Overview

Overview of Complex Networks

Complex Networks CSYS/MATH 303, Spring, 2011

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Office hours:

- 1:00 pm to 3:00 pm, Wednesday;
 Farrell Hall, second floor, Trinity Campus.
- Appointments by email (peter.dodds@uvm.edu).

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- Projects
- Assignments (about 8)
- Assignment 1 appears today and involves:
- dolphins
- a Karate club

- political blogs
- a worm's brain

- the Internet
- jazz musicians

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Three versions (all in pdf):

- 1. Presentation,
- 2. Flat Presentation,
- 3. Handout (2x2).

Presentation versions are navigable and hyperlink are clickable.

► Web links look like this (⊞)

 Hererences in slides link to full citation at end. ⁽⁴⁾
 Citations contain links to papers in pdf (if available
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Bonus materials:

Textbooks:

- Mark Newman (Physics, Michigan) "Networks: An Introduction" (III)
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
 "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" (
)

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Bonus materials:

Review articles:

- S. Boccaletti et al.
 "Complex networks: structure and dynamics" ^[5] Times cited: 1,028 (as of June 7, 2010)
- M. Newman
 "The structure and function of complex networks" ^[16] Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
 "Statistical mechanics of complex networks"^[1] Times cited: 3,995 (as of June 7, 2010)

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Complex: (Latin = with + fold/weave (com + plex)) Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Complex System—Some ingredients:

- No contralized control
 Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- Emergence 'More is Different' [2]
- Many pitenomena can be complex: social, techn informational, geophysical, meteorological, fluidir



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Complex System—Some ingredients:

Distributed system of many interrelated parts

Nonlinear relationships

Existence of feedback loops

- Complex systems are open (out of equilibrium)
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Modular (nested)/multiscale structure

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net•work |'net,wərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

• a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : a trade network.

• a group of people who exchange information, contacts, and experience for professional or social purposes *: a support network.*

• a group of broadcasting stations that connect for the simultaneous broadcast of a program : the introduction of a second TV network | [as adj.] network television.

• a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*

• a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : the stock exchanges have proven to be resourceful in networking these deals.

• link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) networked workstations.

• [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Thesaurus deliciousness:

network

noun

 a network of arteries WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
 a network of lanes MAZE, labyrinth, warren, tangle.
 a network of friends SYSTEM, complex, nexus, web, webwork. Overview

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From Keith Briggs's excellent etymological investigation: (⊞)

Opus reticulatum: A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]



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First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869-: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

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Net and Work are venerable old words:

'Net' first used to mean spider web (King Ælfréd, 888).
 'Work' appears to have long meant purposeful action.





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References

 Network = something built based on the idea of rightral_flexible bittce or web.
 cit; ironwork, stonework, fretwork.



Net and Work are venerable old words:

'Net' first used to mean spider web (King Ælfréd, 888).
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'Network' = something built based on the idea of natural, flexible lattice or web.



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- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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- Many complex systems can be viewed as complex networks of physical or abstract interactions.
 - Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on comp interworks since 1998...
 - ... largely due to your typical theoretical physicis

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- Many complex systems can be viewed as complex networks of physical or abstract interactions.
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Piranha physicus

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- Piranha physicus
- Hunt in packs.

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- Piranha physicus
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

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Popularity (according to ISI)

"Collective dynamics of 'small-world' networks" [23]

- Watts and Strogatz Nature, 1998
- 4677 citations (as of January 18, 2011)
- Over 1100 citations in 2008 alone.

"Emergence of scaling in random networks" [3]

- Barabási and Albert Science, 1999
- Over 1100 citations in 2008 alone.

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Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell^[11]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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Popularity according to books:

Haw Everything In Connected to Exceptions Else and What Is Means for Datasets, Science, and Everyday Life



Albert-László Barabási

Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts^[21]

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Numerous others:

- Complex Social Networks—F. Vega-Redondo^[20]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo^[17]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster^[7]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes^[10]

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But surely networks aren't new...

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- But surely networks aren't new...Graph theory is well established...

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- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...

Answer: Oodles of Easily Accessible Data.
We can now inform (alas) our theories with a much more measurable reality."
Real networks occupy a tiny, low entropy part of a network space and require specific attention.
A worthy goal, establish mechanistic explanations
What kinds of dynamics lead to these real network

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- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
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- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*

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mechanistic explanations

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- What kinds of dynamics lead to these real networks?

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- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- Real networks occupy a tiny, low entropy part of all network space and require specific attention.
- A worthy goal: establish mechanistic explanations.
- What kinds of dynamics lead to these real networks? *If this is upsetting, maybe string theory is for you...

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Web-scale data sets can be overly exciting.

Witness:

The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (I)

- "The Unreasonable Effectiveness of Data," Halevy et al.^[12]
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [24]

For scientists, description is only part of the battle
 We still need to understand.

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Web-scale data sets can be overly exciting.

Witness:

► The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)

For scientists, description is only part of the battle
 We still need to understand.

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- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [24]

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Super basic definitions

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Nodes = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages organisms...

Super basic definitions

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 e.g., people, forks in rivers, proteins, webpages, organisms,...



Basic definitions:

Links = Connections between nodes

links

- may be real and fixed (rivers).
- real and dynamic (airline routes)
- abstract with physical impact (hyperlinks)
- or purely abstract (semantic connections between concepts).

Linksmay be directed or undirected

Links may be binary or weighted.

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Links = Connections between nodes links

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Node degree = Number of links per node

Notation: the average degree of a network = $\langle k \rangle$

For undirected networks, connection between number of edges *m* and average degree:



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$$\langle k \rangle = \frac{2m}{N}$$



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$$\langle k_{\rm out} \rangle = \langle k_{\rm in} \rangle = \frac{m}{N}$$

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For directed networks,

$$\langle k_{\rm out} \rangle = \langle k_{\rm in} \rangle = \frac{m}{N}$$

• Defn: N_i = the set of *i*'s k_i neighbors

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Adjacency matrix:

We represent a graph or network by a matrix A with link weight a_{ii} for nodes i and j in entry (i, j).

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▶ e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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 (n.b., for numerical work, we always use sparse matrices.)

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What passes for a complex network?

- Complex networks are large (in node number)
 Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
 Complex networks can be social, economic, natural informational, abstract.

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Physical networks

River networks

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 Road networks
 Power grids



Distribution (branching) versus redistribution

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Distribution (branching) versus redistribution

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- Neural networks
- Trees and leaves







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(cyclical)



Distribution (branching) versus redistribution



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Interaction networks

- The Blogosphere
 - Gene-protein networks
- Food webs: who eats whom
 - The World Wide

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Interaction networks

- The Blogosphere
- Biochemical networks

eats whom

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 Web (?)



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- Food webs: who eats whom
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- Airline networks



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- Gene-protein networks
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- The World Wide
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- Call networks (AT&T)



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- The Blogosphere
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- Food webs: who eats whom
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- Airline networks
- Call networks (AT&T)
- The Media



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Interaction networks: social networks

Snogging

- Boards and directors
- Organizations
- twitter.com (⊞) facebook.com (⊞

The Structure of Romantic and Scicial Relations at "Jefferson High School"

Each circle represents a student and lines connecting students represent remarks relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pars unconnected to anyone else).

(Bearman et al., 2004)

emotely sensed by: tweets (open), instant essaging, Facebook posts, emails, phone log cough*) Overview

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Relational networks

- Consumer purchases
- Thesauri: Networks of words generated by meanin
 Knowledge/Databases/Ideas
- ► Metadata—Tagging: delicious (⊞) flickr (⊞)

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Relational networks

- Consumer purchases (Wal-Mart: ≈ 2.5 petabyte = 2.5×10^{15} bytes) (\boxplus)
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 - -Tagging: delicious (\boxplus) , flickr (\boxplus)

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Clickworthy Science:



Bollen et al.^[6]; a higher resolution figure is here (\boxplus)

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And even when renderings somehow look good

need to extract digestible, meaningful aspect



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Graphical renderings are often just a big mess.

need to extract digestible, meaningful aspect



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Graphical renderings are often just a big mess.



- ← Typical hairball
- number of nodes N = 500
- number of edges m = 1000
- average degree $\langle k \rangle = 4$

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extract digestible, meaningful aspects

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We need to extract digestible, meaningful aspects.

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Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity

- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness
- Plus coevolution of network structure and processes on networks.
- Degree distribution is the elephant in the room that we are now all very aware of...

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1. degree distribution P_k

 P_{k} is the probability that a randomly selected not has degree k

ex 1: Erdős-Rényi random networks

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1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- k = node degree = number of connections

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- ex 1: Erdős-Rényi random networks:

$$P_k = e^{-\langle k
angle} \langle k
angle^k / k!$$

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 $P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$

Distribution is Poisson

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- 1. degree distribution P_k
 - ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
 - k cost controls skew
 - hubs may facilitate or impede contagion

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- 1. degree distribution P_k
 - ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
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Note:

Erdős-Rényi random networks are a mathematical construct.

 Scale-free networks are growing networks that to according to a plausible mechanism.
 Randomness is out there, just not to the degree or Overview

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Note:

- Erdős-Rényi random networks are a mathematical construct.
- 'Scale-free' networks are growing networks that form according to a plausible mechanism.



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Nutshell



Note:

- Erdős-Rényi random networks are a mathematical construct.
- 'Scale-free' networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.

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Nutshell



2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
 - measure degree-degree correlations.
 Assortative network: [15] similar degree nodes tconnecting to each other.

Disassortative network: high degree nodes connecting to low degree nodes.

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2. Assortativity/3. Homophily:

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2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
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- Assortative network: ^[15] similar degree nodes connecting to each other.
 Often social: company directors, coauthors, actors.
- Disassortative network: high degree nodes connecting to low degree nodes. Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

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Nutshell



Local socialness:

C

d

4. Clustering:

ib

 Your friends tend to know each other
 Two measures (explained following slides): 1. Watts & Strogatz ^[23]

 $C_{1} = \left\langle rac{\sum_{j_{1}j_{2} \in \mathcal{N}_{i}} a_{j_{1}j_{2}}}{k_{i}(k_{i}-1)/2}
ight
angle$

2. Newman^[16] $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$

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- Two measures (explained on following slides):
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Example network:



Calculation of C_1 :



C₁ is the average fraction o bairs of neighbors who are connected

Fraction of pairs of neig who are connected is

 $K_1(K_1 - 1)/2$ there K_1 is node *i* is degree and M_1 is the set of *i* is

Averaging over all nodes, v

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Example network:

Calculation of C_1 :



 C₁ is the average fraction of pairs of neighbors who are connected.

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Averaging over all nodes, v

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Example network:



Calculation of C_1 :



- C₁ is the average fraction of pairs of neighbors who are connected.
- Fraction of pairs of neighbors who are connected is

 $\frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2}$

where k_i is node *i*'s degree, and \mathcal{N}_i is the set of *i*'s neighbors. Overview

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Example network:



Calculation of C_1 :



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where k_i is node *i*'s degree, and \mathcal{N}_i is the set of *i*'s neighbors.

Averaging over all nodes, we have:

$$C_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2}$$

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Example network:



Calculation of C_1 :



- C₁ is the average fraction of pairs of neighbors who are connected.
- Fraction of pairs of neighbors who are connected is

 $\frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2}$

where k_i is node *i*'s degree, and \mathcal{N}_i is the set of *i*'s neighbors.

Averaging over all nodes, we have:

$$C_{1} = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_{1}j_{2} \in \mathcal{N}_{i}} a_{j_{1}j_{2}}}{k_{i}(k_{i}-1)/2} = \left\langle \frac{\sum_{j_{1}j_{2} \in \mathcal{N}_{i}} a_{j_{1}j_{2}}}{k_{i}(k_{i}-1)/2} \right\rangle_{i}$$

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Example network:



Triangles:



Triples:



Nodes 4, 12, and 13 form a triangle if each pair of nodes is connected
 The definition C₂ = 3×#triangles/#triples/measures the fraction of closed triples

The '3' appears because for each triangle, we have 3 close trinles

Social Network Analysis (SN traction of transitive triples.

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Example network:



Triangles:





Nodes i₁, i₂, and i₃ form a triple around i₁ if i₁ is connected to i₂ and i₃.

 Nodes i₁, i₂, and i₃ form a triangle if each pair of nodes is connected

measures the fraction of closed triples The '3' appears because for

Social Network Analysis (S) traction of transitive triples.

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Example network:



Triangles:



Triples:

Nodes i₁, i₂, and i₃ form a triple around i₁ if i₁ is connected to i₂ and i₃.

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Example network:



Triangles:



Triples:

- Nodes i₁, i₂, and i₃ form a triple around i₁ if i₁ is connected to i₂ and i₃.
- Nodes i₁, i₂, and i₃ form a triangle if each pair of nodes is connected
- The definition C₂ = ^{3×#triangles}/_{#triples} measures the fraction of closed triples
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Example network:



Triangles:



Triples:

- Nodes i₁, i₂, and i₃ form a triple around i₁ if i₁ is connected to i₂ and i₃.
- Nodes i₁, i₂, and i₃ form a triangle if each pair of nodes is connected
- The definition C₂ = ^{3×#triangles}/_{#triples} measures the fraction of closed triples
- The '3' appears because for each triangle, we have 3 closed triples.
- Social Network Analysis (SNA): fraction of transitive triples.

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For sparse networks, C₁ tends to discount highly connected nodes.

In general, C₁ ≠ C₂.
 C₁ is a global average of a local ratio.

is a ratio of two global quantities

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For sparse networks, C₁ tends to discount highly connected nodes.

C₂ is a useful and often preferred variant

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- C₂ is a useful and often preferred variant
- ▶ In general, $C_1 \neq C_2$.



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For sparse networks, C₁ tends to discount highly connected nodes.

- C₂ is a useful and often preferred variant
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- For sparse networks, C₁ tends to discount highly connected nodes.
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- C_2 is a ratio of two global quantities.

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5. motifs:

small, recurring functional subnetworks
 e.g., Feed Forward Loop;

Shen-Orr, Uri Alon, *et al.* ^{[18}

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5. motifs:

small, recurring functional subnetworks

Shen-Orr, Oki Alon, *et al*. ^{[18}

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5. motifs:

- small, recurring functional subnetworks
- e.g., Feed Forward Loop:



Shen-Orr, Uri Alon, et al. [18]

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6. modularity and structure/community detection:



Clauset et al., 2006 [9]: NCAA football

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7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple mode
- dynamic property—static networks are not enou
- knowledge of previous contacts crucia
- beware cumulated network data
- Kretzschmar and Morris, 1996^[14]

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8. Horton-Strahler ratios:Metrics for branching networks:

- Number:
- Segment length:
- Area/Volume



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8. Horton-Strahler ratios:

Metrics for branching networks:

Method for ordering streams hierarchically



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8. Horton-Strahler ratios:

Metrics for branching networks:

- Method for ordering streams hierarchically
- Number: $R_n = N_\omega / N_{\omega+1}$



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8. Horton-Strahler ratios:

Metrics for branching networks:

- Method for ordering streams hierarchically
- Number: $R_n = N_{\omega}/N_{\omega+1}$
- Segment length: $R_I = \langle I_{\omega+1} \rangle / \langle I_{\omega} \rangle$



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8. Horton-Strahler ratios:

Metrics for branching networks:

- Method for ordering streams hierarchically
- Number: $R_n = N_{\omega}/N_{\omega+1}$
- Segment length: $R_I = \langle I_{\omega+1} \rangle / \langle I_{\omega} \rangle$
- Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_{\omega} \rangle$



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9. network distances:

(a) shortest path length d_{ii}

Fewest number of steps between nodes *i* and *j*.
 (Also called the chemical distance between *i* and

b) average path length $\langle d_{ij} angle$

Average shortest path length in whole network
 Good algorithms exist for calculation.
 Weighted links can be accommodated.

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9. network distances:(a) shortest path length *d_{ij}*:

Fewest number of steps between nodes i and j.

b) average path length $\langle {m d}_{ij} angle$

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Average shortest path length in whole n Good algorithms exist for calculation Weighted links can be accommodated Overview

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9. network distances:

- network diameter d_{max}: Maximum shortest path length between any two nodes.
 - closeness $d_{
 m cl} = [\sum_{ij} d_{jj}^{-1} / {n \choose 2}]^{-1}$:
 - Gloseness handles disconnected networks (c
 - couly when all nodes are isolated.
 - whaps compresses too much into o

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9. network distances:

- network diameter d_{max}: Maximum shortest path length between any two nodes.
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9. network distances:

- network diameter d_{max}: Maximum shortest path length between any two nodes.
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- Closeness handles disconnected networks ($d_{ij} = \infty$)

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9. network distances:

- network diameter d_{max}: Maximum shortest path length between any two nodes.
- ► closeness $d_{cl} = [\sum_{ij} d_{ij}^{-1} / {n \choose 2}]^{-1}$: Average 'distance' between any two nodes.
- Closeness handles disconnected networks ($d_{ij} = \infty$)
- $d_{cl} = \infty$ only when all nodes are isolated.
- Closeness perhaps compresses too much into one number

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Nutshell



10. centrality:

- Many such measures of a node's 'importance.'
- ▶ ex 1: Degree centrality: k,
- ex 2: Node /'s betweenness
- a = fraction of shortest paths that pass through
- ex 3: Edge &s betweenness
 - fraction of shortest paths that travel along
 - ex 4: Recursive centrality: Hubs and Authorities (J Kleinberg [¹³]

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- ex 4: Recursive centrality: Hubs and Authorities (J

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- ex 3: Edge l's betweenness
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10. centrality:

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- ex 3: Edge l's betweenness
 = fraction of shortest paths that travel along l.
- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg^[13])

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Some important models:

generalized random networks (touched on in 30)
 scale-free networks (⊞) (covered in 300)
 small-world networks (⊞) (covered in 300)

statistical generative models (p⁻

5. generalized affiliation networks (partly covered i

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Nutshell



Some important models:

1. generalized random networks (touched on in 300)

- 2. scale-free networks (⊞) (covered in 300)
- 3. small-world networks (\boxplus) (covered in 300)
- 4. statistical generative models (ho^*
- generalized affiliation networks (partly covered i

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Some important models:

- 1. generalized random networks (touched on in 300)
- 2. scale-free networks (\boxplus) (covered in 300)
- 3. small-world networks (\boxplus) (covered in 300)
- statistical generative models (p^{*})
- 5. generalized affiliation networks (partly covered i

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1. generalized random networks:

- Wire nodes together randomly.
 Create ensemble to test deviations fro randomness.
- Interesting, applicable, rich mathematically
 We will have fun with these guys...

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2. 'scale-free networks':

 $\gamma = 2.5$ $\langle k \rangle = 1.8$ N = 150

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- 2. 'scale-free networks':
 - Introduced by Barabasi and Albert^[3]
 - Generative model
 - model with growth:
 P[attachment to node i].
 Produces P_k ~ k^{-γ} when α = 1.
 Trickiness: other models generate skewed degree distributions.

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3. small-world networks

Introduced by Watts and Strogatz^[23]

 local regularity (an individual's friends know eac other)
 olobal randomness (shortcuts)

Shortcuts allow disease to jump
 Number of infectives increases
 exponentially in time
 Facilitates synchronization

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Models 5. generalized affiliation networks



Bipartite affiliation networks: boards and directors, movies and actors.

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Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Hardened up much thinking about complex systems
 Specific locus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic and (crucially) measurable.
 - Physical (e.g., river networks),
 Interactional (e.g., social networks),
 Abstract (e.g., thesauri),

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Overview Key Points (cont.):

- Obvious connections with the vast extant field of graph theory.
 - physics/stat-mech/comp-sci flavo
 - Description: Characterizing very large network
 Explanation: Micro-story Macro-leatures
 - Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overal, structure,... Still much work to be done, especially with respect dynamics, exciting!

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