Contagion Complex Networks CSYS/MATH 303, Spring, 2011

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Center for Complex Systems
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Contagion

Basic Contagion Models

Global spreading condition

Social Contagion Models

All-to-all networks
Theory







Outline

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Some large questions concerning network contagion:

- For a given spreading mechanism on a given network, what's the probability that there will be global spreading?
- 2. If spreading does take off, how far will it go?
- 3. How do the details of the network affect the outcome?
- 4. How do the details of the spreading mechanism affect the outcome?
- 5. What if the seed is one or many nodes?
 - Next up We'll look at some fundamental kinds of spreading or generalized random networks.

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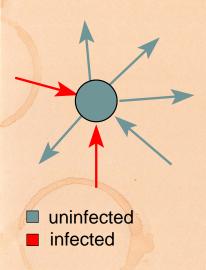
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Spreading mechanisms



 General spreading mechanism:
 State of node *i* depends on history of *i* and *i*'s neighbors' states.

- Doses of entity may be stochastic and
- May have multiple interacting entities spreading at once.

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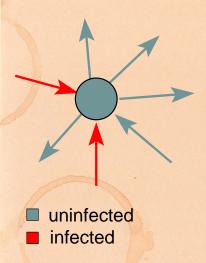
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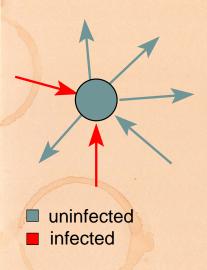
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- For random networks, we know local structure is pure branching.
- Successful spreading is a contingent on single edges infecting nodes.

- Focus on binary case with edges and nodes either infected or not:
- ► First big question: for a given network and contagio process, can global spreading from a single seed

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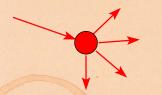


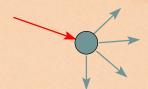


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Failure:





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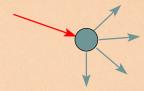


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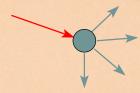


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prob. of connecting to a degree *k* node

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outgoing infected edges

no infection

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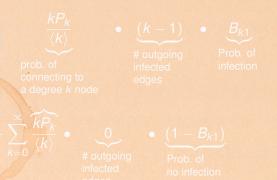
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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle}$$
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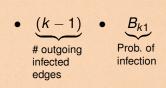


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outgoing

infected

edges

infection

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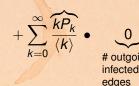






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outgoing infected edges

• $(1 - B_{k1})$ Prob. of no infection

infection

outgoing

infected

edges

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Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

▶ Case 1: If $B_{k1} = 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1$$

▶ Good: This is just our giant component condition again.

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- Analogous phase transition to giant component case but critical value of \(\k \rangle \) is increased.
- ▶ Aka bond percolation (⊞)
- ► Resulting degree distribution P_k

Insert question from assignment $7 (\boxplus)$

▶ We can allow $F_{\mathbb{P}^r}(x) = F_{\mathbb{P}}(\beta x + 1 - \beta)$.

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▶ Case 2: If $B_{k1} = \beta < 1$

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- Possibility: B_{k1} increases with k... unlikely
- Possibility: B_{k1} is not monotonic in k... unlikely
- Possibility: B_{k1} decreases with k... hmmm
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- B_{k1} is a plausible representation of a simple kind of social contagion.
- The story: More well connected people are harder to influence.

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Example: $B_{k1} = 1/k$.

$$\mathbf{B} = \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} = \sum_{k=1}^{\infty} (k-1) \bullet \frac{k P_k}{\langle k \rangle} \bullet \frac{1}{k}$$

$$=\sum_{k=1}^{\infty}\frac{P_k}{\langle k\rangle}\bullet(k-1)=1-\frac{1-P_0}{\langle k\rangle}$$

Since R is always less than 1, no spreading can occur for this mechanism

- \triangleright Decay of B_{k1} is too fast
- Result is independent of degree distribution.

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- ► Example: $B_{k1} = H(\frac{1}{k} \phi)$ where $0 < \phi \le 1$ is a threshold and H is the Heaviside function (\mathbb{H}).
- ► Infection only occurs for nodes with low degree
- Call these nodes vulnerables:
- they flip when only one of their friends flips

$$\mathbf{H} = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet \mathbf{H} \left(\frac{1}{k} - \phi \right)$$

$$=\sum_{k=0}^{\lfloor rac{1}{k}
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 where $\lfloor \cdot
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$$\frac{kP_k}{(k-1)\bullet B_{k1}} = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet H \left(\frac{1}{k} - q\right)$$

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The uniform threshold model global spreading condition:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} (k-1) \bullet \frac{kP_k}{\langle k \rangle} > 1.$$

- ightharpoonup As $\phi
 ightharpoonup$ 1, all nodes become resilient and t
- ightharpoonup As $\phi
 ightharpoonup$ 0, all nodes become vulnerable and the
 - contagion condition matches up with the giant
 - component condition.
 - Key. If we (k) and then vary (k), we may see two
- we will see a cut off in spreading as nodes become

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▶ As $\phi \to 1$, all nodes become resilient and $r \to 0$.

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- As $\phi \to 0$, all nodes become vulnerable and the contagion condition matches up with the giant component condition.
- Key: If we fix ϕ and then vary $\langle k \rangle$, we may see two phase transitions.
- Added to our standard giant component transition, we will see a cut off in spreading as nodes become more connected.

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Social Contagion

Some important models (recap from CSYS 300)

- ► Tipping models—Schelling (1971) [9, 10, 11]
 - Simulation on checker boards.
 - Idea of thresholds
- ▶ Threshold models—Granovetter (1978) [8]
- ► Herding models—Bikhchandani et al. (1992) [1, 2]
 - Social learning theory, Informational cascades,...

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Original work:

"A simple model of global cascades on random networks" D. J. Watts, Proc. Natl. Acad. Sci., 2002 [13]

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- ▶ Mean field Granovetter model → network model
- > Individuals now have a limited view of the world

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- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k, contacts
- Influence on each link is reciprocal and of unit weight
- Each individual / has a fixed threshold.
- Individuals repeatedly poll contacts on network
- Syndhrenous: discrete time updating
- Individual Abecomes active when
- number of active contacts $a_i \ge \phi_i k_i$
- Activation is permanent (SI)

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All nodes have threshold $\phi = 0.2$.

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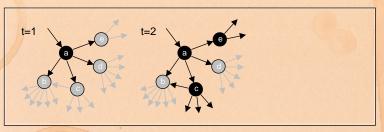
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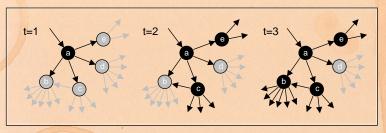
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Vulnerables:

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Vulnerables:

- Recall definition: individuals who can be activated by just one contact being active are vulnerables.
- he vulnerability condition for node i
- Means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$.
- Key: For global spreading events (cascades) on random networks, must have a global component of vulnerables^[13]
- For a uniform threshold φ, our global spreading
 - $\mathbf{R} = \sum_{i=1}^{\lfloor k \rfloor} \frac{k P_k}{i k!} \bullet (k-1) > 1.$

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- For a uniform threshold ϕ , our global spreading condition tells us when such a component exists:

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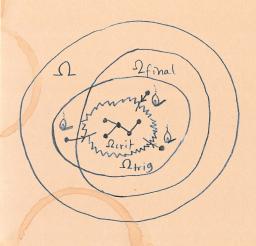
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Example random network structure:



- Ω_{crit} = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

 $\Omega_{crit} \subset \Omega_{trig}$; $\Omega_{crit} \subset \Omega_{final}$; and Ω_{trig} , $\Omega_{final} \subset \Omega$.

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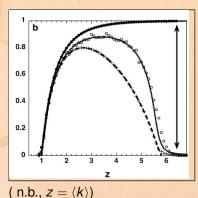
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- ► Top curve: final fraction infected if successful.

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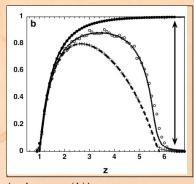












(n.b., $z = \langle k \rangle$)

- ► Top curve: final fraction infected if successful.
- Bottom curve: fractional size of vulnerable subcomponent. [13]

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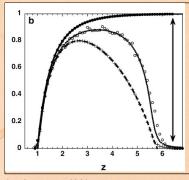












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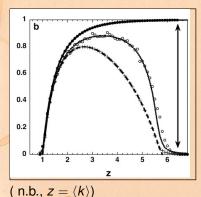
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- Top curve: final fraction infected if successful.
- Middle curve: chance of starting a global spreading event (cascade).
- Bottom curve: fractional size of vulnerable subcomponent. [13]
- Global spreading events occur only if size of vulnerable subcomponent > 0.

➤ System is robust-yet-fragile just below upper

► "Ignorance" facilitates spreading.

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Global spreading condition

Social Contagion Models

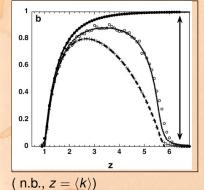
Network version

Theory









- Top curve: final fraction infected if successful.
- Middle curve: chance of starting a global spreading event (cascade).
- Bottom curve: fractional size of vulnerable subcomponent. [13]
- Global spreading events occur only if size of vulnerable subcomponent > 0.
- System is robust-yet-fragile just below upper boundary [3, 4, 12]

Contagion

Basic Contagion Models

Global spreading condition

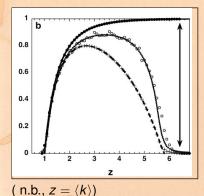
Social Contagion Models

All-to-all networks









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- System is robust-yet-fragile just below upper boundary [3, 4, 12]
- 'Ignorance' facilitates spreading.

Contagion

Basic Contagion Models

Global spreading condition

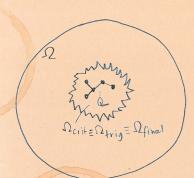
Social Contagion Models

Network version All-to-all network

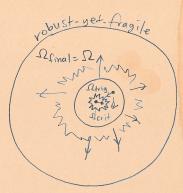








Above lower phase transition



Just below upper phase transition

Contagion

Models

Global spreading

Social Contagion

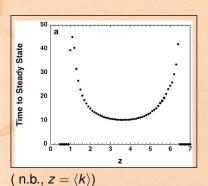
Network version











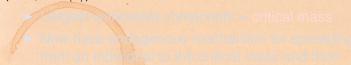
- Time taken for cascade to spread through network. [13]

Contagion

Models

Social Contagion

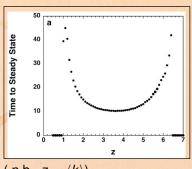
Network version All-to-all networks











- Time taken for cascade to spread through network. [13]
- Two phase transitions.

Contagion

Models

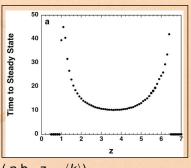
Social Contagion

Network version

$$(n.b., z = \langle k \rangle)$$







- Time taken for cascade to spread through network. [13]
- Two phase transitions.

(n.b.,
$$z = \langle k \rangle$$
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Largest vulnerable component = critical mass.

Contagion

Models

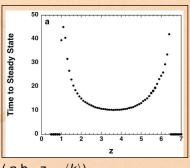
Social Contagion

Network version









- Time taken for cascade to spread through network. [13]
- ► Two phase transitions.

(n.b.,
$$z = \langle k \rangle$$
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- Largest vulnerable component = critical mass.
- Now have endogenous mechanism for spreading from an individual to the critical mass and then beyond.

Contagion

Basic Contagion Models

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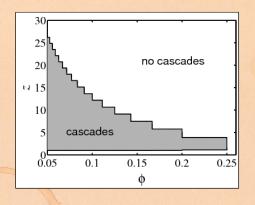
Social Contagion Models

Network version





Cascade window for random networks



(n.b.,
$$z = \langle k \rangle$$
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Outline of cascade window for random networks.

Contagion

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Network version

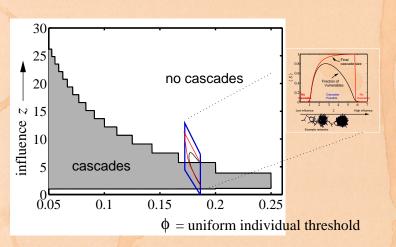
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Cascade window for random networks



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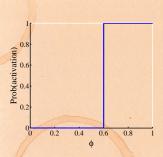
Theory







Granovetter's Threshold model—recap



Assumes deterministic response functions

Contagion

Models

Social Contagion

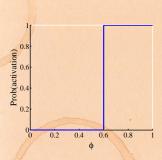
All-to-all networks Theory







Granovetter's Threshold model—recap



- Assumes deterministic response functions
- ϕ_* = threshold of an individual.
- I(φ_n) = distribution of thresholds in a popul
- F(φ_n) = cumulative distribution = Γ^{φ_n} ∘ f(φ)
- \$\overline{\sigma}_t\$ = fraction of people *rioting at time step t.

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All-to-all networks

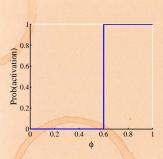
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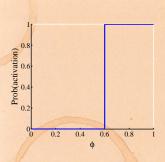
All-to-all networks







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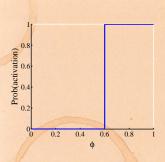
Network version

All-to-all networks





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At time t+1, fraction rioting = fraction with $\phi_* \leq \phi_t$.

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Social Contagion

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Models







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ightharpoonup \Rightarrow Iterative maps of the unit interval [0, 1].

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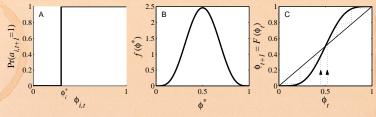
Theory







Action based on perceived behavior of others.



- ► Two states: S and I
- Recover now possible (SIS)
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time, synchronous update (strong accumulation)
- This is a Critical mass model

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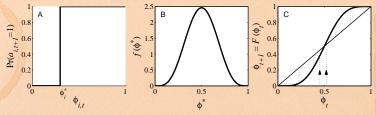
Referen







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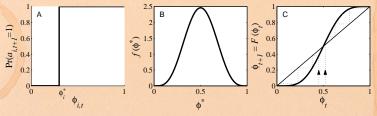
Referer







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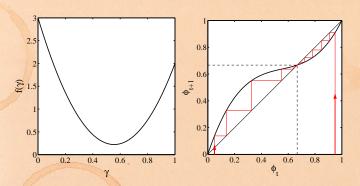
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Example of single stable state model

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Implications for collective action theory:

- Collective uniformity

 individual uniformity
- 2. Small individual changes \Rightarrow large global changes

Next

- Connect mean-field model to network model.
- Single seed for network model: 1/N → 0
- Comparison between network and mean-field model sensible for vanishing seed size for the latter.

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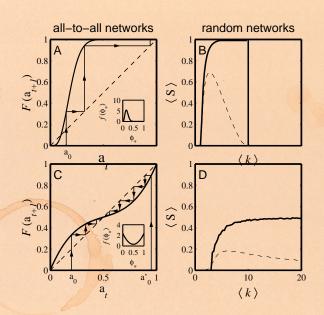
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All-to-all versus random networks



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Three key pieces to describe analytically:

- The fractional size of the largest subcomponent of vulnerable nodes. S_{vuln}
- 2. The chance of starting a global spreading event, $P_{\text{trip}} = S_{\text{trip}}$
- The expected final size of any successful spread. S
 - ightharpoonup n.b., the distribution of S is almost always bimodal.

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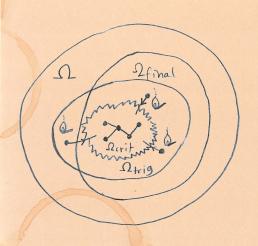
Network version

Theor





Example random network structure:



- $\begin{array}{l} \quad \Omega_{crit} = \Omega_{vuln} = \\ \quad critical \; mass = \\ \quad global \\ \quad vulnerable \\ \quad component \end{array}$
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

 $\Omega_{crit} \subset \Omega_{trig}$; $\Omega_{crit} \subset \Omega_{final}$; and Ω_{trig} , $\Omega_{final} \subset \Omega$.

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First goal: Find the largest component of vulnerable nodes.

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- First goal: Find the largest component of vulnerable nodes.
- Recall that for finding the giant component's size, we had to solve:

$$F_{\pi}(x) = xF_{P}(F_{\rho}(x))$$
 and $F_{\rho}(x) = xF_{R}(F_{\rho}(x))$

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- We'll find a similar result for the subset of nodes that are vulnerable.
- This is a node-based percolation problem.
- For a general monotonic threshold distribution $f(\phi)$, a degree k node is vulnerable with probability

$$B_{k1} = \int_0^{1/k} f(\phi) \mathrm{d}\phi.$$

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 Everything now revolves around the modified generating function:

$$F_P^{(\text{vuln})}(x) = \sum_{k=0}^{\infty} B_{k1} P_k x^k.$$

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Everything now revolves around the modified generating function:

$$F_P^{(\text{vuln})}(x) = \sum_{k=0}^{\infty} B_{k1} P_k x^k.$$

Generating function for friends-of-friends distribution is related in same way as before:

$$F_R^{(\text{vuln})}(x) = \frac{\frac{d}{dx} F_P^{(\text{vuln})}(x)}{\frac{d}{dx} F_P^{(\text{vuln})}(x)|_{x=1}}.$$

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Functional relations for component size g.f.'s are almost the same...

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Functional relations for component size g.f.'s are almost the same...

$$F_{\pi}^{(\text{vuln})}(x) = x F_{P}^{(\text{vuln})} \left(F_{\rho}^{(\text{vuln})}(x) \right)$$

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Functional relations for component size g.f.'s are almost the same...

$$F_{\pi}^{(\text{vuln})}(x) = \underbrace{1 - F_{P}^{(\text{vuln})}(1)}_{\text{central node is not vulnerable}} + x F_{P}^{(\text{vuln})} \left(F_{\rho}^{(\text{vuln})}(x)\right)$$

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► Can now solve as before to find $S_{\text{vuln}} = 1 - F_{\pi}^{(\text{vuln})}(1)$.

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- Second goal: Find probability of triggering largest vulnerable component.

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- Second goal: Find probability of triggering largest vulnerable component.
- Assumption is first node is randomly chosen.

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- Second goal: Find probability of triggering largest vulnerable component.
- Assumption is first node is randomly chosen.
- Same set up as for vulnerable component except now we don't care if the initial node is vulnerable or not:

$$F_{\pi}^{(\mathrm{trig})}(x) = x F_{P}\left(F_{\rho}^{(\mathrm{vuln})}(x)\right)$$

$$F_{\rho}^{(\text{vuln})}(x) = 1 - F_{R}^{\nu)}(1) + x F_{R}^{(\text{vuln})} \left(F_{\rho}^{(\text{vuln})}(x) \right)$$

 \triangleright Solve as before to find $P_{max} = S_{max} = 1 - F_{+}^{(trig)}(1)$.

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Solve as before to find $P_{\text{trig}} = S_{\text{trig}} = 1 - F_{\pi}^{(\text{trig})}(1)$.

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Third goal: Find expected fractional size of spread.

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- Third goal: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- ▶ Difficulty is in figuring out if and when nodes that
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 - Seed size strongly affects cascades on random
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. By 1990 161



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Idea:

▶ Randomly turn on a fraction ϕ_0 of nodes at time t=0

- Now think about what must happen for a specific mode / to become active at time /:
- t = 0: I is one of the seeds (prob =
- t = 1: i was not a seed but enough of i's friends switched on at time t = 0 so that i's threshold is nov
- t=2: enough of its friends and friends-of-friends switched on at time t=0 so that its threshold is no exceeded.
- t = n: enough nodes within n hops of residence on

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Idea:

- ▶ Randomly turn on a fraction ϕ_0 of nodes at time t = 0
- Capitalize on local branching network structure of random networks (again)

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- Now think about what must happen for a specific node *i* to become active at time *t*:

Contagion

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Theory





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- t = n: enough nodes within n hops of i switched on at t = 0 and their effects have propagated to reach i.

Contagion

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Global spreading condition

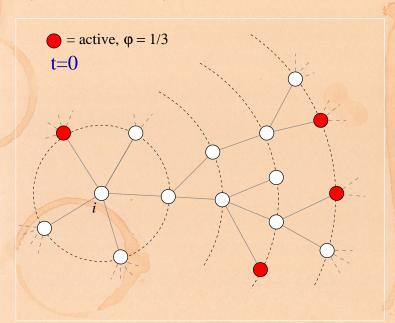
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Network version

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Social Contagion

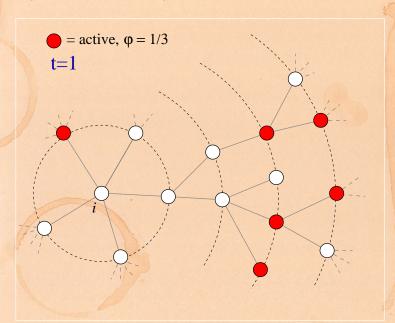
Network version All-to-all networks











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All-to-all networks

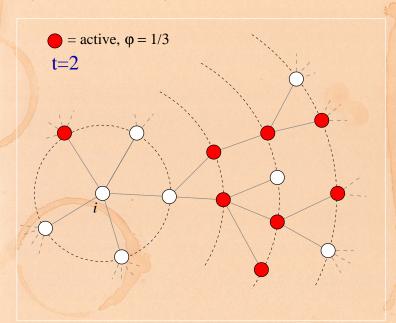
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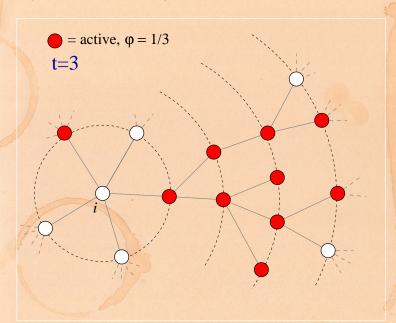
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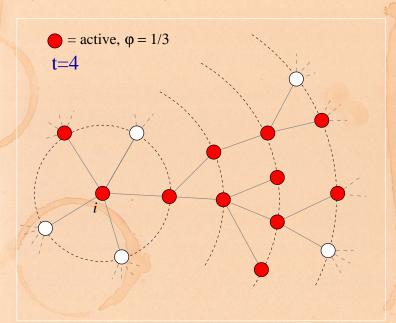
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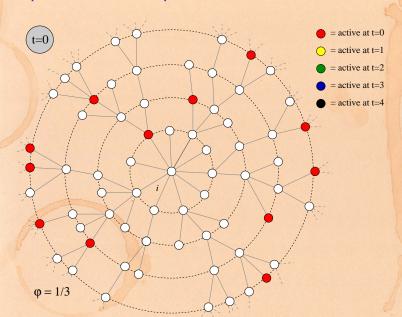
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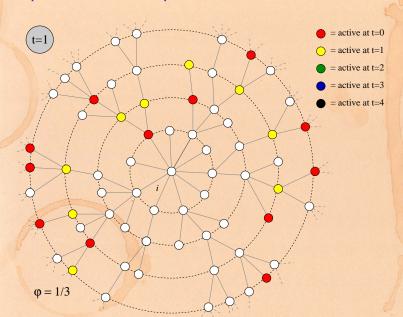
Social Contagion

Network version









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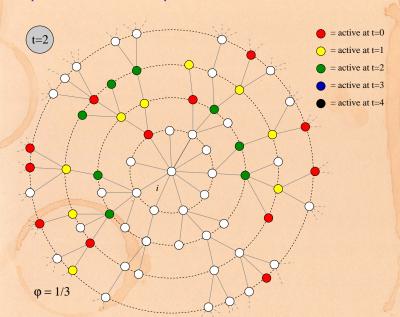
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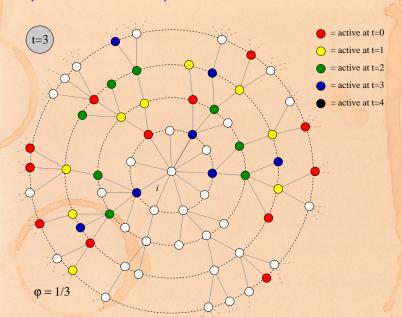
Theory







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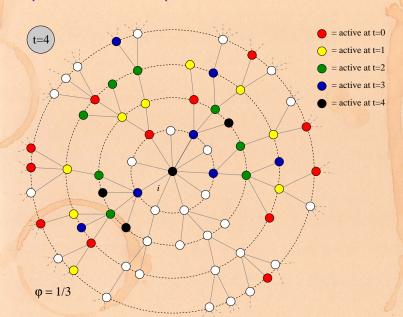
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Notes:

 Calculations are possible nodes do not become inactive (strong restriction).

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Social Contagion









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- ➤ We can analytically determine the entire time evolution, not just the final size.

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- We can in fact determinePr(node of degree k switching on at time t).

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- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determinePr(node of degree k switching on at time t).
- Asynchronous updating can be handled too.

Contagion

Basic Contagion Models

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Theory





Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a mode.

Contagion

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Network version

Theory







Notation:

$$\phi_{k,t} = \mathbf{Pr}(a \text{ degree } k \text{ node is active at time } t).$$

Our starting point:
$$\phi_{k,\alpha}$$

$$\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j} = \Pr(j \text{ of a degree } k \text{ node's})$$

Contagion

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► Notation:

 $\phi_{k,t} = \mathbf{Pr}(\text{a degree } k \text{ node is active at time } t).$

Notation: $B_{kj} = \mathbf{Pr}$ (a degree k node becomes active if j neighbors are active).

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Contagion

Basic Contagion Models

Global spreading condition

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- Probability a degree k node was a seed at t = 0 is ϕ_0 (as above).
- Probability a degree k node was not a seed at t = 0 is $(1 \phi_0)$.
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k {k \choose j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}.$$

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Basic Contagion Models

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Social Contagion Models

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Theory





- ► For general *t*, we need to know the probability an edge coming into a degree *k* node at time *t* is active.
 - Notation: call this probability
- ightharpoonup We already know $\theta_0 = \phi_0$.
 - Story analogous to t = 1 case:

$$\phi_{k,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{m} {k_i \choose j} \theta_t^j (1 - \theta_t)^{k_j - j} B_{k_0}$$

 $\phi_0 = \phi_0 + (1 - \phi_0) \sum_{k=1}^{K} P_k \sum_{j=1}^{K} {k \choose j} \theta_t^j (1 - \theta_t)^{k-j} B$

► So we need to compute 0.

Contagion

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Average over all nodes to obtain expression for ϕ_{t+1} :

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} {k \choose j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}.$$

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So we need to compute θ_t ...

Contagion

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So we need to compute θ_t ... massive excitement...

Contagion

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Theory







First connect θ_0 to θ_1 :

 $\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} {k-1 \choose j} \theta_0^{j} (1 - \theta_0)^{k-1-j} B_{kj}$$

- $ightharpoonup rac{kP_k}{\langle k \rangle} = R_k = \mathbf{Pr}$ (edge connects to a degree k node).
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- ϕ_0 and $(1 \phi_0)$ terms account for state of node at time t = 0.

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- ▶ See this all generalizes to give θ_{t+1} in terms of θ_t ...

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Theory





Two pieces: edges first, and then nodes

1.
$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$$

$$+(1-\phi_0)\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_t^{\ j}(1-\theta_t)^{k-1-j}B_{kj}$$
social effects

with $\theta_0 = \phi_0$.

2.
$$\phi_{t+1} =$$

$$\underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}}_{\text{exogenous}}.$$

social effects

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Network version

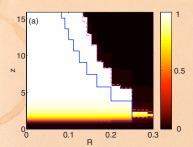
Theory

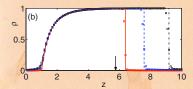






Comparison between theory and simulations





From Gleeson and Cahalane [7]

- Pure random networks with simple threshold responses
- ► R = uniform threshold (our ϕ_*); z = average degree; $\rho = \phi$; $q = \theta$; $N = 10^5$.
- $\phi_0 = 10^{-3}, 0.5 \times 10^{-2},$ and $10^{-2}.$
- ► Cascade window is for $\phi_0 = 10^{-2}$ case.
- Sensible expansion of cascade window as ϕ_0 increases.

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Theory







▶ Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \rightarrow 0$.

lends on map $\theta_{t+1} = G(\theta_t, \phi_0)$.

First: If self-starters are present, some activation in assured.

 $G(0,\phi_0) = \sum_{k} rac{k P_k}{\langle k
angle} ullet B_{k0}$

meaning $B_{k0}>0$ for at least one value of k

If $\theta=0$ is a fixed point of G (i.e., $G(0,\phi_0)=0$) then spreading occurs if

 $G(0, \rho_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \circ (k-1) \circ B_{k1} > 1.$

Insert question from assignment 8 (⊞)

Contagion

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All-to-all networks

Theory







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Contagion

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Reference

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Contagion

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Theory

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In words:

- If $G(0; \phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If G has an unstable fixed point at $\theta = 0$, then passages are also always possible

Non-vanishing seed case:

- ▶ Cascade condition is more complicated for $\phi_0 > 0$.
- If G has a stable fixed point at $\theta=0$, and an unstable fixed point for some $0<\theta_*<1$, then for $\theta_0>\theta_*$, spreading takes off.
- ► Tricky point: G depends on ϕ_0 , so as we change ϕ_0 , we also change G.

Contagion

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Theory







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Theory







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Contagion

Basic Contagion Models

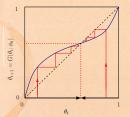
Global spreading condition

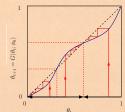
Social Contagion Models

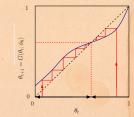
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- ▶ Given $\theta_0(=\phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.
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 - Important: Actual form of G depends on
- So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G.

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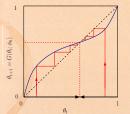
Network version

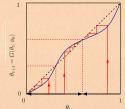
Theory

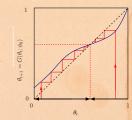












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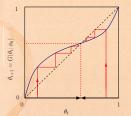
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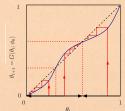
Theory

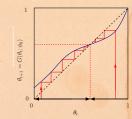












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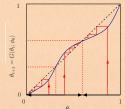
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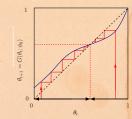












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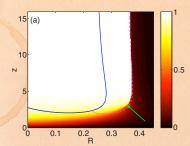
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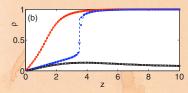
Theory





Comparison between theory and simulations





From Gleeson and Cahalane [7]

Now allow thresholds to be distributed according to a Gaussian with mean R.

R = 0.2, 0.362, and0.38; $\sigma = 0.2$.

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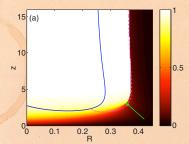
Theory

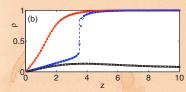






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Now allow thresholds to be distributed according to a Gaussian with mean B.

- $R = 0.2, 0.362, and 0.38; \sigma = 0.2.$
- $\phi_0 = 0$ but some nodes have thresholds ≤ 0 so effectively $\phi_0 > 0$.
 - discontinuous phase

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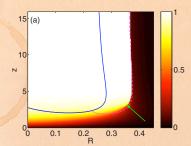
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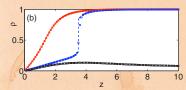
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- ► R = 0.2, 0.362, and 0.38; $\sigma = 0.2$.
- $\phi_0 = 0$ but some nodes have thresholds ≤ 0 so effectively $\phi_0 > 0$.
- Now see a (nasty) discontinuous phase transition for low (k).

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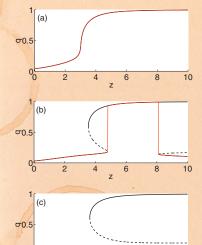
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From Gleeson and Cahalane [7]

- Plots of stability points for $\theta_{t+1} = G(\theta_t; \phi_0)$.
- n.b.: 0 is not a fixed point here: $\theta_0 = 0$ always takes off.
- ► Top to bottom: *R* = 0.35, 0.371, and 0.375.
- for (b) and (c) lead to
- Saddle node
 bifurcations appear and

merge (b and c).



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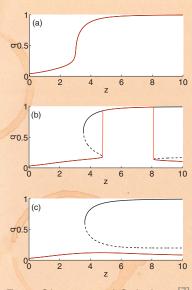
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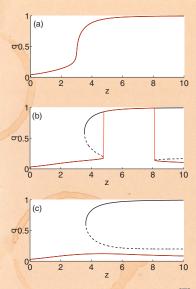
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From Gleeson and Cahalane [7]

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Bridging to single seed case:

- Consider largest vulnerable component as initial set of seeds.
- Not quite right as spreading must move through
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Bridging to single seed case:

- Consider largest vulnerable component as initial set of seeds.
- Not quite right as spreading must move through vulnerables.
- But we can usefully think of the vulnerable component as activating at time t = 0 because order doesn't matter.
- Rebuild ϕ_t and θ_t expressions...

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Two pieces modified for single seed:

1. $\theta_{t+1} = \theta_{\text{vuln}} + \theta_{\text{vuln}}$

$$(1 - \theta_{\text{vuln}}) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} {k-1 \choose j} \theta_t^{j} (1 - \theta_t)^{k-1-j} B_{kj}$$

with $\theta_0 = \theta_{\text{vuln}} = \mathbf{Pr}$ an edge leads to the giant vulnerable component (if it exists).

2.
$$\phi_{t+1} = S_{\text{vuln}} +$$

$$(1 - S_{\text{vuln}}) \sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} {k \choose j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}.$$

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Synchronous update

Done: Evolution of φ_t and θ_t given exactly by the maps we have derived.

Asynchronous updates

- ▶ Update nodes with probability α .
- As $\alpha \to 0$, updates become effectively independent.
- Now can talk about $\phi(t)$ and $\theta(t)$.

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