## Random Networks <br> Complex Networks CSYS/MATH 303, Spring, 2011

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## Random networks

Pure, abstract random networks:

- Consider set of all networks with $N$ labelled nodes and $m$ edges.


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- Standard random network = one randomly chosen network from this set.

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## Random networks

 one randomly chosen network from this set.- To be clear: each network is equally probable.

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- Sometimes equiprobability is a good assumption, but it is always an assumption.

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## Random networks

Pure, abstract random networks:

- Consider set of all networks with $N$ labelled nodes and $m$ edges.
- Standard random network = one randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

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## Random network generator for $N=3$ :



- Get your own exciting generator here ( $\boxplus$ ).
- As $N$, our polyhedral die rapidly becomes a ball...

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## Random networks-basic features:

- Number of possible edges:

$$
0 \leq m \leq\binom{ N}{2}=\frac{N(N-1)}{2}
$$

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- Number of possible networks with $N$ labelled nodes:

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2\binom{N}{2} \sim e^{\frac{\ln 2}{2} N^{2}} .
$$

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- Given $m$ edges, there are $\left(\begin{array}{c}\left(\begin{array}{c}N \\ 2 \\ m\end{array}\right)\end{array}\right)$ different possible networks.

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- Crazy factorial explosion for $1 \ll m \ll\binom{N}{2}$.
- Real world: links are usually costly so real networks are almost always sparse.

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## Random networks

## How to build standard random networks:

- Given $N$ and $m$.


## (we'll see a third later on)

## - Useful for theoretical work.

> -Algorithm: Randomly choose a pair of nodes $i$ and $j$, $i \neq j$, and connect if unconnected; repeat until all $m$ edges are;allocated.
> - Best for adding relatively small numbers of links (most cases).
> $>1$ and 2 are effectively equivalent for large $N$.

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## Random networks

How to build standard random networks:

- Given $N$ and $m$.
- Two probablistic methods


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- Given $N$ and $m$.
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1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability $p$.

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- Given $N$ and $m$.
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1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability $p$.
2. Take $N$ nodes and add exactly $m$ links by selecting edges without replacement.

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## Random networks

A few more things:

- For method 1, \# links is probablistic:

$$
\langle m\rangle=p\binom{N}{2}
$$

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## Random networks

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- For method 1, \# links is probablistic:

$$
\langle m\rangle=p\binom{N}{2}=p \frac{1}{2} N(N-1)
$$

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- So the expected or average degree is

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\langle k\rangle=\frac{2\langle m\rangle}{N}
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=\frac{2}{N} p \frac{1}{2} N(N-1)=\frac{2}{N} p \frac{1}{2} N(N-1)
\end{gathered}
$$

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$$
\begin{gathered}
\langle k\rangle=\frac{2\langle m\rangle}{N} \\
=\frac{2}{N} p \frac{1}{2} N(N-1)=\frac{2}{X} p \frac{1}{2} N(N-1)=p(N-1) .
\end{gathered}
$$

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- Which is what it should be...

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\end{gathered}
$$

- Which is what it should be...
- If we keep $\langle k\rangle$ constant then $p \propto 1 / N \rightarrow 0$ as $N \rightarrow \infty$.

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- $N=500$



## Random networks：examples

Next slides：
Example realizations of random networks
－$N=500$
－Vary $m$ ，the number of edges from 100 to 1000.

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## Random networks: examples

Next slides:
Example realizations of random networks

- $N=500$
- Vary $m$, the number of edges from 100 to 1000.
- Average degree $\langle k\rangle$ runs from 0.4 to 4 .

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## Random networks: examples

Next slides:
Example realizations of random networks

- $N=500$
- Vary $m$, the number of edges from 100 to 1000.
- Average degree $\langle k\rangle$ runs from 0.4 to 4 .
- Look at full network plus the largest component.


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## Random networks: examples

entire network:

largest component:

$N=500$, number of edges $m=100$ average degree $\langle k\rangle=0.4$

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## Random networks: examples

entire network:


## largest component:


$N=500$, number of edges $m=200$ average degree $\langle k\rangle=0.8$

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## Random networks: examples

## largest component:


$N=500$, number of edges $m=230$ average degree $\langle k\rangle=0.92$

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## Random networks: examples

## largest component:


$N=500$, number of edges $m=240$ average degree $\langle k\rangle=0.96$

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## Random networks: examples

entire network:


## largest component:


$N=500$, number of edges $m=250$ average degree $\langle k\rangle=1$

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## Random networks: examples

## largest component:

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$N=500$, number of edges $m=260$ average degree $\langle k\rangle=1.04$

## Random networks: examples

## largest component:


$N=500$, number of edges $m=280$ average degree $\langle k\rangle=1.12$

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## Random networks: examples

entire network:

largest component:


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$N=500$, number of edges $m=300$ average degree $\langle k\rangle=1.2$

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## Random networks：examples

largest component：

$N=500$ ，number of edges $m=500$ average degree $\langle k\rangle=2$

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entire network:


## largest component:


$N=500$, number of edges $m=1000$ average degree $\langle k\rangle=4$

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## Random networks: examples for $N=500$


$m=230$
$\langle k\rangle=0.92$
$\langle k\rangle=0.8$
$m=100$
$\langle k\rangle=0.4$


$$
\begin{array}{ll}
m=260 & m=280 \\
\langle k\rangle=1.04 & \langle k\rangle=1.12
\end{array}
$$

$m=300$
$\langle k\rangle=1.2$
$m=500$
$\langle k\rangle=2$
$m=240$
$\langle k\rangle=0.96$
$m=250$
$\langle k\rangle=1$

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$$
m=1000
$$

$\langle k\rangle=4$


## Random networks: largest components

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$m=100$
$\langle k\rangle=0.4$

$$
m=200
$$

$\langle k\rangle=0.8$

$$
\begin{aligned}
& m=230 \\
& \langle k\rangle=0.92
\end{aligned}
$$


$m=260$
$m=280$
$\langle k\rangle=1.04$
$\langle k\rangle=1.12$
$m=300$
$\langle k\rangle=1.2$

$$
m=240
$$

$\langle k\rangle=0.96$
$m=250$
$\langle k\rangle=1$

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$m=1000$
$\langle k\rangle=4$
$\langle k\rangle=2$


## Random networks: examples for $N=500$

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$m=250$
$\langle k\rangle=1$

$m=250$
$m=250$
$\langle k\rangle=1$
$\langle k\rangle=1$
$m=250$
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## Random networks: largest components

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$$
m=250
$$

$\langle k\rangle=1$

$$
m=250
$$

$$
\langle k\rangle=1
$$

$$
\begin{aligned}
& m=250 \\
& \langle k\rangle=1
\end{aligned}
$$

$$
\begin{aligned}
& m=250 \\
& \langle k\rangle=1
\end{aligned}
$$

$$
m=250
$$

$$
\langle k\rangle=1
$$



$$
m=250
$$

$$
m=250
$$

$m=250$

$$
\langle k\rangle=1
$$

$$
\langle k\rangle=1
$$

$m=250$
$\langle k\rangle=1$
$\langle k\rangle=1$
$m=250$
$\langle k\rangle=1$

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## Clustering in random networks:

- For method 1, what is the clustering coefficient for a finite network?


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## Clustering in random networks:

- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient: ${ }^{[1]}$

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

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## Clustering in random networks：

－For method 1，what is the clustering coefficient for a finite network？
－Consider triangle／triple clustering coefficient：

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

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－Recall：$C_{2}=$ probability that two friends of a node are also friends．


## Clustering in random networks:

- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient:

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

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- Recall: $C_{2}=$ probability that two friends of a node are also friends.
- Or: $C_{2}=$ probability that a triple is part of a triangle.



## Clustering in random networks:

- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient:

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

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- Recall: $C_{2}=$ probability that two friends of a node are also friends.
- Or: $C_{2}=$ probability that a triple is part of a triangle.
- For standard random networks, we have simply that


$$
C_{2}=p .
$$

## Other ways to compute clustering：

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－Expected number of triples in entire network：

$$
\frac{1}{2} N(N-1)(N-2) p^{2}
$$

（Double counting dealt with by $\frac{1}{2}$ ．）

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## Other ways to compute clustering:

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$$
\frac{1}{6} N(N-1)(N-2) p^{3}
$$

(Over-counting dealt with by $\frac{1}{6}$.)

$$
\frac{1}{2} N(N-1)(N-2) p^{2}
$$

(Double counting dealt with by $\frac{1}{2}$.)

- Expected number of triangles in entire network:


## Other ways to compute clustering:

- Expected number of triples in entire network:

$$
\frac{1}{2} N(N-1)(N-2) p^{2}
$$

(Double counting dealt with by $\frac{1}{2}$.)

- Expected number of triangles in entire network:

$$
\frac{1}{6} N(N-1)(N-2) p^{3}
$$

(Over-counting dealt with by $\frac{1}{6}$.)

$$
C_{2}=\frac{3 \times \text { \#triangles }}{\# \text { triples }}=\frac{3 \times \frac{1}{6} N(N-1)(N-2) p^{3}}{\frac{1}{2} N(N-1)(N-2) p^{2}}=p .
$$

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## Other ways to compute clustering:

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## Other ways to compute clustering:

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## Other ways to compute clustering:

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- Triangle occurs with probability $p^{3}$. $p^{2} \times(1-p)+p^{2} \times p=p^{2}$.

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## Other ways to compute clustering:

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- Triangle occurs with probability $p^{3}$.
- Therefore,

$$
C_{2}=\frac{p^{3}}{p^{2}}=p
$$

## Clustering in random networks:

- So for large random networks ( $N \rightarrow \infty$ ), clustering drops to zero.


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## Clustering in random networks:

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- Key structural feature of random networks is that they locally look like
pure branching networks
- So for large random networks ( $N \rightarrow \infty$ ), clustering drops to zero.



## Clustering in random networks:

- Key structural feature of random networks is that they locally look like
pure branching networks
- No small loops.

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## Degree distributions



## Random networks <br> Degree distribution：

－Recall $P_{k}=$ probability that a randomly selected node has degree $k$ ．

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## Random networks

## Degree distribution:

- Recall $P_{k}=$ probability that a randomly selected node has degree $k$.
- Consider method 1 for constructing random networks: each possible link is realized with probability $p$.

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## Random networks

## Degree distribution:

- Recall $P_{k}=$ probability that a randomly selected node has degree $k$.
- Consider method 1 for constructing random networks: each possible link is realized with probability $p$.
- Now consider one node: there are ' $N-1$ choose $k$ ' ways the node can be connected to $k$ of the other $N-1$ nodes.


## Random networks

## Degree distribution:

- Recall $P_{k}=$ probability that a randomly selected node has degree $k$.
- Consider method 1 for constructing random networks: each possible link is realized with probability $p$.
- Now consider one node: there are ' $N-1$ choose $k$ ' ways the node can be connected to $k$ of the other $N-1$ nodes.
- Each connection occurs with probability $p$, each non-connection with probability $(1-p)$.


## Random networks

Degree distribution:

- Recall $P_{k}=$ probability that a randomly selected node has degree $k$.
- Consider method 1 for constructing random networks: each possible link is realized with probability $p$.


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- Now consider one node: there are ' $N-1$ choose $k$ ' ways the node can be connected to $k$ of the other $N-1$ nodes.
- Each connection occurs with probability $p$, each non-connection with probability $(1-p)$.
- Therefore have a binomial distribution:

$$
P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}
$$

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## Limiting form of $P(k ; p, N)$ :

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## Random networks

Limiting form of $P(k ; p, N)$ :

- Our degree distribution: $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$.

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## Random networks

## Limiting form of $P(k ; p, N)$ ：

－Our degree distribution： $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$ ．
－What happens as $N \rightarrow \infty$ ？

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## Random networks

## Limiting form of $P(k ; p, N)$ ：

－Our degree distribution： $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$ ．
－What happens as $N \rightarrow \infty$ ？

## Structure

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－We must end up with the normal distribution right？

$\left|\begin{array}{l}0 \\ 0\end{array}\right|$
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## Random networks

## Limiting form of $P(k ; p, N)$ ：

－Our degree distribution： $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$ ．
－What happens as $N \rightarrow \infty$ ？
－We must end up with the normal distribution right？
－If $p$ is fixed，then we would end up with a Gaussian with average degree $\langle k\rangle \simeq p N \rightarrow \infty$ ．



## Random networks

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- We must end up with the normal distribution right?
- If $p$ is fixed, then we would end up with a Gaussian with average degree $\langle k\rangle \simeq p N \rightarrow \infty$.
- But we want to keep $\langle k\rangle$ fixed...
- So examine limit of $P(k ; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k\rangle=p(N-1)=$ constant.


## Limiting form of $P(k ; p, N)$ :

- Our degree distribution: $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$.
- What happens as $N \rightarrow \infty$ ?


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## Limiting form of $P(k ; p, N)$ :

- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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## Limiting form of $P(k ; p, N)$ :

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- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
& \quad=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ :

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- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
\begin{gathered}
P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
=\frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{gathered}
$$

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## Limiting form of $P(k ; p, N)$ :

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- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{N^{k}\left(1-\frac{1}{N}\right) \cdots\left(1-\frac{k}{N}\right)}{k!N^{k}} \frac{\langle k\rangle^{k}}{\left(1-\frac{1}{N}\right)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ :

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- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{A^{k}\left(1-\frac{1}{N}\right) \cdots\left(1-\frac{k}{N}\right)}{k!A^{k}} \frac{\langle k\rangle^{k}}{\left(1-\frac{1}{N}\right)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ :

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- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
& \simeq \frac{A^{k}\left(1-\frac{1}{N}\right) \cdots\left(1-\frac{k}{N}\right)}{k!A^{k}} \frac{\langle k\rangle^{k}}{\left(1-\frac{y}{N}\right)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ :

- We are now here:

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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## (Use l'Hôpital's rule to prove.)

## Limiting form of $P(k ; p, N)$ :

- We are now here:

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

- Now use the excellent result:

$$
\lim _{n \rightarrow \infty}\left(1+\frac{x}{n}\right)^{n}=e^{x}
$$

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## Limiting form of $P(k ; p, N)$ :

- We are now here:

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

- Now use the excellent result:

$$
\lim _{n \rightarrow \infty}\left(1+\frac{x}{n}\right)^{n}=e^{x}
$$

(Use l'Hôpital's rule to prove.)

- Identifying $n=N-1$ and $x=-\langle k\rangle$ :

$$
P(k ;\langle k\rangle) \simeq \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}\left(1-\frac{\langle k\rangle}{N-1}\right)^{-k}
$$

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## Limiting form of $P(k ; p, N)$ :

- We are now here:

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

- Now use the excellent result:

$$
\lim _{n \rightarrow \infty}\left(1+\frac{x}{n}\right)^{n}=e^{x}
$$

(Use l'Hôpital's rule to prove.)

- Identifying $n=N-1$ and $x=-\langle k\rangle$ :

$$
P(k ;\langle k\rangle) \simeq \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}\left(1-\frac{\langle k\rangle}{N-1}\right)^{-k} \rightarrow \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
$$

- This is a Poisson distribution ( $\boxplus$ ) with mean $\langle k\rangle$.

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## Poisson basics：

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$$
P(k ; \lambda)=\frac{\lambda^{k}}{k!} e^{-\lambda}
$$


－$\lambda>0$
－$k=0,1,2,3, \ldots$
－Classic use：probability that an event occurs $k$ times in a given time period，given an average rate of occurrence．
－e．g．： phone calls／minute， horse－kick deaths．
－＇Law of small numbers＇

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## Poisson basics：

－Normalization：we must have

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

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## Poisson basics:

- Normalization: we must have

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

- Checking:


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$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
$$



## Poisson basics:

- Normalization: we must have
- Checking:

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

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$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
$$

$$
=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!}
$$

## Poisson basics:

- Normalization: we must have
- Checking:

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

$$
\begin{gathered}
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \\
=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} \\
=e^{-\langle k\rangle} e^{\langle k\rangle}
\end{gathered}
$$

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## Poisson basics:

- Normalization: we must have

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

- Checking:

$$
\begin{gathered}
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \\
=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} \\
=e^{-\langle k\rangle} e^{\langle k\rangle}=1 \checkmark
\end{gathered}
$$

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## Poisson basics:

- Mean degree: we must have

$$
\langle k\rangle=\sum_{k=0}^{\infty} k P(k ;\langle k\rangle)
$$

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## Poisson basics:

- Mean degree: we must have

$$
\langle k\rangle=\sum_{k=0}^{\infty} k P(k ;\langle k\rangle)
$$

- Checking:

$$
\sum_{k=0}^{\infty} k P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} k \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
$$

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- Checking:

$$
\begin{gathered}
\sum_{k=0}^{\infty} k P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} k \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \\
=e^{-\langle k\rangle} \sum_{k=1}^{\infty} \frac{\langle k\rangle^{k}}{(k-1)!}
\end{gathered}
$$

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\begin{aligned}
& =e^{-\langle k\rangle} \sum_{k=1}^{\infty} \frac{\langle k\rangle^{k}}{(k-1)!} \\
& =\langle k\rangle e^{-\langle k\rangle} \sum_{k=1}^{\infty} \frac{\langle k\rangle^{k-1}}{(k-1)!}
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## Poisson basics:

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=\langle k\rangle e^{-\langle k\rangle} \sum_{k=1}^{\infty} \frac{\langle k\rangle^{k-1}}{(k-1)!} \\
=\langle k\rangle e^{-\langle k\rangle} \sum_{i=0}^{\infty} \frac{\langle k\rangle^{i}}{i!}=\langle k\rangle e^{-\langle k\rangle} e^{\langle k\rangle}=\langle k\rangle \checkmark
\end{gathered}
$$

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## Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.


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## Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding $\langle k\rangle$ to find the second moment:

$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
$$

## Random Networks

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## Poisson basics：

－The variance of degree distributions for random networks turns out to be very important．
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## Structure

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Degree distributions
－Variance is then

$$
\sigma^{2}=\left\langle k^{2}\right\rangle-\langle k\rangle^{2}
$$


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## Structure

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｜ö

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Clustering
Degree distributions
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- Variance is then

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\sigma^{2}=\left\langle k^{2}\right\rangle-\langle k\rangle^{2}=\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle^{2}=\langle k\rangle
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- So standard deviation $\sigma$ is equal to $\sqrt{\langle k\rangle}$.




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- Variance is then

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$$

- So standard deviation $\sigma$ is equal to $\sqrt{\langle k\rangle}$.
- Note: This is a special property of Poisson distribution and can trip us up...



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## Configuration model



## General random networks

- So... standard random networks have a Poisson degree distribution


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## General random networks

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution $P_{k}$.

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## General random networks

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution $P_{k}$.
- Also known as the configuration model. ${ }^{[1]}$

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## General random networks

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution $P_{k}$.
- Also known as the configuration model.
- Can generalize construction method from ER random networks.

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## General random networks

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution $P_{k}$.
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- Assign each node a weight $w$ from some distribution $P_{w}$ and form links with probability
$P($ link between $i$ and $j) \propto w_{i} w_{j}$.



## General random networks

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- But we'll be more interested in


## General random networks

－So．．．standard random networks have a Poisson degree distribution
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－Can generalize construction method from ER random networks．
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－But we＇ll be more interested in
1．Randomly wiring up（and rewiring）already existing nodes with fixed degrees．

## General random networks

- So... standard random networks have a Poisson degree distribution
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$$

- But we'll be more interested in

1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
2. Examining mechanisms that lead to networks with certain degree distributions.

## Random networks：examples

Coming up：
Example realizations of random networks with power law degree distributions：

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## Random networks：examples

Coming up：
Example realizations of random networks with power law degree distributions：
－$N=1000$ ．

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## Random networks：examples

Coming up：
Example realizations of random networks with power law degree distributions：
－$N=1000$ ．
－$P_{k} \propto k^{-\gamma}$ for $k \geq 1$ ．

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## Random networks: examples

Coming up:
Example realizations of random networks with power law degree distributions:

- $N=1000$.
- $P_{k} \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_{0}=0$ (no isolated nodes).


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## Random networks: examples

Coming up:
Example realizations of random networks with power law degree distributions:

- $N=1000$.
- $P_{k} \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_{0}=0$ (no isolated nodes).
- Vary exponent $\gamma$ between 2.10 and 2.91.

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## Random networks: examples

Coming up:
Example realizations of random networks with power law degree distributions:

- $N=1000$.
- $P_{k} \propto k^{-\gamma}$ for $k \geq 1$.
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- Vary exponent $\gamma$ between 2.10 and 2.91.
- Again, look at full network plus the largest component.

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## Random networks: examples

## Coming up:

Example realizations of random networks with power law degree distributions:

- $N=1000$.

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- $P_{k} \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_{0}=0$ (no isolated nodes).
- Vary exponent $\gamma$ between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.


## Random networks: examples for $N=1000$

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$\gamma=2.1$
$\langle k\rangle=3.448$

$\gamma=2.55$
$\langle k\rangle=1.712$
$\gamma=2.64$
$\langle k\rangle=1.6$

$\gamma=2.28$
$\langle k\rangle=2.306$

$\gamma=2.73$
$\langle k\rangle=1.862$

$\gamma=2.37$
$\langle k\rangle=2.504$

$\gamma=2.82$
$\langle k\rangle=1.386$

$\gamma=2.46$
$\langle k\rangle=1.856$

$\gamma=2.91$
$\langle k\rangle=1.49$

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## Random networks: largest components

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$$
\begin{array}{ll}
\gamma=2.55 & \gamma=2.64 \\
\langle k\rangle=1.712 & \langle k\rangle=1.6
\end{array}
$$

$\gamma=2.19$
$\langle k\rangle=2.986$

$\gamma=2.37$
$\langle k\rangle=2.504$
$\gamma=2.28$
$\langle k\rangle=2.306$

$\gamma=2.73$
$\langle k\rangle=1.862$

$\gamma=2.82$
$\langle k\rangle=1.386$

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## The edge-degree distribution:

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## The edge-degree distribution:

- The degree distribution $P_{k}$ is fundamental for our description of many complex networks


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## The edge-degree distribution:

- The degree distribution $P_{k}$ is fundamental for our description of many complex networks
- Again: $P_{k}$ is the degree of randomly chosen node.


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## The edge-degree distribution:

- The degree distribution $P_{k}$ is fundamental for our description of many complex networks
- Again: $P_{k}$ is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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- Define $Q_{k}$ to be the probability the node at a random end of a randomly chosen edge has degree $k$.



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- Now choosing nodes based on their degree (i.e., size):

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Q_{k} \propto k P_{k}
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- Normalized form:

$$
Q_{k}=\frac{k P_{k}}{\sum_{k^{\prime}=0}^{\infty} k^{\prime} P_{k^{\prime}}}
$$



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- Normalized form:

$$
Q_{k}=\frac{k P_{k}}{\sum_{k^{\prime}=0}^{\infty} k^{\prime} P_{k^{\prime}}}=\frac{k P_{k}}{\langle k\rangle} .
$$



## The edge-degree distribution:

- For random networks, $Q_{k}$ is also the probability that a friend (neighbor) of a random node has $k$ friends.


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## The edge-degree distribution:

- For random networks, $Q_{k}$ is also the probability that a friend (neighbor) of a random node has $k$ friends.
- Useful variant on $Q_{k}$ :
$R_{k}=$ probability that a friend of a random node has $k$ other friends.

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$R_{k}=$ probability that a friend of a random node has $k$ other friends.

$$
R_{k}=\frac{(k+1) P_{k+1}}{\sum_{k^{\prime}=0}\left(k^{\prime}+1\right) P_{k^{\prime}+1}}
$$

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## Simple,



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$$

- Equivalent to friend having degree $k+1$.


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## The edge－degree distribution：

－For random networks，$Q_{k}$ is also the probability that a friend（neighbor）of a random node has $k$ friends．
－Useful variant on $Q_{k}$ ：
$R_{k}=$ probability that a friend of a random node has $k$ other friends．

$$
R_{k}=\frac{(k+1) P_{k+1}}{\sum_{k^{\prime}=0}\left(k^{\prime}+1\right) P_{k^{\prime}+1}}=\frac{(k+1) P_{k+1}}{\langle k\rangle}
$$

－Equivalent to friend having degree $k+1$ ．
－Natural question：what＇s the expected number of other friends that one friend has？


## The edge-degree distribution:

- Given $R_{k}$ is the probability that a friend has $k$ other friends, then the average number of friends' other friends is

$$
\langle k\rangle_{R}=\sum_{k=0}^{\infty} k R_{k}
$$

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## The edge－degree distribution：

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$$
\langle k\rangle_{R}=\sum_{k=0}^{\infty} k R_{k}=\sum_{k=0}^{\infty} k \frac{(k+1) P_{k+1}}{\langle k\rangle}
$$

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## The edge-degree distribution:

- Given $R_{k}$ is the probability that a friend has $k$ other friends, then the average number of friends' other friends is

$$
\begin{aligned}
\langle k\rangle_{R} & =\sum_{k=0}^{\infty} k R_{k}=\sum_{k=0}^{\infty} k \frac{(k+1) P_{k+1}}{\langle k\rangle} \\
& =\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}
\end{aligned}
$$

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&=\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty}\left((k+1)^{2}-(k+1)\right) P_{k+1}
\end{aligned}
$$

(where we have sneakily matched up indices)

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\end{aligned}
$$

(where we have sneakily matched up indices)

$$
=\frac{1}{\langle k\rangle} \sum_{j=0}^{\infty}\left(j^{2}-j\right) P_{j} \quad(\text { using } j=k+1)
$$

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## The edge－degree distribution：

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&=\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1} \\
&=\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty}\left((k+1)^{2}-(k+1)\right) P_{k+1}
\end{aligned}
$$

（where we have sneakily matched up indices）

$$
\begin{gathered}
=\frac{1}{\langle k\rangle} \sum_{j=0}^{\infty}\left(j^{2}-j\right) P_{j} \quad(\text { using } j=k+1) \\
=\frac{1}{\langle k\rangle}\left(\left\langle k^{2}\right\rangle-\langle k\rangle\right)
\end{gathered}
$$

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## The edge-degree distribution:

- Note: our result, $\langle k\rangle_{R}=\frac{1}{\langle k\rangle}\left(\left\langle k^{2}\right\rangle-\langle k\rangle\right)$, is true for all random networks, independent of degree distribution.


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－Again，neatness of results is a special property of the Poisson distribution．

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- So friends on average have $\langle k\rangle$ other friends, and $\langle k\rangle+1$ total friends...


## Two reasons why this matters

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Reason \#1:
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## Two reasons why this matters

Reason \＃1：
－Average \＃friends of friends per node is

$$
\left\langle k_{2}\right\rangle=\langle k\rangle \times\langle k\rangle_{R}
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## Two reasons why this matters

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- Key: Average depends on the 1st and 2nd moments of $P_{k}$ and not just the 1st moment.
- Three peculiarities:

1. We might guess $\left\langle k_{2}\right\rangle=\langle k\rangle(\langle k\rangle-1)$ but it's actually $\langle k(k-1)\rangle$.

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(e.g., in the case of a power-law distribution)


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2. If $P_{k}$ has a large second moment, then $\left\langle k_{2}\right\rangle$ will be big.
(e.g., in the case of a power-law distribution)
3. Your friends really are different from you...

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## Two reasons why this matters

More on peculiarity \#3:

- A node's average \# of friends: $\langle k\rangle$

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More on peculiarity \#3:

- A node's average \# of friends: $\langle k\rangle$
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## Two reasons why this matters

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- A node's average \# of friends: $\langle k\rangle$
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- Comparison:


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$$
\frac{\left\langle k^{2}\right\rangle}{\langle k\rangle}=\langle k\rangle \frac{\left\langle k^{2}\right\rangle}{\langle k\rangle^{2}}
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- Comparison:

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\frac{\left\langle k^{2}\right\rangle}{\langle k\rangle}=\langle k\rangle \frac{\left\langle k^{2}\right\rangle}{\langle k\rangle^{2}}=\langle k\rangle \frac{\sigma^{2}+\langle k\rangle^{2}}{\langle k\rangle^{2}}
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- So only if everyone has the same degree (variance $=\sigma^{2}=0$ ) can a node be the same as its friends.



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- So only if everyone has the same degree (variance $=\sigma^{2}=0$ ) can a node be the same as its friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.


## Two reasons why this matters

(Big) Reason \#2:

- $\langle k\rangle_{R}$ is key to understanding how well random networks are connected together.


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## Two reasons why this matters

(Big) Reason \#2:

- $\langle k\rangle_{R}$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.

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- $\langle k\rangle_{R}$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As $N \rightarrow \infty$, does our network have a giant component?

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- Defn: Component = connected subnetwork of nodes such that $\exists$ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.

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- Defn: Component = connected subnetwork of nodes such that $\exists$ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Giant component = component that comprises a non-zero fraction of a network as $N \rightarrow \infty$.

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- Defn: Giant component = component that comprises a non-zero fraction of a network as $N \rightarrow \infty$.
- Note: Component = Cluster


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## Largest component

## Giant component

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## Structure of random networks

## Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.


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## Structure of random networks

## Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.

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- All of this is the same as requiring $\langle k\rangle_{R}>1$.



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- Equivalently, expect exponential growth in node number as we move out from a random node.
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- Giant component condition (or percolation condition):

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1
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- Again, see that the second moment is an essential part of the story.

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- Again, see that the second moment is an essential part of the story.
- Equivalent statement: $\left\langle k^{2}\right\rangle>2\langle k\rangle$

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## Giant component

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## Standard random networks:

- Recall $\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle$.



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## Giant component

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## Random Networks

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## Giant component

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－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．


## Giant component

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- Therefore when $\langle k\rangle>1$, standard random networks have a giant component.
- When $\langle k\rangle<1$, all components are finite.

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## Giant component

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－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
－When $\langle k\rangle<1$ ，all components are finite．
－Fine example of a continuous phase transition（ $\boxplus$ ）．


## Giant component

## Standard random networks:

- Recall $\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle$.
- Condition for giant component:

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\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}=\langle k\rangle
$$

- Therefore when $\langle k\rangle>1$, standard random networks have a giant component.
- When $\langle k\rangle<1$, all components are finite.
- Fine example of a continuous phase transition ( $\boxplus$ ).
- We say $\langle k\rangle=1$ marks the critical point of the system.



## Giant component

Random networks with skewed $P_{k}$ :

- e.g, if $P_{k}=c k^{-\gamma}$ with $2<\gamma<3, k \geq 1$, then

$$
\left\langle k^{2}\right\rangle=c \sum_{k=1}^{\infty} k^{2} k^{-\gamma}
$$

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## Giant component

Random networks with skewed $P_{k}$ :

- e.g, if $P_{k}=c k^{-\gamma}$ with $2<\gamma<3, k \geq 1$, then

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\left\langle k^{2}\right\rangle=c \sum_{k=1}^{\infty} k^{2} k^{-\gamma} \\
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\end{gathered}
$$

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\end{gathered}
$$

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\end{gathered}
$$

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\end{gathered}
$$

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- So giant component always exists for these kinds of networks.



## Giant component

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\sim \int_{x=1}^{\infty} x^{2-\gamma} d x \\
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- Cutoff scaling is $k^{-3}$ : if $\gamma>3$ then we have to look harder at $\langle k\rangle_{R}$.


## Giant component

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- Cutoff scaling is $k^{-3}$ : if $\gamma>3$ then we have to look harder at $\langle k\rangle_{R}$.
- How about $P_{k}=\delta_{k k_{0}}$ ?

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## Giant component

And how big is the largest component？
－Define $S_{1}$ as the size of the largest component．

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## Giant component

## And how big is the largest component?

- Define $S_{1}$ as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k\rangle$.


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## Giant component

## And how big is the largest component?

- Define $S_{1}$ as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k\rangle$.
- Let's find $S_{1}$ with a back-of-the-envelope argument.


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## Giant component

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- Define $S_{1}$ as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k\rangle$.
- Let's find $S_{1}$ with a back-of-the-envelope argument.
- Define $\delta$ as the probability that a randomly chosen node does not belong to the largest component.


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$\left|\begin{array}{|c|}0 \\ 0\end{array}\right|$
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## Giant component

## And how big is the largest component?

- Define $S_{1}$ as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k\rangle$.
- Let's find $S_{1}$ with a back-of-the-envelope argument.
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- Simple connection: $\delta=1-S_{1}$.



## Giant component

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- Define $S_{1}$ as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k\rangle$.
- Let's find $S_{1}$ with a back-of-the-envelope argument.
- Define $\delta$ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta=1-S_{1}$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

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## Giant component

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- Define $S_{1}$ as the size of the largest component.
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- Simple connection: $\delta=1-S_{1}$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.
- So

$$
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}
$$

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## Giant component

## And how big is the largest component?

- Define $S_{1}$ as the size of the largest component.
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- So

$$
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}
$$



- Substitute in Poisson distribution...


## Giant component

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$$
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}
$$

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$$
=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{(\langle k\rangle \delta)^{k}}{k!}
$$

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$$
\begin{aligned}
\delta= & \sum_{k=0}^{\infty} P_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k} \\
& =e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{(\langle k\rangle \delta)^{k}}{k!} \\
= & e^{-\langle k\rangle} e^{\langle k\rangle \delta}
\end{aligned}
$$

## Giant component

- Carrying on:

$$
\begin{aligned}
& \delta= \sum_{k=0}^{\infty} P_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k} \\
&=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{(\langle k\rangle \delta)^{k}}{k!} \\
&=e^{-\langle k\rangle} e^{\langle k\rangle \delta}=e^{-\langle k\rangle(1-\delta)}
\end{aligned}
$$

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$$
=e^{-\langle k\rangle} e^{\langle k\rangle \delta}=e^{-\langle k\rangle(1-\delta)}
$$

－Now substitute in $\delta=1-S_{1}$ and rearrange to obtain：

$$
S_{1}=1-e^{-\langle k\rangle S_{1}} .
$$

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## Giant component

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- We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$.


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## Giant component

- We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$.
- First, we can write $\langle k\rangle$ in terms of $S_{1}$ :

$$
\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}}
$$

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## Giant component

- We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$.
- First, we can write $\langle k\rangle$ in terms of $S_{1}$ :

$$
\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}} .
$$

## Structure

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- As $\langle k\rangle \rightarrow 0, s_{1} \rightarrow 0$.


## Giant component

- We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$.
- First, we can write $\langle k\rangle$ in terms of $S_{1}$ :

$$
\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}} .
$$

## Structure

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- As $\langle k\rangle \rightarrow 0, s_{1} \rightarrow 0$.
- As $\langle k\rangle \rightarrow \infty, s_{1} \rightarrow 1$.


## Giant component

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- We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$.
- First, we can write $\langle k\rangle$ in terms of $S_{1}$ :

$$
\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}}
$$

## Structure

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## Largest component

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- As $\langle k\rangle \rightarrow 0, s_{1} \rightarrow 0$.
- As $\langle k\rangle \rightarrow \infty, s_{1} \rightarrow 1$.
- Notice that at $\langle k\rangle=1$, the critical point, $S_{1}=0$.


## Giant component

- As $\langle k\rangle \rightarrow 0, s_{1} \rightarrow 0$.
- As $\langle k\rangle \rightarrow \infty, s_{1} \rightarrow 1$.
- Notice that at $\langle k\rangle=1$, the critical point, $S_{1}=0$.
- Only solvable for $S_{1}>0$ when $\langle k\rangle>1$.

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## Random friends are strange

- As $\langle k\rangle \rightarrow 0, s_{1} \rightarrow 0$.
- As $\langle k\rangle \rightarrow \infty, s_{1} \rightarrow 1$.
- Notice that at $\langle k\rangle=1$, the critical point, $S_{1}=0$.
- Only solvable for $S_{1}>0$ when $\langle k\rangle>1$.
- Really a transcritical bifurcation. ${ }^{[2]}$

$$
\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}}
$$



## Giant component

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## Giant component

## Turns out we were lucky．．．

－Our dirty trick only works for ER random networks．

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## Giant component

Turns out we were lucky．．．
－Our dirty trick only works for ER random networks．
－The problem：We assumed that neighbors have the same probability $\delta$ of belonging to the largest component．

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## Giant component

## Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability $\delta$ of belonging to the largest component.
- But we know our friends are different from us...

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## Giant component

## Turns out we were lucky．．．

－Our dirty trick only works for ER random networks．
－The problem：We assumed that neighbors have the same probability $\delta$ of belonging to the largest component．
－But we know our friends are different from us．．．
－Works for ER random networks because $\langle k\rangle=\langle k\rangle_{R}$ ．


## Giant component

## Turns out we were lucky．．．

－Our dirty trick only works for ER random networks．
－The problem：We assumed that neighbors have the same probability $\delta$ of belonging to the largest component．
－But we know our friends are different from us．．．
－Works for ER random networks because $\langle k\rangle=\langle k\rangle_{R}$ ．
－We need a separate probability $\delta^{\prime}$ for the chance that an edge leads to the giant（infinite）component．


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## Giant component

Turns out we were lucky．．．
－Our dirty trick only works for ER random networks．
－The problem：We assumed that neighbors have the same probability $\delta$ of belonging to the largest component．
－But we know our friends are different from us．．．
－Works for ER random networks because $\langle k\rangle=\langle k\rangle_{R}$ ．
－We need a separate probability $\delta^{\prime}$ for the chance that an edge leads to the giant（infinite）component．
－We can sort many things out with sensible probabilistic arguments．．．

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## Giant component

Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability $\delta$ of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because $\langle k\rangle=\langle k\rangle_{R}$.
- We need a separate probability $\delta^{\prime}$ for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a spot of Generatingfunctionology. ${ }^{[3]}$

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[2] S. H. Strogatz.
Nonlinear Dynamics and Chaos.
Addison Wesley, Reading, Massachusetts, 1994.
[3] H. S. Wilf.
Generatingfunctionology.
A K Peters, Natick, MA, 3rd edition, 2006. pdf ( $\boxplus$ )

