the normal distribution right?

But we want to keep (k) fixed...
 So examine limit of P(k, p, N) when p → 0 and N → ∞ with (k) = p(N − 1) = constant

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Pegree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Limiting form of P(k; p, N):

- Our degree distribution:  $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}.$
- What happens as  $N \to \infty$ ?
- We must end up with the normal distribution right?
- If p is fixed, then we would end up with a Gaussian with average degree ⟨k⟩ ≃ pN → ∞.

Nmit of P(k, p, N) when  $p \to 0$  and  $\langle k \rangle = p(N-1) = \text{constant.}$ 

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Pegree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





## Limiting form of P(k; p, N):

- Our degree distribution:  $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}.$
- What happens as  $N \to \infty$ ?
- We must end up with the normal distribution right?
- If p is fixed, then we would end up with a Gaussian with average degree (k) ≃ pN → ∞.
- But we want to keep  $\langle k \rangle$  fixed...
- So examine limit of P(k; p, N) when  $p \to 0$  and  $N \to \infty$  with  $\langle k \rangle = p(N-1) = \text{constant.}$

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated

References





2 0 0 34 of 65

Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated





Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)(N-2)\cdots(N-k)}{k!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$



Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

**Bandom Networks** 





Da @ 34 of 65

Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$= \frac{(N-1)(N-2)\cdots(N-k)}{k!} \frac{\langle k \rangle^k}{(N-1)^k} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$
$$= \frac{N^k (1 - \frac{1}{N})\cdots(1 - \frac{k}{N})}{k!N^k} \frac{\langle k \rangle^k}{(1 - \frac{1}{N})^k} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





20 C 34 of 65

Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$= \frac{(N-1)(N-2)\cdots(N-k)}{k!} \frac{\langle k \rangle^k}{(N-1)^k} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$
$$= \frac{\mathcal{M}^k (1 - \frac{1}{N})\cdots(1 - \frac{k}{N})}{k! \mathcal{M}^k} \frac{\langle k \rangle^k}{(1 - \frac{1}{N})^k} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model

Random friends are strange Largest component Simple, physically-motivated analysis

References





Da @ 34 of 65

Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$= \frac{(N-1)(N-2)\cdots(N-k)}{k!} \frac{\langle k \rangle^k}{(N-1)^k} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$
$$\simeq \frac{\mathcal{M}^k(1-\frac{1}{N})\cdots(1-\frac{k}{N})}{k!\mathcal{M}^k} \frac{\langle k \rangle^k}{(1-\frac{1}{N})^k} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





2 C 34 of 65

We are now here:

$$P(k; p, N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

(Use l'Hôpital's rule to prove.

Poisson distribution (⊞)

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





2 C 35 of 65

We are now here:

$$P(k; p, N) \simeq rac{\langle k 
angle^k}{k!} \left(1 - rac{\langle k 
angle}{N-1}
ight)^{N-1-k}$$

Now use the excellent result:

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=e^x.$$

**Random Networks** 

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





Poisson distribution (⊞)

We are now here:

$$P(k; p, N) \simeq rac{\langle k 
angle^k}{k!} \left(1 - rac{\langle k 
angle}{N-1}
ight)^{N-1-k}$$

Now use the excellent result:

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=e^x$$

(Use l'Hôpital's rule to prove.)

• Identifying n = N - 1 and  $x = -\langle k \rangle$ :

$$P(k;\langle k\rangle) \simeq \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle} \left(1 - \frac{\langle k\rangle}{N-1}\right)^{-k}$$

Poisson distribution (⊞)

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions Configuration model Random friends are strange Largest component Simple.

physically-motivated analysis





We are now here:

$$P(k; p, N) \simeq rac{\langle k 
angle^k}{k!} \left(1 - rac{\langle k 
angle}{N-1}
ight)^{N-1-k}$$

Now use the excellent result:

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=e^x$$

(Use l'Hôpital's rule to prove.)

• Identifying n = N - 1 and  $x = -\langle k \rangle$ :

$$P(k;\langle k\rangle) \simeq \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle} \left(1 - \frac{\langle k\rangle}{N-1}\right)^{-k} \to \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle}$$

▶ This is a Poisson distribution ( $\boxplus$ ) with mean  $\langle k \rangle$ .

### Random Networks

#### Basics

Definitions How to build

Structure

Clustering

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated





$$P(k;\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$



- ▶ λ > 0
- ▶ *k* = 0, 1, 2, 3, ...
- Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
- e.g.: phone calls/minute, horse-kick deaths.
- 'Law of small numbers'

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated





Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analycie





Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$=e^{-\langle k
angle}\sum_{k=0}^{\infty}rac{\langle k
angle^k}{k!}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analycie





Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated

References





DQ @ 37 of 65

Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!}$$

$$=e^{-\langle k\rangle}e^{\langle k\rangle}=1\checkmark$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k P(k; \langle k \rangle).$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions

Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k P(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k P(k; \langle k \rangle) = \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k \mathcal{P}(k;\langle k 
angle) = \sum_{k=0}^{\infty} k rac{\langle k 
angle^k}{k!} e^{-\langle k 
angle}$$

$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle) = \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!}$$

 $a = \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} = \langle k \rangle \checkmark$ 

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle) = \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{i=0}^{\infty} \frac{\langle k \rangle^{i}}{i!}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple.

physically-motivated analysis

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k P(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle) = \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{i=0}^{\infty} \frac{\langle k \rangle^{i}}{i!} = \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle) = \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

$$= \langle \boldsymbol{k} \rangle \boldsymbol{e}^{-\langle \boldsymbol{k} \rangle} \sum_{k=1}^{\infty} \frac{\langle \boldsymbol{k} \rangle^{k-1}}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{i=0}^{\infty} \frac{\langle k \rangle^i}{i!} = \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} = \langle k \rangle \checkmark$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple.

physically-motivated analysis

References





Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k \mathcal{P}(k; \langle k \rangle).$$

Checking:

$$\sum_{k=0}^{\infty} k P(k; \langle k \rangle) = \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{i=0}^{\infty} \frac{\langle k \rangle^{i}}{i!} = \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} = \langle k \rangle \checkmark$$

Note: We'll get to a better and crazier way of doing this...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strat Largest component Simple, hysically-motivated

References





DQ @ 38 of 65

The variance of degree distributions for random networks turns out to be very important.

### second moment

Variance is then

 $\langle k 
angle^2 = \langle k 
angle^2 + \langle k 
angle - \langle k 
angle^2 = \langle k 
angle$ 

So standard deviation # is equal to \sqrt{k}.
 Note: This is a special property of Poissor distribution and can trip us up...

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component Simple,

physically-motivated analysis

References





2 C 39 of 65
- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

 $\langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k 
angle.$ 

So standard deviation - is equal to (-(-k)).
 Note: This is a special property of Poisson distribution and can trip us up...

## Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis





- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Variance is then

 $\sigma^2 = \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle^2$ 

So standard deviation σ is equal to √(k).
 Note: This is a special property of Poissor distribution and can trip us up...

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis





- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Variance is then

 $\sigma^{2} = \langle k^{2} \rangle - \langle k \rangle^{2} = \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle^{2}$ 

So standard deviation *σ* is equal to √(*k*).
 Note: This is a special property of Poissor distribution and can trip us up...

## Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis





- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle \mathbf{k}^2 \rangle = \langle \mathbf{k} \rangle^2 + \langle \mathbf{k} \rangle.$$

Variance is then

 $\sigma^{2} = \langle k^{2} \rangle - \langle k \rangle^{2} = \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle^{2} = \langle k \rangle.$ 

So standard deviation *σ* is equal to √ (k).
 Note: This is a special property of Poissor distribution and can trip us up...

## Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis





- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle \mathbf{k}^2 \rangle = \langle \mathbf{k} \rangle^2 + \langle \mathbf{k} \rangle.$$

Variance is then

$$\sigma^{2} = \langle \mathbf{k}^{2} \rangle - \langle \mathbf{k} \rangle^{2} = \langle \mathbf{k} \rangle^{2} + \langle \mathbf{k} \rangle - \langle \mathbf{k} \rangle^{2} = \langle \mathbf{k} \rangle.$$

So standard deviation  $\sigma$  is equal to  $\sqrt{\langle k \rangle}$ .

## Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

Simple, physically-motivated analysis





- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle \mathbf{k}^2 \rangle = \langle \mathbf{k} \rangle^2 + \langle \mathbf{k} \rangle.$$

Variance is then

$$\sigma^{2} = \langle \mathbf{k}^{2} \rangle - \langle \mathbf{k} \rangle^{2} = \langle \mathbf{k} \rangle^{2} + \langle \mathbf{k} \rangle - \langle \mathbf{k} \rangle^{2} = \langle \mathbf{k} \rangle.$$

- So standard deviation  $\sigma$  is equal to  $\sqrt{\langle k \rangle}$ .
- Note: This is a special property of Poisson distribution and can trip us up...

### Random Networks

Basics

Definitions How to build Some visual examples

Structure

Degree distributions Configuration model Random friends are strange Largest component

simple, physically-motivated analysis





# Outline

## Basics

Definitions How to build Some visual examples

## Structure

Configuration model

Largest convolent Simple, physically-motivated analysis

# References

## Random Networks

### Basics

Definitions

Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple,

physically-motivated analysis

References





2 C 40 of 65

- So... standard random networks have a Poisson degree distribution
  - Also known as the configuration model.<sup>[1]</sup> Can generalize construction method from ER trandom networks.
  - Assign each node a weight w from some distribution
     P<sub>w</sub> and form links with probability
    - link between *i* and *j*)  $\propto$  *w*,*w*<sub>*j*</sub>
    - 1. Randomly wiring up (and rewiring) already exi
      - Examining mechanisms that lead to networks with certain degree distributions.

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution P<sub>k</sub>.

Can generalize construction method from ER random networks.

Assign each node a weight w from some distribution
 *P<sub>w</sub>* and form links with probability

link between *i* and *j*)  $\propto$  *w*,*w* 

 Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
 Examining mechanisms that lead to networks with exterior of the time of the time.

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





Da @ 41 of 65

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model.<sup>[1]</sup>

- Assign each node a weight w from some distribution
   P<sub>w</sub> and form links with probability
  - ink between i and  $j) \propto w_i w_j$
  - Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
     Examining mechanisms that lead to networks with certain degree distributions

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model.<sup>[1]</sup>
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution
   P<sub>w</sub> and form links with probability
  - $P(\text{link between } i \text{ and } j) \propto w_i w_j$ .

 Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
 Examining mechanisms that lead to networks with

certain degree distributions.

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





na (~ 41 of 65

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model.<sup>[1]</sup>
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P<sub>w</sub> and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_i$ .

1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

Examining mechanisms that lead to networks with certain degree distributions.

## Random Networks

### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model.<sup>[1]</sup>
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P<sub>w</sub> and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_i$ .

But we'll be more interested in

## Random Networks

Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





2 C 41 of 65

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model.<sup>[1]</sup>
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P<sub>w</sub> and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_i$ .

- But we'll be more interested in
  - 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

## Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model.<sup>[1]</sup>
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P<sub>w</sub> and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_i$ .

- But we'll be more interested in
  - 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
  - Examining mechanisms that lead to networks with certain degree distributions.

## Random Networks

### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





## Coming up:

Example realizations of random networks with power law degree distributions:

Set P<sub>0</sub> = 0 (no isolated nodes)
 Vary exponent < between 2.10 and 2.91.</li>
 As the last of full permark plue the last of the la

mponent

Apart from degree distribution, wiring is random

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





2 C 42 of 65

## Coming up:

Example realizations of random networks with power law degree distributions:

► *N* = 1000.

Set P<sub>0</sub> = 0 (no isolated nodes).
 Vary exponent < between 2.10 and 2.91.</li>
 Again, look at full network plus the largest component.
 Apart from degree distribution, wiring is ran

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple,

References





2 C 42 of 65

# Coming up:

Example realizations of random networks with power law degree distributions:

- ► *N* = 1000.
- $P_k \propto k^{-\gamma}$  for  $k \ge 1$ .

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple,

analysis





# Coming up:

Example realizations of random networks with power law degree distributions:

- ► *N* = 1000.
- $P_k \propto k^{-\gamma}$  for  $k \ge 1$ .
- Set  $P_0 = 0$  (no isolated nodes).

## Random Networks

### Basics

Definitions How to build Some visual examp

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple,

analysis





# Coming up:

Example realizations of random networks with power law degree distributions:

- ► *N* = 1000.
- $P_k \propto k^{-\gamma}$  for  $k \ge 1$ .
- Set  $P_0 = 0$  (no isolated nodes).
- Vary exponent  $\gamma$  between 2.10 and 2.91.

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple,

physically-motivated analysis





# Coming up:

Example realizations of random networks with power law degree distributions:

- ► *N* = 1000.
- $P_k \propto k^{-\gamma}$  for  $k \ge 1$ .
- Set  $P_0 = 0$  (no isolated nodes).
- Vary exponent  $\gamma$  between 2.10 and 2.91.
- Again, look at full network plus the largest component.

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple,

physically-motivated analysis





# Coming up:

Example realizations of random networks with power law degree distributions:

- ► *N* = 1000.
- $P_k \propto k^{-\gamma}$  for  $k \ge 1$ .
- Set  $P_0 = 0$  (no isolated nodes).
- Vary exponent γ between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

## Random Networks

### Basics

Definitions How to build Some visual examp

Structure

Clustering

Degree distributions

Configuration model Random friends are strange

Largest component Simple, physically-motivated





# Random networks: examples for N=1000











Simple. physically-motivated

 $\gamma = 2.1$ (k) = 3.448

 $\gamma = 2.55$ 

 $\langle k \rangle = 1.712$ 

 $\gamma = 2.19$  $\langle k \rangle = 2.986$ 

 $\gamma = 2.64$ 

 $\langle k \rangle = 1.6$ 

 $\gamma = 2.28$  $\langle k \rangle = 2.306$   $\gamma = 2.37$ (k) = 2.504

 $\gamma = 2.82$ 

 $\langle k \rangle = 1.386$ 

 $\gamma = 2.46$  $\langle k \rangle = 1.856$ 







 $\gamma = 2.73$ 

 $\langle k \rangle = 1.862$ 





 $\gamma = 2.91$ 

 $\langle k \rangle = 1.49$ 





Da @ 43 of 65

## **Bandom Networks**

Definitions Some visual examples

Degree distributions

Configuration model

# Random networks: largest components











 $\gamma = 2.1$ (k) = 3.448

 $\gamma = 2.19$  $\langle k \rangle = 2.986$ 



 $\gamma = 2.37$ (k) = 2.504  $\gamma = 2.46$  $\langle k \rangle = 1.856$ 





 $\langle k \rangle = 1.49$ 

## **Bandom Networks**

Definitions Some visual examples

Degree distributions

Configuration model

Simple.

physically-motivated





 $\gamma = 2.55$  $\langle k \rangle = 1.712$ 

 $\gamma = 2.64$  $\langle k \rangle = 1.6$ 



 $\gamma = 2.73$  $\langle k \rangle = 1.862$ 

 $\gamma = 2.82$  $\langle k \rangle = 1.386$ 

 $\gamma = 2.91$ 





# Outline

## Basics

Definitions How to build Some visual examples

## Structure

Clustering Degree distributions Configuration model

## Random friends are strange

Simple, physically-motivated analysis

## References

## Random Networks

#### Basics

Definitions How to build

Some visual examples

Structure

Clustering

Degree distributions

Configuration model

## Random friends are strange

Largest component

physically-motivated analysis

References





2 C 45 of 65

The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
Again: P<sub>k</sub> is the degree of randomly chosen node.
A second very important distribution arises from choosing randomly on edges rather than on nodes.
Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
Now choosing nodes based on their degree (i.e., and the second secon



### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions

Random friends are strange

Largest component Simple, physically-motivated analysis





The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks

A second very important distribution arises from choosing randomly on edges rather than on nodes.
Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
Now choosing nodes based on their degree (i.e., size).



### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Degree distributions

Configuration mode

Random friends are strange

Largest component Simple, physically-motivated analysis

References





20 46 of 65

- The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
- Again:  $P_k$  is the degree of randomly chosen node.

A second very important distribution arises from choosing randomly on edges rather than on nodes.
Define *Q<sub>k</sub>* to be the probability the node at a random end of a randomly chosen edge has degree *k*.
Now choosing nodes based on their degree (i.e., size).



### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

## Random friends are strange

Largest component Simple, physically-motivated analysis

References





20 46 of 65

- The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
- Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
   Now choosing nodes based on their degree (i.e.,



### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

## Random friends are strange

Largest component Simple, physically-motivated analysis





- The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
- Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration mode

### Random friends are strange

Largest component Simple, physically-motivated analysis





- The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
- Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):



## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple, physically-motivated





- The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
- Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):



Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}} - \frac{k!}{k!}$$

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distribution

Configuration model

### Random friends are strange

Largest component Simple, physically-motivated analysis





- The degree distribution P<sub>k</sub> is fundamental for our description of many complex networks
- Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):



Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

## Random friends are strange

Largest component Simple, physically-motivated analysis

References





20 0 46 of 65

For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.

 $R_k$  = probability that a friend of a random node hak to the friends.

 $rac{(k+1)P_{k+1}}{\langle k
angle}$ 

Equivalent for there having degree X = 1.
 Natural question: what's the expected number other friends that one friend has?

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

### Random friends are strange

Largest component Simple, physically-motivated analysis

References





n a c 47 of 65

- For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.
   Useful variant on Q<sub>k</sub>:
  - $R_k$  = probability that a friend of a random node has k other friends.

 Natural question, what's the expected number of other friends that one friend has?

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component Simple, physically-motivated analysis

References





2 C 47 of 65

For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q<sub>k</sub>:

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

 Natural question what's the expected number of other friends that one friend has?

## Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple, physically-motivated analysis

References





na (~ 47 of 65
For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q<sub>k</sub>:

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Natural question what's the expected number of other friends that one triend has?

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple, physically-motivated analysis





For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q<sub>k</sub>:

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.

Natural question

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple, physically-motivated analysis





For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q<sub>k</sub>:

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

- Equivalent to friend having degree k + 1.
- Natural question: what's the expected number of other friends that one friend has?

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering

Degree distributions

Configuration model

#### Random friends are strange

Largest component Simple, physically-motivated analysis





Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_R = \sum_{k=0}^{\infty} k R_k$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

### Random friends are strange

Largest component Simple,

analysis

References





n a (~ 48 of 65

Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_{R} = \sum_{k=0}^{\infty} kR_{k} = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

#### Random friends are strange

argest component

physically-motivated analysis

References





2 C 48 of 65

Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_{R} = \sum_{k=0}^{\infty} k R_{k} = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle}$$
$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1)P_{k+1}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration mode

#### Random friends are strange

argest component Simple,

physically-motivated analysis

References





n a (~ 48 of 65

Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_{R} = \sum_{k=0}^{\infty} k R_{k} = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

$$=rac{1}{\langle k
angle}\sum_{k=1}k(k+1)P_{k+1}$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}\left((k+1)^2-(k+1)\right)P_{k+1}$$

(where we have sneakily matched up indices)

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component

physically-motivated analysis

References





2 C 48 of 65

Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_R = \sum_{k=0}^{\infty} k R_k = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}k(k+1)P_{k+1}$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}\left((k+1)^2-(k+1)\right)P_{k+1}$$

(where we have sneakily matched up indices)

$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j \quad \text{(using j = k+1)}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributio

Configuration model

#### Random friends are strange

Largest component Simple

physically-motivated analysis

References





2 C 48 of 65

Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_{R} = \sum_{k=0}^{\infty} k R_{k} = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{k}k(k+1)P_{k+1}$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}\left((k+1)^2-(k+1)\right)P_{k+1}$$

(where we have sneakily matched up indices)

$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j \quad \text{(using j = k+1)}$$
$$= \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distribution

Configuration model

#### Random friends are strange

Largest component

physically-motivated analysis

References





DQ @ 48 of 65

Note: our result, ⟨k⟩<sub>R</sub> = 1/⟨k⟩ (⟨k²⟩ - ⟨k⟩), is true for all random networks, independent of degree distribution.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

#### Random friends are strange

Largest component Simple.

physically-motivated analysis

References





- Note: our result, ⟨k⟩<sub>R</sub> = 1/⟨k⟩ (⟨k²⟩ ⟨k⟩), is true for all random networks, independent of degree distribution.
- For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple

physically-motivated analysis

References





- ▶ Note: our result,  $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle \langle k \rangle)$ , is true for all random networks, independent of degree distribution.
- For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Therefore:

$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left( \langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right)$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering

Degree distributions

Configuration model

### Random friends are strange

Largest component

Simple, physically-motivated analysis

References





- ▶ Note: our result,  $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle \langle k \rangle)$ , is true for all random networks, independent of degree distribution.
- For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Therefore:

$$\langle k \rangle_{R} = \frac{1}{\langle k \rangle} \left( \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component

Simple, physically-motivated analysis

References





- Note: our result, ⟨k⟩<sub>R</sub> = 1/⟨k⟩ (⟨k²⟩ ⟨k⟩), is true for all random networks, independent of degree distribution.
- For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Therefore:

$$\langle k \rangle_{R} = \frac{1}{\langle k \rangle} \left( \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

 Again, neathess of results is a special property of the Poisson distribution.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple,

physically-motivated analysis





▶ Note: our result,  $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$ , is true for all random networks, independent of degree distribution.

For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Therefore:

$$\langle k \rangle_{R} = \frac{1}{\langle k \rangle} \left( \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

- Again, neathess of results is a special property of the Poisson distribution.
- So friends on average have  $\langle k \rangle$  other friends, and  $\langle k \rangle + 1$  total friends...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distribution

Configuration mode

### Random friends are strange

Largest component

physically-motivated analysis

References





Dac 49 of 65

Reason #1:

 $k_{R}=\langle k
angle rac{1}{\langle k
angle }\left(\langle k^{2}
angle -\langle k
angle 
ight) =\langle k^{2}
angle -\langle k
angle$ 

 Key: Average depends on the 1st and 2nd moments of and not just the 1st moment.

We might guess ⟨k<sub>2</sub>⟩ = ⟨k⟩(⟨k⟩ − 1) but it's actually ⟨k(k − 1)⟩.
 If P<sub>k</sub> has a large second moment, then ⟨k<sub>2</sub>⟩ will be big. (e.g., in the case of a power-law distribution)
 Your friends really are different from you...

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration mode

Random friends are strange

Simple, physically-motivated

References





2 C 50 of 65

## Reason #1:

Average # friends of friends per node is

$$\langle \mathbf{k}_2 \rangle = \langle \mathbf{k} \rangle \times \langle \mathbf{k} \rangle_R$$

Key: Average depends on the 1st and 2nd moments of and not just the 1st moment.

We might guess ⟨k₂⟩ = ⟨k⟩(⟨k⟩ − 1) but it's actually ⟨k(k − 1)⟩.
 If P<sub>k</sub> has a large second moment, then ⟨k₂⟩ will be big. (e.g., in the case of a power-law distribution)
 Your friends really are different from you...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Simple, physically-motivated analysis

References





20 0 50 of 65

## Reason #1:

Average # friends of friends per node is

$$\langle \mathbf{k}_2 \rangle = \langle \mathbf{k} \rangle \times \langle \mathbf{k} \rangle_{\mathbf{R}} = \langle \mathbf{k} \rangle \frac{1}{\langle \mathbf{k} \rangle} \left( \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle \right) \qquad (\mathbf{k}_2)$$

Key: Average depends on the 1st and 2nd moments of and not just the 1st moment.

We might guess ⟨k<sub>2</sub>⟩ = ⟨k⟩(⟨k⟩ − 1) but it's actually ⟨k(k − 1)⟩.
 If P<sub>k</sub> has a large second moment, then ⟨k<sub>2</sub>⟩ will be big. (e.g., in the case of a power-law distribution)
 Your friends really are different from you...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Simple,

References





na @ 50 of 65

## Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left( \langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

Key: Average depends on the 1st and 2nd moments of a and not just the 1st moment.

We might guess ⟨k<sub>2</sub>⟩ = ⟨k⟩(⟨k⟩ − 1) but it's actually ⟨k(k − 1)⟩.
 If P<sub>k</sub> has a large second moment, then ⟨k<sub>2</sub>⟩ will be big. (e.g., in the case of a power-law distribution)
 Your friends really are different from you...

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering

Degree distributions

Configuration model

### Random friends are strange

Largest component Simple, physically-motivated

References





20 0 50 of 65

## Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left( \langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

Key: Average depends on the 1st and 2nd moments of P<sub>k</sub> and not just the 1st moment.

We might guess ⟨k<sub>2</sub>⟩ = ⟨k⟩(⟨k⟩ − 1) but it's actually ⟨k(k − 1)⟩.
 If P<sub>k</sub> has a large second moment, then ⟨k<sub>2</sub>⟩ will be big. (e.g., in the case of a power-law distribution)
 Your friends really are different from you...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component Simple,

physically-motivated analysis

References





20 0 50 of 65

# Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left( \langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

- Key: Average depends on the 1st and 2nd moments of P<sub>k</sub> and not just the 1st moment.
- Three peculiarities:
  - 1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$  but it's actually  $\langle k(k-1) \rangle$ .

(e.g., in the case of a power-law distribution

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component Simple, physically-motivated





# Reason #1:

Average # friends of friends per node is

$$\langle \mathbf{k}_2 \rangle = \langle \mathbf{k} \rangle \times \langle \mathbf{k} \rangle_R = \langle \mathbf{k} \rangle \frac{1}{\langle \mathbf{k} \rangle} \left( \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle \right) = \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle.$$

## Three peculiarities:

- 1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$  but it's actually  $\langle k(k-1) \rangle$ .
- 2. If  $P_k$  has a large second moment, then  $\langle k_2 \rangle$  will be big.

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration mode

### Random friends are strange Largest component

Simple, physically-motivated analysis





# Reason #1:

Average # friends of friends per node is

$$\langle \mathbf{k}_2 \rangle = \langle \mathbf{k} \rangle \times \langle \mathbf{k} \rangle_R = \langle \mathbf{k} \rangle \frac{1}{\langle \mathbf{k} \rangle} \left( \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle \right) = \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle.$$

## Three peculiarities:

- 1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$  but it's actually  $\langle k(k-1) \rangle$ .
- If *P<sub>k</sub>* has a large second moment, then ⟨*k*<sub>2</sub>⟩ will be big. (e.g., in the case of a power-law distribution)

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component

physically-motivated analysis





# Reason #1:

Average # friends of friends per node is

$$\langle \mathbf{k}_2 \rangle = \langle \mathbf{k} \rangle \times \langle \mathbf{k} \rangle_R = \langle \mathbf{k} \rangle \frac{1}{\langle \mathbf{k} \rangle} \left( \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle \right) = \langle \mathbf{k}^2 \rangle - \langle \mathbf{k} \rangle.$$

## Three peculiarities:

- 1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$  but it's actually  $\langle k(k-1) \rangle$ .
- 2. If *P<sub>k</sub>* has a large second moment, then ⟨*k*<sub>2</sub>⟩ will be big.
  (e.g., in the case of a power-law distribution)
- 3. Your friends really are different from you...

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component Simple, physically-motivated





## More on peculiarity #3:

A node's average # of friends: (k)

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple.

physically-motivated analysis

References





# More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Simple, physically-motivated analysis

References





# More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

## Random Networks

Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Random friends are strange

Largest component Simple,

physically-motivated analysis

References





# More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle \mathbf{k}^2 \rangle}{\langle \mathbf{k} \rangle} = \langle \mathbf{k} \rangle \frac{\langle \mathbf{k}^2 \rangle}{\langle \mathbf{k} \rangle^2} = \langle \mathbf{k} \rangle \frac{\sigma^2 + \langle \mathbf{k} \rangle^2}{\langle \mathbf{k} \rangle^2}$$

(variance = b<sup>2</sup> = 0) can a node be the same as its hierds.
Intuition: for random networks, the more connecte node, the more likely it is to be chosen as a friend

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Random friends are strange

Largest component

simple, physically-motivated analysis

References





2 C 51 of 65

# More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle \mathbf{k}^2 \rangle}{\langle \mathbf{k} \rangle} = \langle \mathbf{k} \rangle \frac{\langle \mathbf{k}^2 \rangle}{\langle \mathbf{k} \rangle^2} = \langle \mathbf{k} \rangle \frac{\sigma^2 + \langle \mathbf{k} \rangle^2}{\langle \mathbf{k} \rangle^2} = \langle \mathbf{k} \rangle \left( 1 + \frac{\sigma^2}{\langle \mathbf{k} \rangle^2} \right)$$

(variance: v<sup>2</sup> = 0) can a node be the same as its friends.
 Intuition: for random networks, the more connected node, the more likely it is to be chosen as a friend.

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Random friends are strange

Largest component

physically-motivated analysis

References





990 51 of 65

# More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left( 1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \ge \langle k \rangle$$

## Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component

Simple, physically-motivated analysis

References

N.M.



## More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle \mathbf{k}^2 \rangle}{\langle \mathbf{k} \rangle} = \langle \mathbf{k} \rangle \frac{\langle \mathbf{k}^2 \rangle}{\langle \mathbf{k} \rangle^2} = \langle \mathbf{k} \rangle \frac{\sigma^2 + \langle \mathbf{k} \rangle^2}{\langle \mathbf{k} \rangle^2} = \langle \mathbf{k} \rangle \left( 1 + \frac{\sigma^2}{\langle \mathbf{k} \rangle^2} \right) \ge \langle \mathbf{k} \rangle$$

So only if everyone has the same degree (variance=  $\sigma^2 = 0$ ) can a node be the same as its friends.

### Random Networks

#### Basics

Definitions How to build Some visual examp

Structure

Clustering

Degree distributions

Random friends are strange

Simple, physically-motivated analysis

References





na ~ 51 of 65

## More on peculiarity #3:

- A node's average # of friends: (k)
- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left( 1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \ge \langle k \rangle$$

- So only if everyone has the same degree (variance=  $\sigma^2 = 0$ ) can a node be the same as its friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

## Random Networks

#### Basics

Definitions How to build Some visual examp

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange Largest component Simple,

References





# (Big) Reason #2:

- \$\langle k \rangle\_R\$ is key to understanding how well random networks are connected together.
- Defn: Component connected subnetwork of node such that L path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Giant component component that comprise a non-seto fraction of a network as N – ∞.
- Note: Component = Cluster

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Largest component Simple, physically-motivated

References





Da @ 52 of 65

# (Big) Reason #2:

- \$\langle k \rangle\_R\$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.

As  $N \rightarrow \infty$ , does our network have a giant component?

- Defn: Component = connected subnetwork of node such that d path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Glant component component that comprise a non-zero fraction of a network as N – ∞.
- Note: Component = Cluster

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Configuration model

## Random friends are strange

argest component

ohysically-motivated analysis

References





2 C 52 of 65

# (Big) Reason #2:

- \$\langle k \rangle\_R\$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As N → ∞, does our network have a giant component?
- Defn: Component = connected subnetwork of node such that I path between each pair of nodes in the subnetwork, and no node outside of the subnetwork in agenteeted to it.
- Defn: Giant component component that comprise a non-zero fraction of a network as N – ∞.
- Note: Component = Cluster

## Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model

## Random friends are strange

argest component

hysically-motivated nalysis

References





2 0 0 52 of 65

# (Big) Reason #2:

- \$\langle k \rangle\_R\$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As N → ∞, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

Simple,

ohysically-motivated analysis

References





2 C 52 of 65
# Two reasons why this matters

# (Big) Reason #2:

- \$\langle k \rangle\_R\$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As N → ∞, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- ▶ Defn: Giant component = component that comprises a non-zero fraction of a network as  $N \rightarrow \infty$ .

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure

Clustering

Degree distributions

Random friends are strange

argest component Simple,

physically-motivated analysis





# Two reasons why this matters

# (Big) Reason #2:

- \$\langle k \rangle\_R\$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As N → ∞, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- ▶ Defn: Giant component = component that comprises a non-zero fraction of a network as  $N \rightarrow \infty$ .
- Note: Component = Cluster

### Random Networks

#### Basics

Definitions How to build Some visual example

Structure

Clustering

Degree distributions

Configuration model

Random friends are strange

argest component Simple,





# Outline

### Basics

Definitions How to build Some visual examples

### Structure

Clustering Degree distributions Configuration model Barreconsciences are strange

### Largest component

Simple, physically-motivated analysis

References

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis







### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strang

Largest component Simple, physically-motivated analysis





# Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- All of this is the same as requiring (k)<sub>B</sub> > 1.

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated

References





n < ?> 55 of 65

# Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.

Giant component condition (or percolation condition)

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated

References





n a c 55 of 65

# Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring  $\langle k \rangle_R > 1$ .

Giant component condition (or perc

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange

### Largest component Simple,

analysis





# Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring  $\langle k \rangle_R > 1$ .
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = rac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated





### Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring  $\langle k \rangle_R > 1$ .
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = rac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

Again, see that the second moment is an essential part of the story.

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated





# Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring  $\langle k \rangle_R > 1$ .
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = rac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

- Again, see that the second moment is an essential part of the story.
- Equivalent statement:  $\langle k^2 \rangle > 2 \langle k \rangle$

### Random Networks

#### Basics

Definitions How to build Some visual examples

#### Structure

Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated





### Standard random networks:

• Recall  $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$ .

 $\frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$ 

 Therefore when (k) > 1, standard random networks have a giant component.
 When (k) > 1, all components are finite.

► Fine example of a continuous phase transition (⊞)

 $\mathbf{k} \in \mathbf{W}$  we say  $\langle k \rangle = 1$  marks the critical point of the system

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated





### Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

Therefore when (k) > 1, standard random networks have a giant component.
 When (k) \_\_\_\_\_\_ all components are finite.
 Fine example of a continuous phase transition (⊞)
 We say (k) = 1-marks the critical point of the system

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated

References





20 0 56 of 65

Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle}$$

Therefore when (k) ≥ 1, standard random network have a giant component.
 When (k) 1, all components are finite.
 Fine example of a continuous phase transition (⊞).
 We say (k) = 1-marks the critical point of the system.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple,

physically-motivated analysis





Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Therefore when (k) = 1, standard random network have a grant component.
 When (k) = 1, all components are finite.
 Fine example of a continuous phase transition (⊞).

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple,

References





2 C 56 of 65

Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Therefore when (k) > 1, standard random networks have a giant component.

Fine example of a continuous phase transition  $(\boxplus)$ 

### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- Therefore when (k) > 1, standard random networks have a giant component.
- When  $\langle k \rangle < 1$ , all components are finite.

......

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated





Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- Therefore when (k) > 1, standard random networks have a giant component.
- When  $\langle k \rangle < 1$ , all components are finite.
- ► Fine example of a continuous phase transition (⊞).

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

#### Largest component Simple, physically-motivated





Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- Therefore when (k) > 1, standard random networks have a giant component.
- When  $\langle k \rangle < 1$ , all components are finite.
- ► Fine example of a continuous phase transition (⊞).
- We say  $\langle k \rangle = 1$  marks the critical point of the system.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strang

### Largest component Simple, physically-motivated





• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

 $=\infty \quad (\gg \langle k \rangle)$ 

### So giant component always exists for these kinds

ing is  $k^{-3}$  if  $\gamma > 3$  then we have to lo

### **Random Networks**

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





na @ 57 of 65

# Giant component Random networks with skewed *P<sub>k</sub>*:

• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

 $=\infty ~(\gg \langle k 
angle).$ 

### **Random Networks**

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





20 0 57 of 65

• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle)$$

ent always exists for these kinds

 $^{3}$ ; if j > 3 then we have to lo

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

### Largest component Simple,

Deferences





20 0 57 of 65

• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle$$

onent always exists for these kinc

 $\mathbb{C}^3$  if  $\gamma > 3$  then we have to lo

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple,

References





20 0 57 of 65

• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

### **Random Networks**

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

### Largest component Simple,

ohysically-motivated analysis

References





• D Q ( ~ 57 of 65

• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

So giant component always exists for these kinds of networks.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





na (~ 57 of 65

• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

- So giant component always exists for these kinds of networks.
- Cutoff scaling is k<sup>-3</sup>: if γ > 3 then we have to look harder at ⟨k⟩<sub>R</sub>.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





• e.g, if  $P_k = ck^{-\gamma}$  with  $2 < \gamma < 3$ ,  $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

- So giant component always exists for these kinds of networks.
- Cutoff scaling is k<sup>-3</sup>: if γ > 3 then we have to look harder at ⟨k⟩<sub>R</sub>.

• How about 
$$P_k = \delta_{kk_0}$$
?

### Random Networks

#### **Basics**

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

### Largest component Simple, physically-motivated

References





20 C 57 of 65

# Giant component And how big is the largest component?

Define S<sub>1</sub> as the size of the largest component.

- Define  $\delta$  as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection:  $\delta = 1 S$
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
  - Define  $\delta$  as the probability that a randomly chosen nod does not belong to the largest component.

Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- Let's find S<sub>1</sub> with a back-of-the-envelope argument.

Dirty trick: If a randomly chosen node is not part of t targest component, then none of its neighbors are.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- Let's find S<sub>1</sub> with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.

Dirty-trick: If a randomly chosen node is not part of lamest component, then none of its neighbors are

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- Let's find S<sub>1</sub> with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection:  $\delta = 1 S_1$ .

Dirty trick

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- Let's find S<sub>1</sub> with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection:  $\delta = 1 S_1$ .
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- Let's find S<sub>1</sub> with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection:  $\delta = 1 S_1$ .
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

So

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- Let's find S<sub>1</sub> with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection:  $\delta = 1 S_1$ .
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.
- So

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

Substitute in Poisson distribution...

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





Carrying on:

# $\delta = \sum_{k=0}^{\infty} P_k \delta^k$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





20 C 59 of 65

Carrying on:

$$\boldsymbol{\delta} = \sum_{k=0}^{\infty} \boldsymbol{P}_k \boldsymbol{\delta}^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} \boldsymbol{e}^{-\langle k \rangle} \boldsymbol{\delta}^k$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





2 C 59 of 65
Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





20 C 59 of 65

Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$=e^{-\langle k
angle}\sum_{k=0}^{\infty}rac{(\langle k
angle \delta)^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

### Largest component Simple, physically-motivated

------





2 C 59 of 65

Carrying on:

$$\boldsymbol{\delta} = \sum_{k=0}^{\infty} \boldsymbol{P}_k \boldsymbol{\delta}^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} \boldsymbol{e}^{-\langle k \rangle} \boldsymbol{\delta}^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$=e^{-\langle k\rangle}e^{\langle k\rangle\delta}=e^{-\langle k\rangle(1-\delta)}$$

### Random Networks

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





2 C 59 of 65

Carrying on:

$$\boldsymbol{\delta} = \sum_{k=0}^{\infty} \boldsymbol{P}_k \boldsymbol{\delta}^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} \boldsymbol{e}^{-\langle k \rangle} \boldsymbol{\delta}^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$=e^{-\langle k\rangle}e^{\langle k\rangle\delta}=e^{-\langle k\rangle(1-\delta)}$$

Now substitute in  $\delta = 1 - S_1$  and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}$$

### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

### **Random Networks**

#### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





n a (~ 60 of 65

• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k 
angle = rac{1}{S_1} \ln rac{1}{1-S_1}.$$

### Random Networks

### Basics

Definitions How to build Some visual examples

### Structure Clustering Degree distributions Configuration model Bandom friends are strange

Largest component Simple, physically-motivated

References





20 0 60 of 65

• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k 
angle = rac{1}{S_1} \ln rac{1}{1-S_1}.$$

• As 
$$\langle k \rangle \rightarrow 0$$
,  $S_1 \rightarrow 0$ .

### Random Networks

### Basics

Definitions How to build Some visual examples

### Structure Clustering Degree distributions Configuration model Bandom friends are strange

Largest component Simple, physically-motivated

References





20 0 60 of 65

• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k 
angle = rac{1}{S_1} \ln rac{1}{1-S_1}.$$

• As 
$$\langle k \rangle \rightarrow 0$$
,  $S_1 \rightarrow 0$ .  
• As  $\langle k \rangle \rightarrow \infty$ ,  $S_1 \rightarrow 1$ .

### **Random Networks**

### Basics

Definitions How to build Some visual examples

### Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated





• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k 
angle = rac{1}{S_1} \ln rac{1}{1-S_1}.$$

• As 
$$\langle k \rangle \rightarrow 0$$
,  $S_1 \rightarrow 0$ .

• As  $\langle k \rangle \to \infty$ ,  $S_1 \to 1$ .

Notice that at  $\langle k \rangle = 1$ , the critical point,  $S_1 = 0$ .

### Random Networks

#### Basics

Definitions How to build Some visual examples

### Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated

References





n a (~ 60 of 65

• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k 
angle = rac{1}{S_1} \ln rac{1}{1-S_1}.$$

- As  $\langle k \rangle \rightarrow 0, S_1 \rightarrow 0.$
- As  $\langle k \rangle \to \infty$ ,  $S_1 \to 1$ .
- Notice that at  $\langle k \rangle = 1$ , the critical point,  $S_1 = 0$ .
- Only solvable for  $S_1 > 0$  when  $\langle k \rangle > 1$ .

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated





• We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k 
angle = rac{1}{S_1} \ln rac{1}{1-S_1}.$$

- As  $\langle k \rangle \rightarrow 0, S_1 \rightarrow 0.$
- As  $\langle k \rangle \to \infty$ ,  $S_1 \to 1$ .

Notice that at  $\langle k \rangle = 1$ , the critical point,  $S_1 = 0$ .

- Only solvable for  $S_1 > 0$  when  $\langle k \rangle > 1$ .
- Really a transcritical bifurcation.<sup>[2]</sup>

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated







### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strang

Largest component Simple, physically-motivated analysis

References





20 C 61 of 65

### Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors hav same probability a of belonging to the largest component.
- But we know our friends are different from us.
- Works for ER random networks because (k
- We need a separate probability & for the chance t an edge leads to the diant (infinite) component.
- We can sort many things out with sensible probabilistic arguments ...
- More detailed investigations will profit from a spot Generatingfunctionology.<sup>[3]</sup>

### Random Networks

### **Basics**

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





2 C 62 of 65

Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability  $\delta$  of belonging to the largest component.
- Works for ER random networks because (k) = (k
   We need a separate probability 8' for the chance to an edge leads to the giant (infinite) component.
   We can sort many things out with sensible probabilistic arguments...
  - Generatingfunctionology <sup>[3]</sup>

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





2 C 62 of 65

Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability  $\delta$  of belonging to the largest component.
- But we know our friends are different from us...

We need a separate probability of for the chance an edge leads to the giant (infinite) component. We can sort many things out with sensible probabilistic arguments... More detailed investigations will profit from a sp

Generatingfunctionology. <sup>[3</sup>

### Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability  $\delta$  of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because  $\langle k \rangle = \langle k \rangle_R$ .

an edge leads to the grant (infinite) component We can solve quary things out with sensible probabilistic arguments ... More detailed investigations will profit from a s Random Networks

Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis

References





Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability  $\delta$  of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because  $\langle k \rangle = \langle k \rangle_R$ .
- We need a separate probability  $\delta'$  for the chance that an edge leads to the giant (infinite) component.

probabilistic arguments

Generatingfunctionology.<sup>[3</sup>

### Random Networks

### **Basics**

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability  $\delta$  of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because  $\langle k \rangle = \langle k \rangle_R$ .
- We need a separate probability  $\delta'$  for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...

Generatingfunctionology. <sup>[3</sup>

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability  $\delta$  of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because  $\langle k \rangle = \langle k \rangle_R$ .
- We need a separate probability  $\delta'$  for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a spot of Generatingfunctionology.<sup>[3]</sup>

### Random Networks

### **Basics**

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange

Largest component Simple, physically-motivated analysis





# Outline

### Basics

Definitions How to build Some visual examples

### Structure

Clustering Degree distributions Configuration model Bandom kiends are strange

Simple, physically-motivated analysis

### References

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering Degree distributions Configuration model

Random friends are strange

Largest component

Simple, physically-motivated analysis

References





20 63 of 65

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure

Clustering Degree distributions Configuration model

Random friends are strang

Largest component

Simple, physically-motivated analysis

References





2 C 64 of 65

# **References** I

# M. E. J. Newman. The structure and function of complex networks. SIAM Review, 45(2):167–256, 2003. pdf (⊞)

# S. H. Strogatz. <u>Nonlinear Dynamics and Chaos</u>. Addison Wesley, Reading, Massachusetts, 1994.

## [3] H. S. Wilf. Generatingfunctionology.

A K Peters, Natick, MA, 3rd edition, 2006. pdf  $(\boxplus)$ 

### Random Networks

### Basics

Definitions How to build Some visual examples

Structure Clustering Degree distributions Configuration model Random friends are strange Largest component Simple, physically-motivated analysis

References





2 C 65 of 65