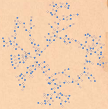
Overview of Complex Networks

Complex Networks
CSYS/MATH 303, Spring, 2011

Prof. Peter Dodds

Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont



Outline

Class admin

Basic definitions

Popularity

Examples of Complex Networks

Properties of Complex Networks

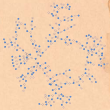
Modelling Complex Networks

Nutshell

References

Class Admin

- Office hours:
  - 1:00 pm to 3:00 pm, Wednesday;
    Farrell Hall, second floor, Trinity Campus.
  - Appointments by email (peter.dodds@uvm.edu).
- Course outline
- Projects
- Assignments (about 8)
- Assignment 1 appears today and involves:
  - dolphins
  - a Karate club
  - political blogs
  - a worm's brain
  - the Internet
  - jazz musicians



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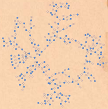

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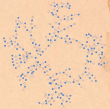

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- Three versions (all in pdf):
  1. Presentation,
  2. Flat Presentation,
  3. Handout (2x2).
- Presentation versions are navigable and hyperlinks are clickable.
- Web links look like this (⊞).
- References in slides link to full citation at end.[2]
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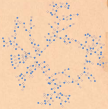

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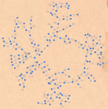

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Bonus materials:

Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction" (⊞)
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" (⊞)

Bonus materials:

Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [5]
  Times cited: 1,028 (as of June 7, 2010)
- M. Newman
  "The structure and function of complex networks" [16]
  Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 3,995 (as of June 7, 2010)

Basic definitions:

Complex: (Latin = with + fold/weave (com + plex))

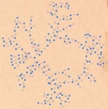
Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Basic definitions:

Complex System—Some ingredients:

- Distributed system of many interrelated parts
- No centralized control
- Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- Emergence—'More is Different' [2]
- Many phenomena can be complex: social, technical, informational, geophysical, meteorological, fluidic, ...



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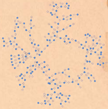

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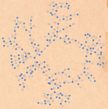

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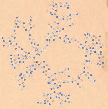

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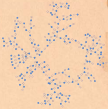

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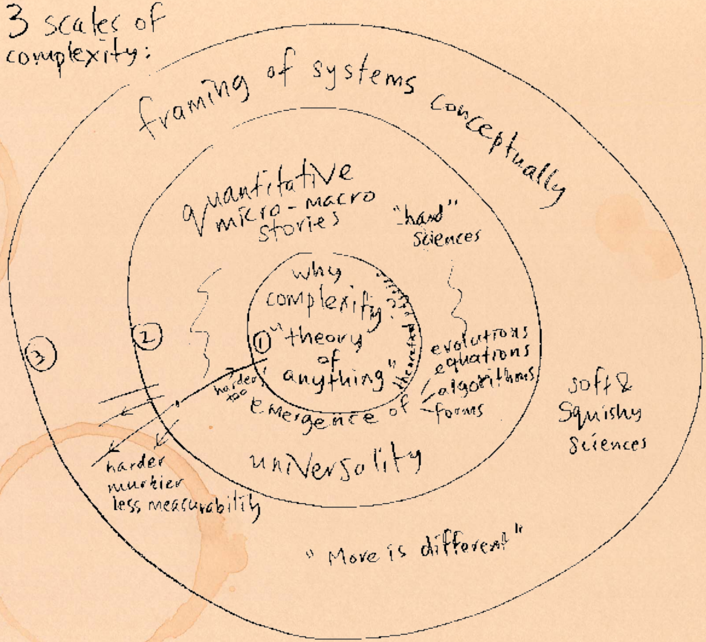

3 scales of complexity:
framing of systems conceptually
quantitative micro-macro stories
"hard" sciences
why complexify?
① "theory of anything"
harder to
emergence of
evolutions
equations
algorithms
forms
②
③
soft & squishy sciences
universality
harder
murkier
less measurability
" More is different "

net•work |ˈnetˌwərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation: (⊞)

- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

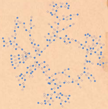
Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks



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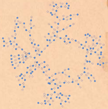

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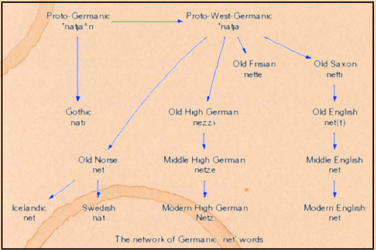
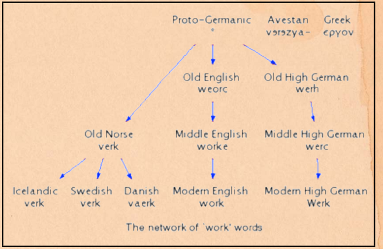
Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appears to have long meant purposeful action.



The network of Germanic 'net' words



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- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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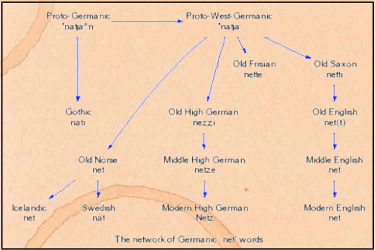

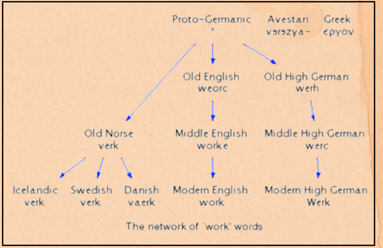

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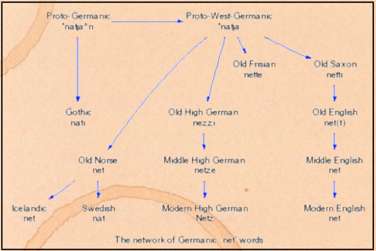

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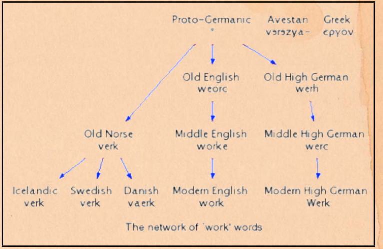

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Key Observation:

- Many complex systems
  can be viewed as complex networks
  of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
- ... largely due to your typical theoretical physicist:

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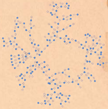

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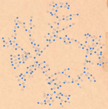

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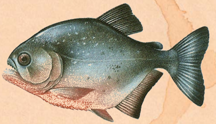

- *Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

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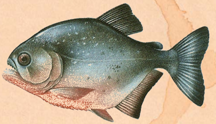

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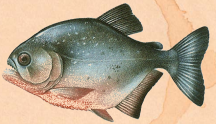

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Popularity (according to ISI)

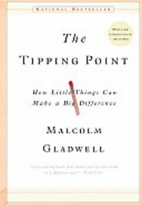
"Collective dynamics of 'small-world' networks" [23]

- Watts and Strogatz
  Nature, 1998
- $\approx$ 4677 citations (as of January 18, 2011)
- Over 1100 citations in 2008 alone.

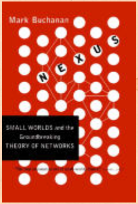
"Emergence of scaling in random networks" [3]

- Barabási and Albert
  Science, 1999
- $\approx$ 5270 citations (as of January 18, 2011)
- Over 1100 citations in 2008 alone.

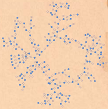
Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [11]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan



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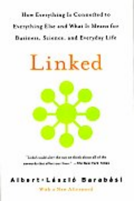
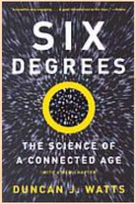


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [21]

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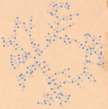
Numerous others:

- Complex Social Networks—F. Vega-Redondo [20]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [17]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [7]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [10]

More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories
  with a much more measurable reality.*
- Real networks occupy a tiny, low entropy part of all network space and require specific attention.
- A worthy goal: establish mechanistic explanations.
- What kinds of dynamics lead to these real networks?



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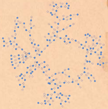

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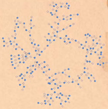

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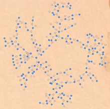

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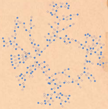

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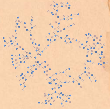

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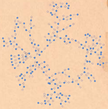

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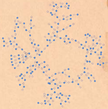
* *If this is upsetting, maybe string theory is for you...*



More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)
- "The Unreasonable Effectiveness of Data," Halevy et al. [12]
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [24]

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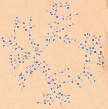
- For scientists, description is only part of the battle.
- We still need to understand.



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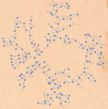

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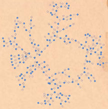

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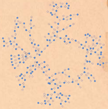

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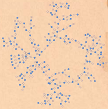
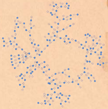


Super basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...



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Links = Connections between nodes

- links
  - may be real and fixed (rivers),
  - real and dynamic (airline routes),
  - abstract with physical impact (hyperlinks),
  - or purely abstract (semantic connections between concepts).
- Links may be directed or undirected.
- Links may be binary or weighted.

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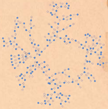

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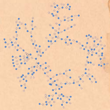

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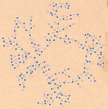
- Notation: Node i's degree $= k_i$.
- $k_i = 0,1,2,\ldots$.
- Notation: the average degree of a network $= \langle k \rangle$
- For undirected networks, connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}$$

- For directed networks,

$$\langle k_{\rm out} \rangle = \langle k_{\rm in} \rangle = \frac{m}{N}$$

- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors



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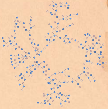

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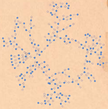

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Adjacency matrix:

- We represent a graph or network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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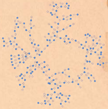

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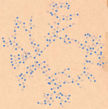

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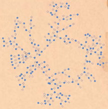

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What passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
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- Complex networks can be social, economic, natural, informational, abstract, ...

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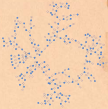

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Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids

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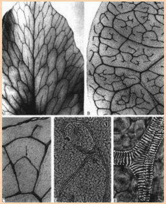

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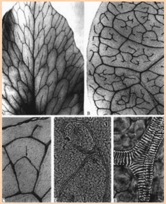

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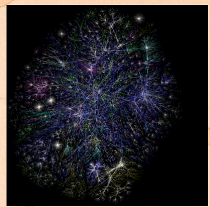

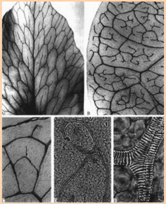

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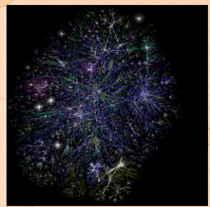

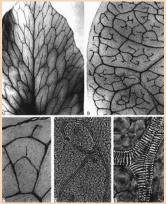

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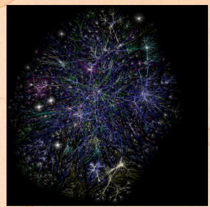

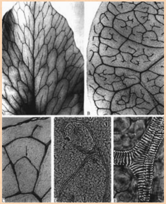

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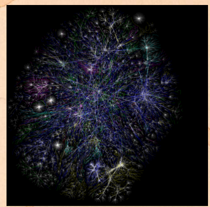

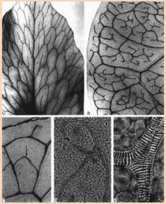

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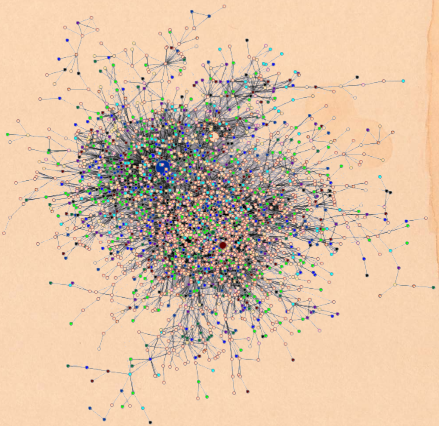

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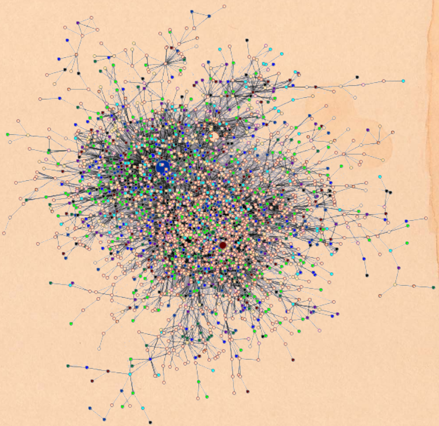

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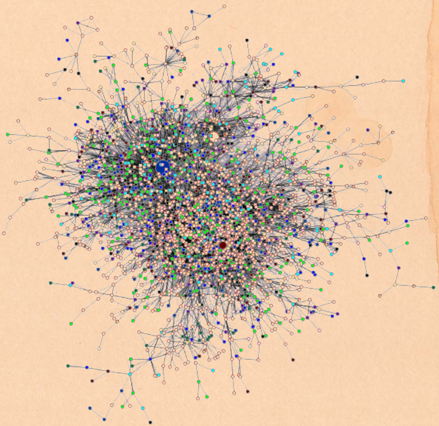

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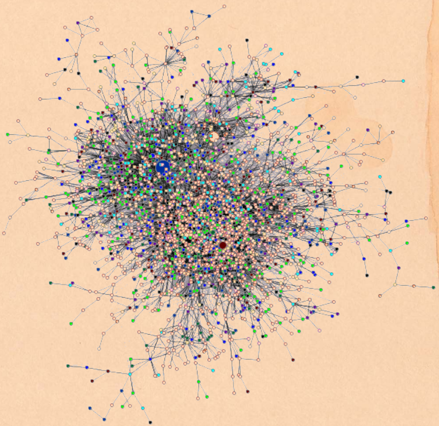

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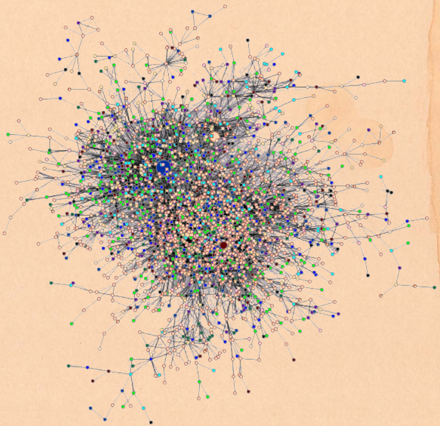

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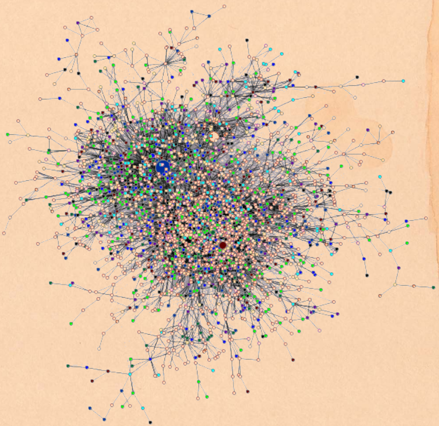

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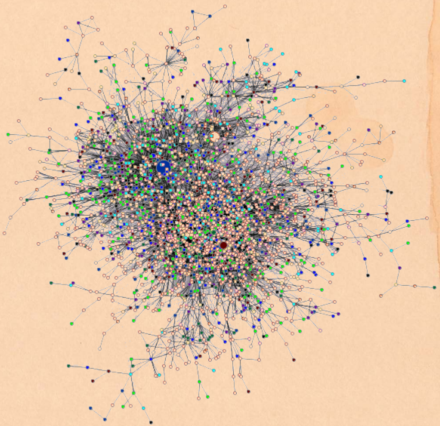

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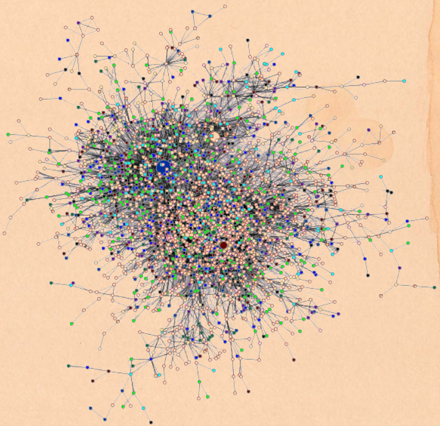

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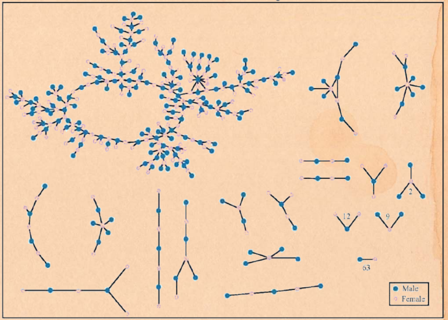

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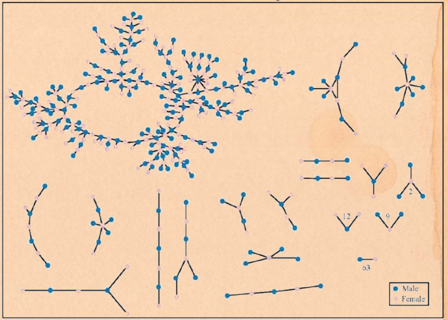

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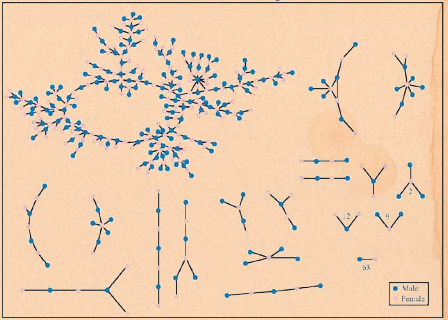

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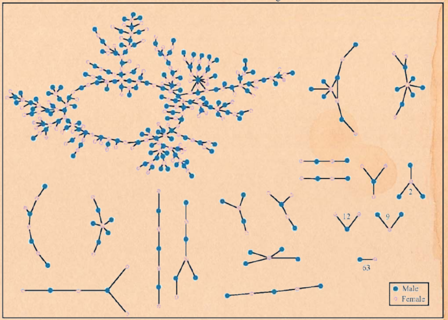

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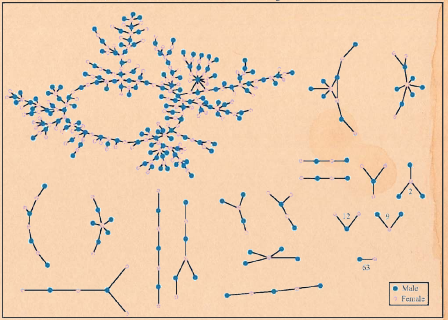

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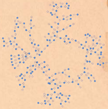

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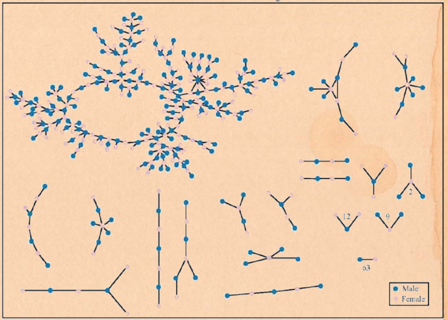

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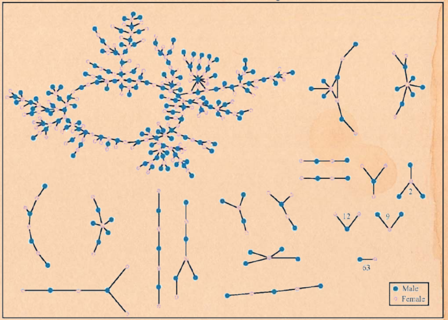

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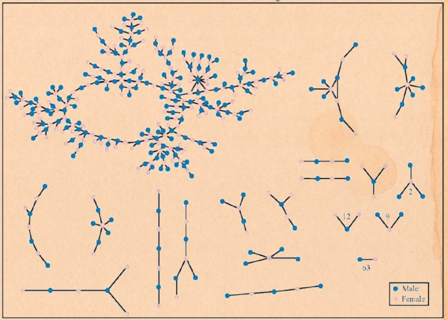

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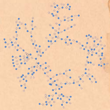

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Relational networks

- Consumer purchases (⊞)
- Thesauri: Networks of words generated by meanings
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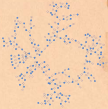

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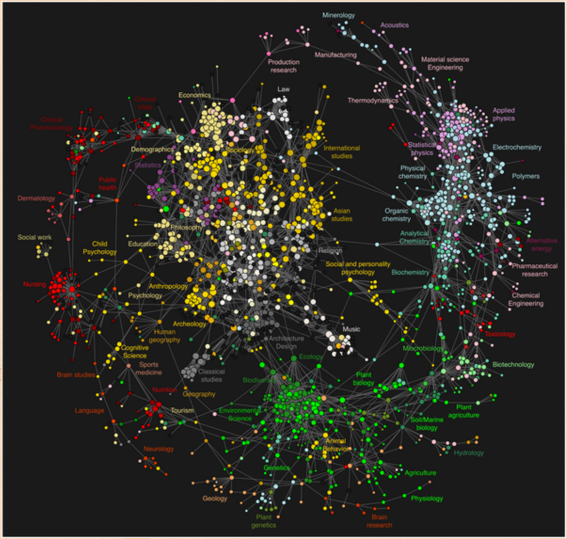
Clickworthy Science:



Bollen et al. [6]; a higher resolution figure is here (⊞)

A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.

- And even when renderings somehow look good:

- We need to extract digestible, meaningful aspects.

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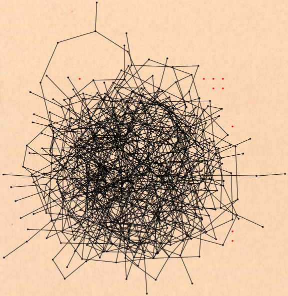

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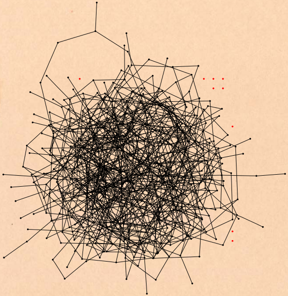

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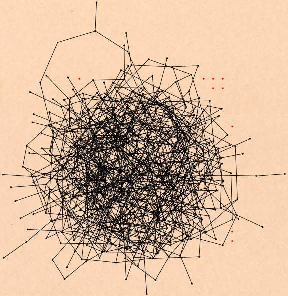

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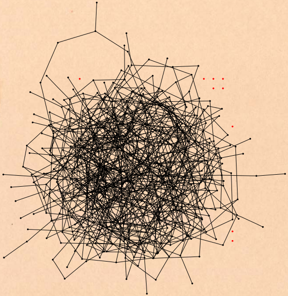

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Properties

Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

- Plus coevolution of network structure and processes on networks.

∗ Degree distribution is the elephant in the room that we are now all very aware of...

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Properties

1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- k = node degree = number of connections
- ex 1: Erdős-Rényi random networks:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

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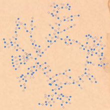

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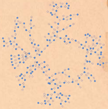

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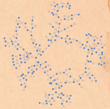

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Note:

- Erdős-Rényi random networks are a *mathematical construct*.
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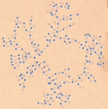

Properties

2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[15] similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.

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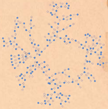

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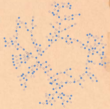

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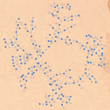

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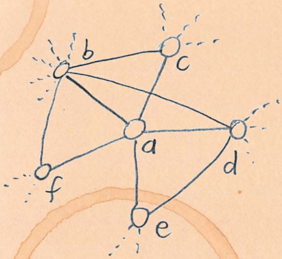
Local socialness:

4. Clustering:



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- Two measures (explained on following slides):
  1. Watts & Strogatz [23]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

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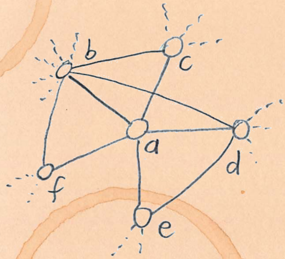

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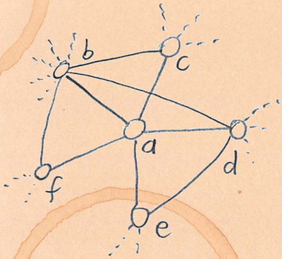

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First clustering measure:

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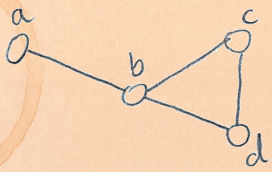

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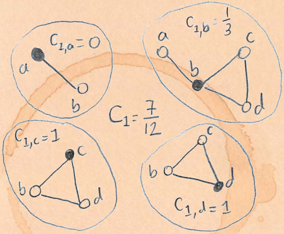

- C_1 is the average fraction of pairs of neighbors who are connected.
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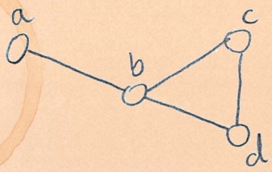

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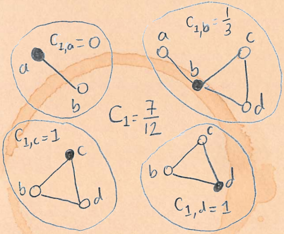

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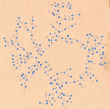

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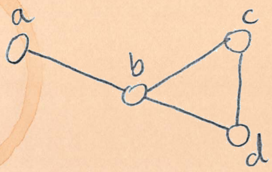

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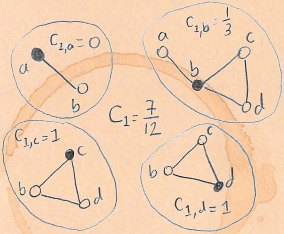

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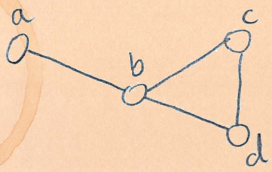

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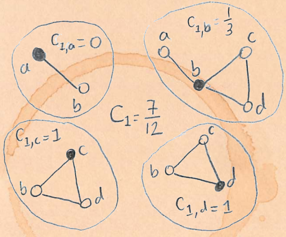

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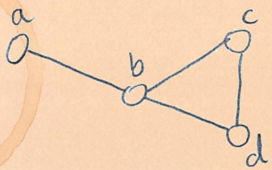

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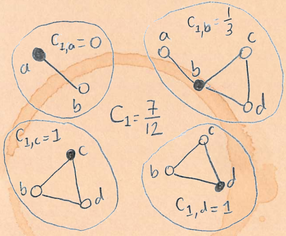

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Triples and triangles

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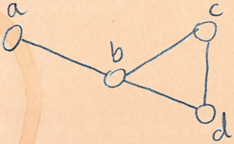

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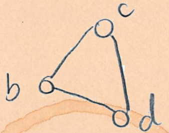

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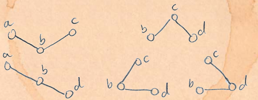

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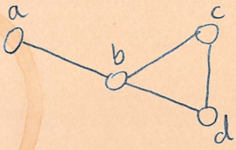

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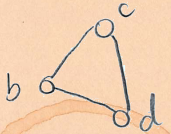

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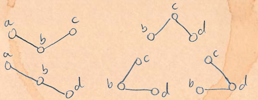

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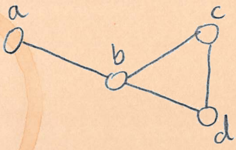

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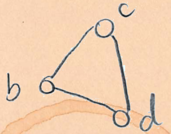

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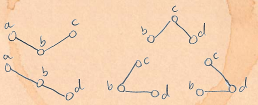

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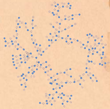

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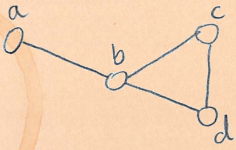

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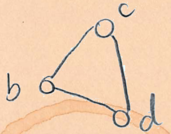

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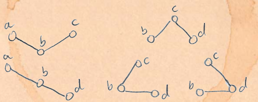

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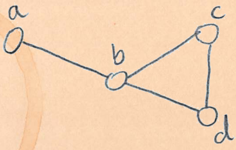

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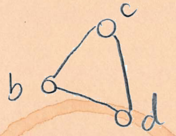

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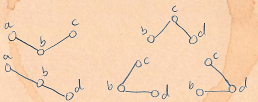

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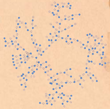

Properties

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- C_1 is a global average of a local ratio.
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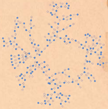

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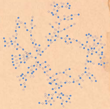

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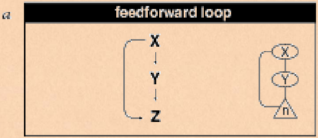

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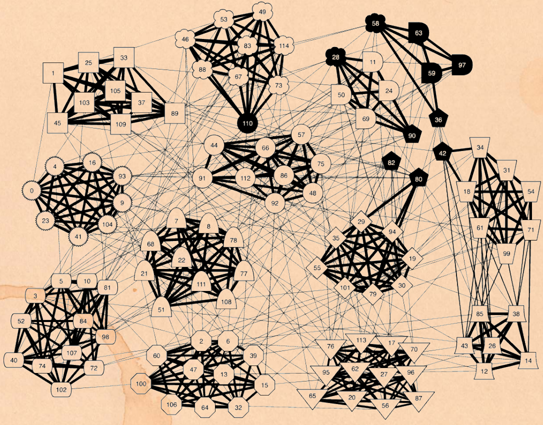
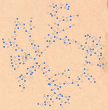
Properties

6. modularity and structure/community detection:



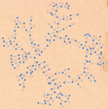
Clauset *et al.*, 2006 [9]: NCAA football



Properties

7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [14]



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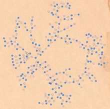

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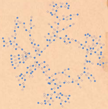

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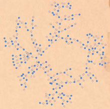

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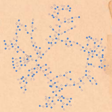

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8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
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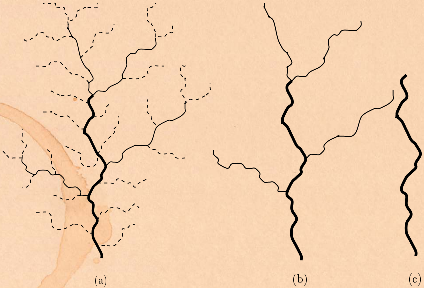

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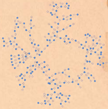

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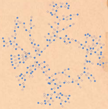

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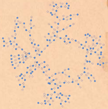

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- closeness $d_{cl} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:
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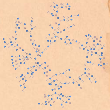

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- Many such measures of a node's 'importance.'
- ex 1: Degree centrality: k_i.
- ex 2: Node i's betweenness
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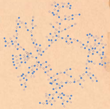

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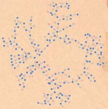

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Some important models:

1. generalized random networks (touched on in 300)
2. scale-free networks (⊞) (covered in 300)
3. small-world networks (⊞) (covered in 300)
4. statistical generative models (p^*)
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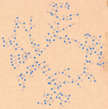

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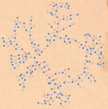

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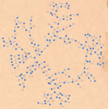

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   - Arbitrary degree distribution P_k.
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   - Interesting, applicable, rich mathematically.
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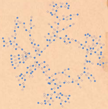

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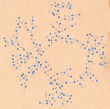

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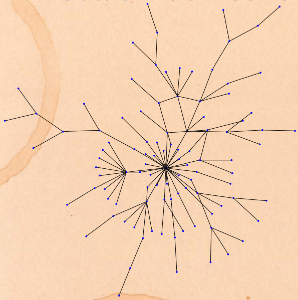

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$\langle k \rangle = 1.8$
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- Generative model
- Preferential attachment model with growth:
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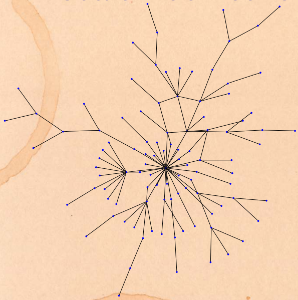

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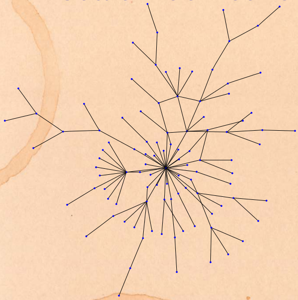

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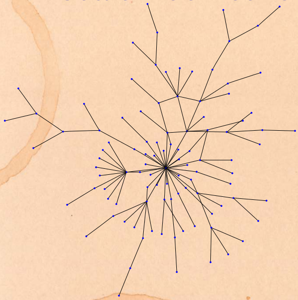

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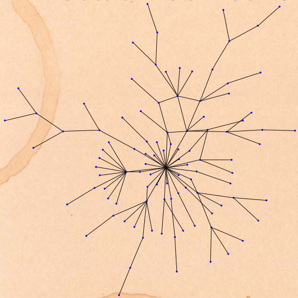

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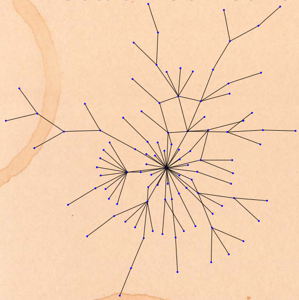

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  - global randomness (shortcuts).
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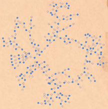

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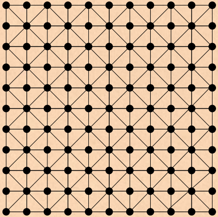

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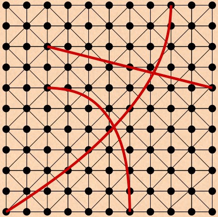

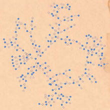

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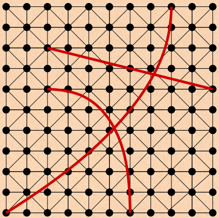

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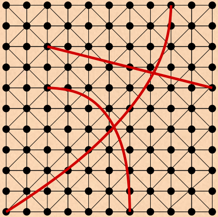

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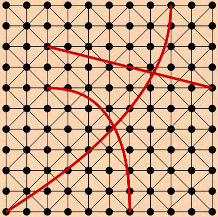
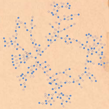
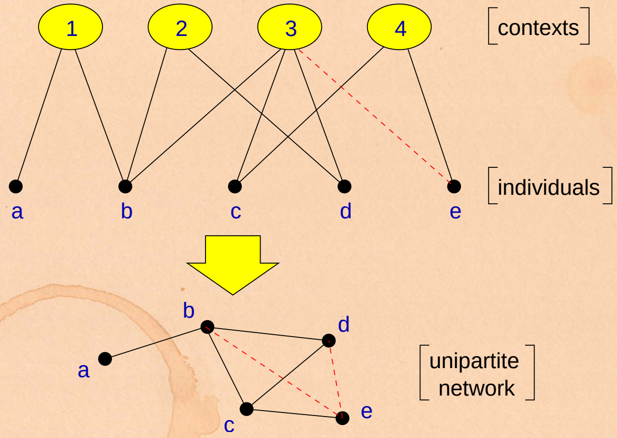


- Shortcuts allow disease to jump
- Number of infectives increases exponentially in time
- Facilitates synchronization



Models

5. generalized affiliation networks



Bipartite affiliation networks: boards and directors, movies and actors.

Models

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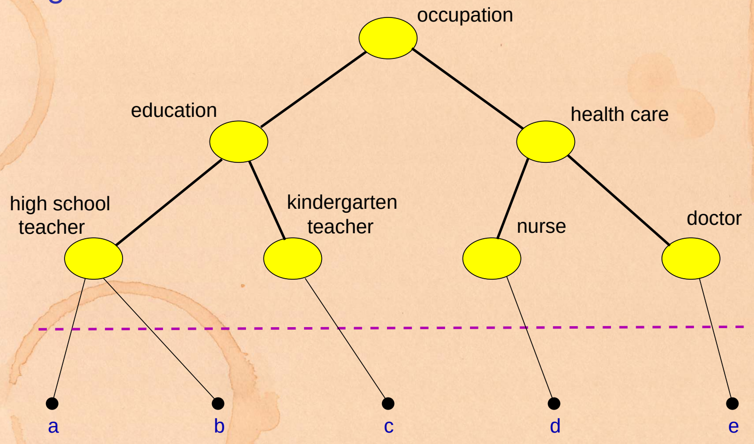

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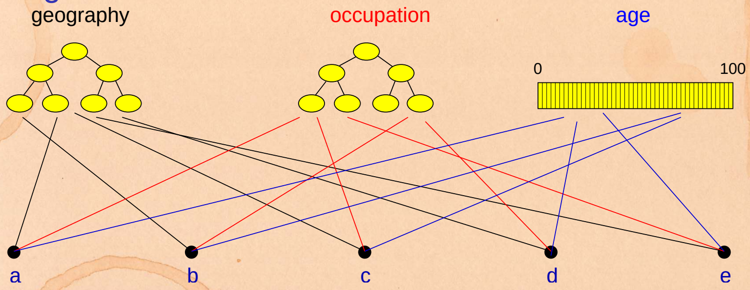

- Blau & Schwartz [4], Simmel [19], Breiger [8], Watts *et al.* [22]

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
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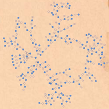

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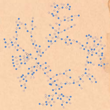

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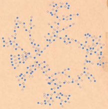

The UNIVERSITY of VERMONT

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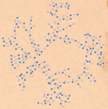

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Overview Key Points (cont.):

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- But focus on dynamics is more of a physics/stat-mech/comp-sci flavor.
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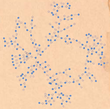

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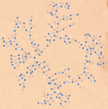

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