Measures of centrality Complex Networks CSYS/MATH 303, Spring, 2011

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- Basic question: how 'important' are specific nodes and edges in a network?
- An important node or edge might.
 - handle a relatively large amount of the network's traffic (e.g., cars, information);
 - bridge two or more distinct groups (e.g., liason, interpreter);
 - 3. be a source of important ideas, knowledge, or judgments (e.g., supreme court decisions, an employee who 'knows where everything is').
 - So how do we quantify such a slippery concept as
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- We generate ad hoc, reasonable measures, and examine their utility...

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- One possible reflection of importance is centrality.
 - sense) in the middle of a network are important for the network's function
- Idea of centrality comes from social networks literature ^[7].
- ▶ Many flavors of centrality.
 - Many are topological and quasi-dynamical,
 Some are based on dynamics (e.g., traffic).
- We will define and examine a few.
- (Later: see centrality useful in identifying communities in networks.)

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- Don: doesn't take in any non-local information

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- Betweenness centrality is based on shortest paths in a network.
- Idea: If the quickest way between any two nodes or a network disproportionately involves certain nodes then they are 'important' in terms of global cohesion
- For each node /, count how many shortest paths pass through *i*.
- In the case of ties, or divide counts between paths.
- ➤ Call requency of shortest paths passing through node *i* the betweenness of *i*, *B_i*.
- Note: Exclude shortest paths between i and othe nodes.
- Note: works for weighted and unweighted networks.

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- Consider a network with N nodes and m edges (possibly weighted).
- Computational goal: Find (⅓) shortest paths (⊞)
 - Maditionally use Floyd-Warshall (⊞) algorithm
- Computation time grows as O(N³)
- See also
 - Dijkstra's algorithm (H) for finding shortest path between two specific nodes.
 - 2. and Johnson's algorithm (⊞) which outperforms Floyd-Warshall for sparse networks:
 - $-O(mN+N^2\log N)$
 - Newman (2001) [4, 5] and Brandes (2001) [1]
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- ► Computation times grow as:
 - 1. O(mN) for unweighted graphs;
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- Consider unweighted networks.
- Use breadth-first search:
 - 1. Start at node i, giving it a distance d = 0 from itself.
 - 2. Create a list of all of i's neighbors and label the being at a distance d = 1.
 - Go through list of most recently visited nodes and find all of their neighbors.
 - Exclude any nodes already assigned a distance.
 - 5. Increment distance d by 1
 - 6. Label newly reached nodes as being at distance d.
 - Repeat steps 3 through 6 until all nodes are visited.
- Record which nodes link to which nodes moving out from i (former are 'predecessors' with respect to i's shortest path structure).
- ▶ Runs in O(m) time and gives N 1 shortest paths
- ➤ Find all shortest paths in O(mN) time
- ▶ Much, much better than naive estimate of O(mN²).

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 - 2. Create a list of all of i's neighbors and label them being at a distance d = 1.
 - Go through list of most recently visited nodes and find all of their neighbors.
 - 4. Exclude any nodes already assigned a distance.

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- Consider unweighted networks.
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 - 7. Repeat steps 3 through 6 until all nodes are visited.

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- Record which nodes link to which nodes moving out from *i* (former are 'predecessors' with respect to *i*'s shortest path structure).

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- ▶ Runs in O(m) time and gives N-1 shortest paths.
- ► Find all shortest paths in O(mN) time
- Much, much better than naive estimate of $O(mN^2)$.

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- 1. Set all nodes to have a value $c_{ij} = 0$, j = 1, ..., N (c for count).
- 3. Find shortest paths to all other N 1 nodes up
- 4. Record # equal shortest paths reaching each node.
- Move through nodes according to their distance from it starting with the furthest.
- 6. Travel back towards *i* from each starting node *j*, along shortest path(s), adding 1 to every value of *c_i*, at seath node along the way.
 - Whenever more than one possibility exists, apportion according to total number of short paths coming through predecessors.
- 8. Exclude starting node / and / from increment.
- 9. Repeat steps 2-8 for every node in

in betweenness as $B_i = \sum_{i=1}^{N} c_{ii}$.

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- 1. Set all nodes to have a value $c_{ij} = 0$, j = 1, ..., N (c for count).
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- 2. Select one node i.
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Travel back towards i from each starting node

Whenever more than one possibility exists, apportion according to total number of short paths coming

Exclude starting node / and / from increment.



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- 6. Travel back towards i from each starting node j

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Whenever more than one possibility exists, apportion

according to total number of short paths coming

through predecessors

Exclude starting node j and i from increment.

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- 8. Exclude starting node *j* and *i* from increment.
- 9. Repeat steps 2–8 for every node *i* and obtain betweenness as $B_j = \sum_{i=1}^{N} c_{ij}$.

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► For a pure tree network, c_{ij} is the number of nodes beyond j from i's vantage point.

Same algorithm for computing drainage area in rive

- For edge betweenness, use exact same algorithm but now
 - 1. *j* indexes edges,
 - 2. and we add one to each edge as we traverse it
- ▶ For both algorithms, computation time grows as



 For sparse networks with relatively small averages degree, we have a fairly digestible time growth of Measures of centrality

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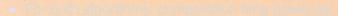
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 For sparse networks with relatively small average degree, we have a fairly digestible time growth of

 $O(N^2)$

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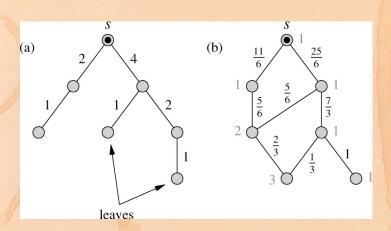
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- Define x_i as the 'importance' of node i
- Idea: x, depends (somehow) on x, if y is a neighbor of i.
- Recursive: importance is transmitted through network.
- Simplest possibility is a linear combination

$$x_i \propto \sum a_{ji} x_j$$

 Assume further that constant of proportionality, c, is independent of i.

Above give

 $\mathbf{c}\mathbf{A}^{\mathsf{T}}\mathbf{X}$ or

$$\mathbf{A}^{\mathrm{T}}\vec{x} = c^{-1}\vec{x} = \lambda\vec{x}$$

- Eigenvalue equation based on adjacency matrix.
- eigenvalue [7] Lose sight of original assumption's non-physicality.

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Eigenvalue equation based on adjacency mains

non-physicality

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- Note: Lots of despair over size of the largest eigenvalue. [7]

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- So... solve $\mathbf{A}^{\mathrm{T}}\vec{\mathbf{x}} = \lambda \vec{\mathbf{x}}$.
- But which eigenvalue and eigenvector
- ► We, the people, would like
 - 1. A unique solution
 - 2. λ to be real
 - 3. Entries of \vec{x} to be real.
 - 4. Entries of \vec{x} to be non-negative.
 - 5. λ to actually mean something..
 - 6. Values of x_i to mean something (what does an observation that $x_3 = 5x_7$ mean?) (maybe only ordering is informative...)
 - 7. λ to equal 1 would be nice...
 - 8. Ordering of \vec{x} entries to be robust to reasonable modifications of linear assumption

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 - 6 Values of v. to mean comething
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 - C. Making of the many name thin
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 - 3. Entries of \vec{x} to be real.
 - 4. Entries of \vec{x} to be non-negative.
 - 5. λ to actually mean something...
 - 6. Values of x_i to mean something (what does an observation that $x_3 = 5x_7$ mean?) (maybe only ordering is informative...)

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- So... solve $\mathbf{A}^{\mathrm{T}}\vec{x} = \lambda \vec{x}$.
- But which eigenvalue and eigenvector?
- ▶ We, the people, would like:
 - 1. A unique solution.
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 (maybe too much)
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- So... solve $\mathbf{A}^{\mathrm{T}}\vec{\mathbf{x}} = \lambda \vec{\mathbf{x}}$.
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If an $N \times N$ matrix A has non-negative entries then:

A has a real eigenvalue $\lambda_1 \geq |\lambda_i|$ for $i=2,\ldots$

 A₁ corresponds to left and right 1-d eigenspaces fo which we can choose a basis vector that has

/ non-negative entries.

3. The dominant real eigenvalue λ_1 is bounded by the minimum and maximum row sums of A

 $\min \sum a_{ij} \le \lambda_1 \le \max_i \sum a_{ij}$

All other eldervectors have one or more negative

entries.

5. Note: Proof is relatively short for symmetric matrices that are structly positive [6] and just non-negative [3]

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Perron-Frobenius theorem: (H)

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Perron-Frobenius theorem: (⊞)

If an $N \times N$ matrix A has non-negative entries then:

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Assuming our network is irreducible (⊞), meaning there is only one component, is reasonable:

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Assuming our network is irreducible (⊞), meaning there is only one component, is reasonable: just consider one component at a time if more than one exists.

Irreducibility means largest eigenvalue's eigenvector

Analogous to notion of ergodicity: every state is

(Another term: Primitive graphs and matrices

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Primitive graphs and matrices.)

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- Generalize eigenvalue centrality to allow nodes to have two attributes:
 - Authority: how much knowledge, information, etc., held by a node on a topic.
 - 2. Hubness (or Hubosity or Hubbishness): how well'a node 'knows' where to find information on a given
- Original work due to the legendary Jon Kleinberg.
- Best hubs point to best authorities
- ▶ Recursive: nodes can be both hubs and authorities.
- More: look for dense links between sets of good hubs pointing to sets of good authorities.
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- Give each node two scores:
 - 1. x_i = authority score for node
 - 2. y_i = hubtasticness score for node
- As for eigenvector centrality, we connect the scores of neighboring nodes.
 - New story f. a good authority is linked to by good hubs:
- ▶ Means x_i should increase as $\sum_{i=1}^{N} a_{ii} y_i$ increases
- ▶ Note: indices are ji meaning i has a directed link to i
- ▶ Newstory II. good hubs point to good authorities
- Means y_i should increase as $\sum_{i=1}^{N} a_{ii} x_i$ increases
- ► Linearity assumption

 $P_{X \sim A}^T \vec{v}$ and $\vec{v} \sim A\vec{v}$

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 $\vec{A} \times \vec{A} \times \vec{A} \vec{V}$ and $\vec{V} \times \vec{A} \vec{X}$

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 $\vec{A} \propto \vec{A}' \vec{y}$ and $\vec{y} \propto \vec{A} \vec{x}$

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So let's say we have

$$\vec{x} = c_1 A^T \vec{y}$$
 and $\vec{y} = c_2 A \vec{x}$

where c_1 and c_2 must be positive.

Above equations combine to give

$$\vec{X} = c_1 A^T c_2 A \vec{X} = \lambda A^T A \vec{X}$$

► It's all good: We have the heart of singular value decomposition before us...

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So let's say we have

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where $\lambda = c_1 c_2 > 0$.

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So let's say we have

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where c_1 and c_2 must be positive.

Above equations combine to give

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where $\lambda = c_1 c_2 > 0$.

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- \rightarrow A^TA is symmetric.
 - A^t A is semi-positive definite so its eigenvalues are all > 0.
 - A' A's eigenvalues are the square of A's singular values.
- A' A's eigenvectors form a joyful orthogonal basis
- ➤ Perron-Frobenius tells us that only the dominant elgenvalue's elgenvector can be chosen to have
 - nenenegative entries
 - So: linear assumption leads to a solvable system.
- What would be very good: find networks where we have independent measures of node 'importance' and see how importance is actually distributed.

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- A^T A is semi-positive definite so its eigenvalues are all > 0.
- A^TA 's eigenvalues are the square of A's singular values.
- A^TA's eigenvectors form a joyful orthogonal basis.

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- Perron-Frobenius tells us that only the dominant eigenvalue's eigenvector can be chosen to have non-negative entries.
- So: linear assumption leads to a solvable system.
- What would be very good: find networks where we have independent measures of node 'importance' and see how importance is actually distributed.

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References I

[1] U. Brandes.
A faster algorithm for betweenness centrality.
J. Math. Sociol., 25:163–177, 2001. pdf (⊞)

[2] J. M. Kleinberg.
Authoritative sources in a hyperlinked environment.

Proc. 9th ACM-SIAM Symposium on Discrete
Algorithms, 1998. pdf (⊞)

[3] K. Y. Lin.

An elementary proof of the perron-frobenius theorem for non-negative symmetric matrices.

Chinese Journal of Physics, 15:283–285, 1977.
pdf (H)

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References II

[4] M. E. J. Newman.
Scientific collaboration networks. II. Shortest paths, weighted networks, and centrality.
Phys. Rev. E, 64(1):016132, 2001. pdf (⊞)

[5] M. E. J. Newman and M. Girvan.
Finding and evaluating community structure in networks.
Phys. Rev. E, 69(2):026113, 2004. pdf (⊞)

[6] F. Ninio. A simple proof of the Perron-Frobenius theorem for positive symmetric matrices. J. Phys. A.: Math. Gen., 9:1281–1282, 1976. pdf (H)

[7] S. Wasserman and K. Faust. Social Network Analysis: Methods and Applications. Cambridge University Press, Cambridge, UK, 1994.

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