Measures of centrality

Complex Networks
CSYS/MATH 303, Spring, 2011

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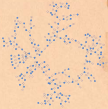
Outline

How big is my node?

- Basic question: how ‘important’ are specific nodes and edges in a network?
- An important node or edge might:
  1. handle a relatively large amount of the network’s traffic (e.g., cars, information);
  2. bridge two or more distinct groups (e.g., liason, interpreter);
  3. be a source of important ideas, knowledge, or judgments (e.g., supreme court decisions, an employee who ‘knows where everything is’).
- So how do we quantify such a slippery concept as importance?
- We generate ad hoc, reasonable measures, and examine their utility...



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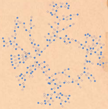

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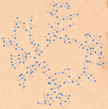

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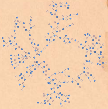

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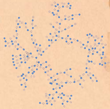

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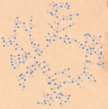
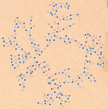


Centrality

- One possible reflection of importance is centrality.
- Presumption is that nodes or edges that are (in some sense) in the middle of a network are important for the network's function.
- Idea of centrality comes from social networks literature [7].
- Many flavors of centrality...
  1. Many are topological and quasi-dynamical;
  2. Some are based on dynamics (e.g., traffic).
- We will define and examine a few...
- (Later: see centrality useful in identifying communities in networks.)



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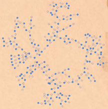

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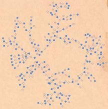

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Centrality measures

Degree centrality

Closeness centrality

Betweenness

Eigenvalue centrality

Hubs and Authorities

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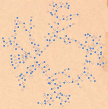
Degree centrality

- Naively estimate importance by node degree. [7]
- Doh: assumes linearity
  (If node i has twice as many friends as node j, it's twice as important.)
- Doh: doesn't take in any non-local information.



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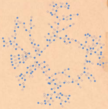

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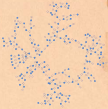
- Idea: Nodes are more central if they can reach other nodes 'easily.'
- Measure average shortest path from a node to all other nodes.
- Define Closeness Centrality for node i as

$$\frac{N-1}{\sum_{j,j\neq i}(\text{distance from } i \text{ to } j).}$$

- Range is 0 (no friends) to 1 (single hub).
- Unclear what the exact values of this measure tells us because of its ad-hocness.
- General problem with simple centrality measures: what do they exactly mean?
- Perhaps, at least, we obtain an ordering of nodes in terms of 'importance.'



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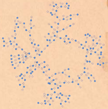

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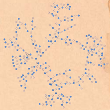

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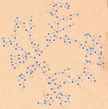

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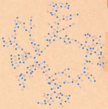

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Photosynth (⊞).

Outline

Betweenness centrality

- Betweenness centrality is based on shortest paths in a network.
- Idea: If the quickest way between any two nodes on a network disproportionately involves certain nodes, then they are 'important' in terms of global cohesion.
- For each node i, count how many shortest paths pass through i.
- In the case of ties, or divide counts between paths.
- Call frequency of shortest paths passing through node i the betweenness of i, B_i.
- Note: Exclude shortest paths between i and other nodes.
- Note: works for weighted and unweighted networks.

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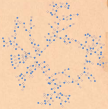

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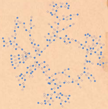

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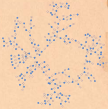

- Consider a network with N nodes and m edges (possibly weighted).
- Computational goal: Find $\binom{N}{2}$ shortest paths (⊞) between all pairs of nodes.
- Traditionally use Floyd-Warshall (⊞) algorithm.
- Computation time grows as $O(N^3)$.
- See also:
  1. Dijkstra's algorithm (⊞) for finding shortest path between two specific nodes,
  2. and Johnson's algorithm (⊞) which outperforms Floyd-Warshall for sparse networks: $O(mN + N^2 \log N)$.
- Newman (2001) [4, 5] and Brandes (2001) [1] independently derive equally fast algorithms that also compute betweenness.
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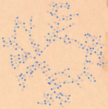

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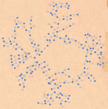

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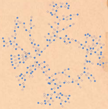

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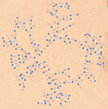

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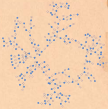

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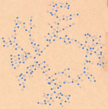

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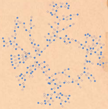

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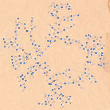

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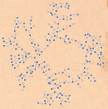

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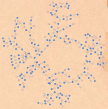

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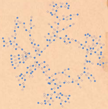

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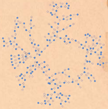

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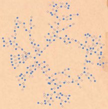

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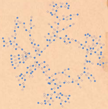

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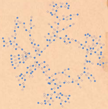

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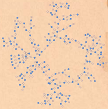

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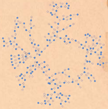

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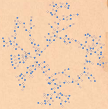

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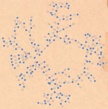

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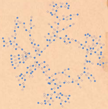

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- For a pure tree network, c_{ij} is the number of nodes beyond j from i's vantage point.
- Same algorithm for computing drainage area in river networks (with 1 added across the board).
- For edge betweenness, use exact same algorithm but now
  1. j indexes edges,
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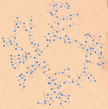

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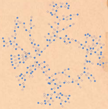

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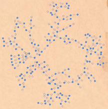

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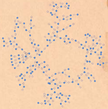

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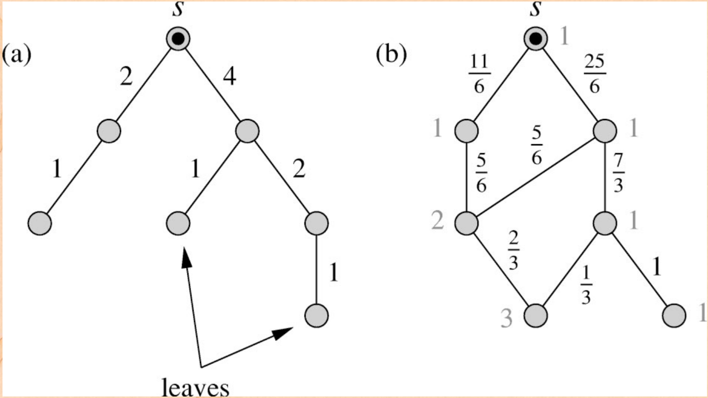

Outline

Background

Centrality measures

- Degree centrality
- Closeness centrality
- Betweenness
- Eigenvalue centrality
- Hubs and Authorities

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- Assume further that constant of proportionality, c, is independent of i.
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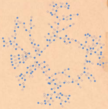

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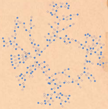

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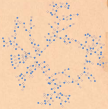

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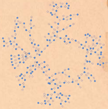

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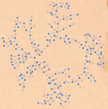

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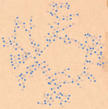

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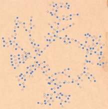

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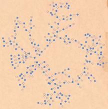

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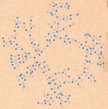

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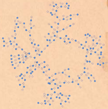

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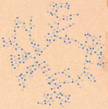

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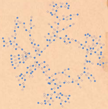

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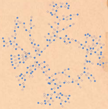

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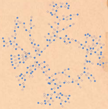

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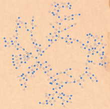

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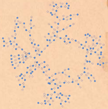

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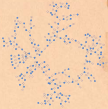

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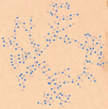

Important nodes have important friends:

- So... solve $\mathbf{A}^{\mathrm{T}}\vec{x} = \lambda\vec{x}$.
- But which eigenvalue and eigenvector?
- We, the people, would like:
  1. A unique solution.
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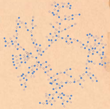

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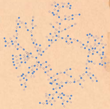

Perron-Frobenius theorem: (⊞)

If an $N \times N$ matrix A has non-negative entries then:

1. A has a real eigenvalue $\lambda_1 \geq |\lambda_i|$ for $i = 2, \ldots, N$.
2. λ_1 corresponds to left and right 1-d eigenspaces for which we can choose a basis vector that has non-negative entries.
3. The dominant real eigenvalue λ_1 is bounded by the minimum and maximum row sums of A:

$$\min_i \sum_{j=1}^{N} a_{ij} \leq \lambda_1 \leq \max_i \sum_{j=1}^{N} a_{ij}$$

4. All other eigenvectors have one or more negative entries.

6. Note: Proof is relatively short for symmetric matrices that are strictly positive [6] and just non-negative [3].

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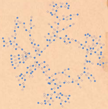

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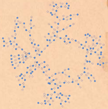

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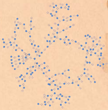
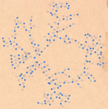


Other Perron-Frobenius aspects:

- Assuming our network is irreducible (⊞), meaning there is only one component, is reasonable: just consider one component at a time if more than one exists.
- Irreducibility means largest eigenvalue's eigenvector has strictly non-negative entries.
- Analogous to notion of ergodicity: every state is reachable.
- (Another term: Primitive graphs and matrices.)



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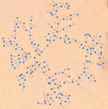

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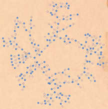

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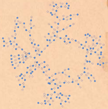
Outline

Background

Centrality measures

- Degree centrality
- Closeness centrality
- Betweenness
- Eigenvalue centrality
- Hubs and Authorities

References



Hubs and Authorities

- Generalize eigenvalue centrality to allow nodes to have two attributes:
  1. Authority: how much knowledge, information, etc., held by a node on a topic.
  2. Hubness (or Hubosity or Hubbishness): how well a node 'knows' where to find information on a given topic.
- Original work due to the legendary Jon Kleinberg. [2]
- Best hubs point to best authorities.
- Recursive: nodes can be both hubs and authorities.
- More: look for dense links between sets of good hubs pointing to sets of good authorities.
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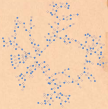

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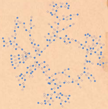

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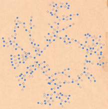

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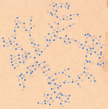

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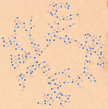

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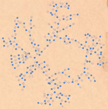

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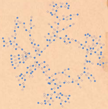

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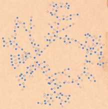

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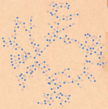

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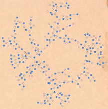

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- Means x_i should increase as $\sum_{j=1}^{N} a_{ji} y_j$ increases.
- Note: indices are ji meaning j has a directed link to i.
- New story II: good hubs point to good authorities.
- Means y_i should increase as $\sum_{j=1}^{N} a_{ij} x_j$ increases.
- Linearity assumption:

$$\vec{x} \propto A^T \vec{y} \text{ and } \vec{y} \propto A \vec{x}$$

Hubs and Authorities

- So let's say we have

$$\vec{x} = c_1 A^T \vec{y} \text{ and } \vec{y} = c_2 A \vec{x}$$

where c_1 and c_2 must be positive.

- Above equations combine to give

$$\vec{x} = c_1 A^T c_2 A \vec{x} = \lambda A^T A \vec{x}.$$

where $\lambda = c_1 c_2 > 0$.

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We can do this:

- $A^T A$ is symmetric.
- $A^T A$ is semi-positive definite so its eigenvalues are all ≥ 0.
- $A^T A$'s eigenvalues are the square of A's singular values.
- $A^T A$'s eigenvectors form a joyful orthogonal basis.
- Perron-Frobenius tells us that only the dominant eigenvalue's eigenvector can be chosen to have non-negative entries.
- So: linear assumption leads to a solvable system.
- What would be very good: find networks where we have independent measures of node 'importance' and see how importance is actually distributed.

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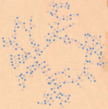

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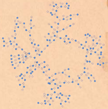

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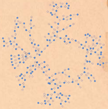

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