

Social Contagion

Principles of Complex Systems CSYS/MATH 300, Fall, 2010

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The
UNIVERSITY
of VERMONT



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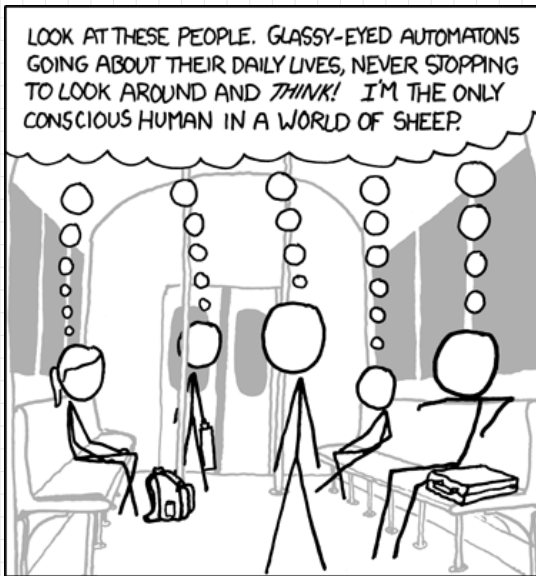
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<http://xkcd.com/610/> (田)



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Social Contagion

Examples abound

- ▶ fashion
- ▶ striking
- ▶ smoking (田) [6]
- ▶ residential segregation [16]
- ▶ ipods
- ▶ obesity (田) [5]
- ▶ Harry Potter
- ▶ voting
- ▶ gossip
- ▶ Rubik's cube 
- ▶ religious beliefs
- ▶ leaving lectures


SIR and SIR_S contagion possible

↳ Classes of behavior versus specific behavior



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SIR and SIR**S** contagion possible

- ▶ Classes of behavior versus specific behavior: **dieting**



Framingham heart study:

Evolving network stories (Christakis and Fowler):

- ▶ The spread of quitting smoking (田) [6]
- ▶ The spread of spreading (田) [5]
- ▶ Also: happiness (田) [8], loneliness, ...
- ▶ The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (田)

Controversy:

- ▶ Are your friends making you fat? (田) (Clive Thompson, NY Times, September 10, 2009).
- ▶ Everything is contagious (田) — Boothis about the social plague still in the human superorganism (Dave Johns, Slate, April 5, 2010).



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Social Contagion

Two focuses for us

- ▶ Widespread media influence
- ▶ Word-of-mouth influence



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We need to understand influence

- ▶ Who influences whom?
- ▶ What kinds of influence response functions are there?
- ▶ Are some individuals super influencers?
- ▶ The infectious idea of opinion leaders (Katz and Lazarsfeld) ^[13]



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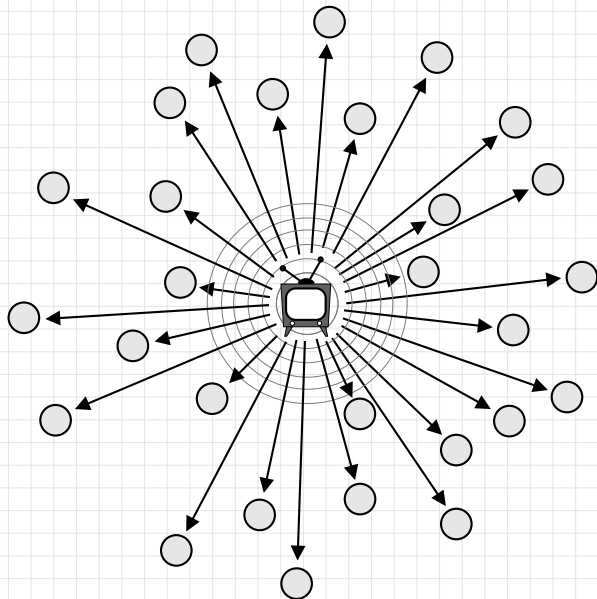


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The hypodermic model of influence



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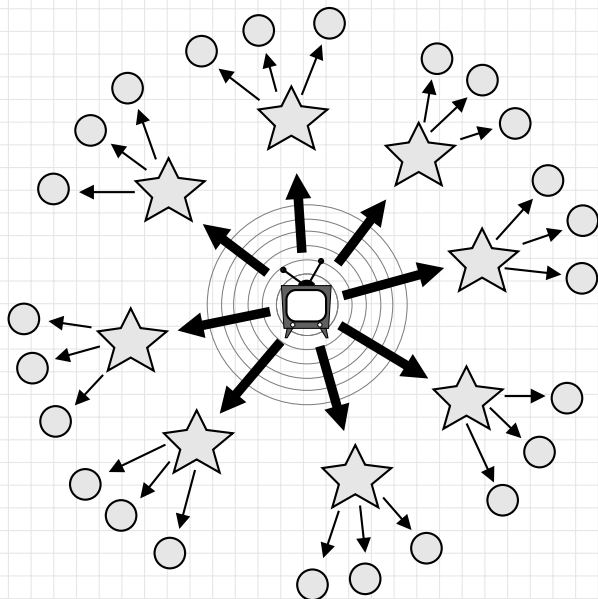
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The two step model of influence [13]



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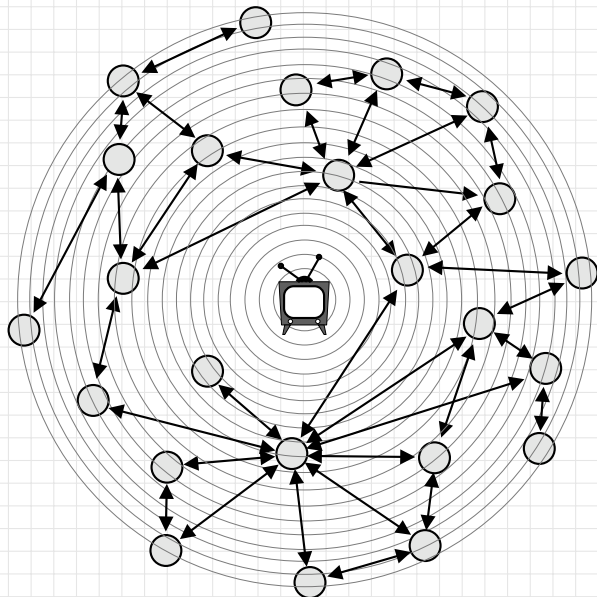
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The general model of influence



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Why do things spread?

- ▶ Because of properties of special individuals?
- ▶ Or system level properties?
- ▶ Is the match that lights the fire important?
- ▶ Yes. But only because we are narrative-making machines...
- ▶ We like to think things happened for reasons...
- ▶ Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- ▶ System/group properties harder to understand
- ▶ Always good to examine what is said before and after the fact...



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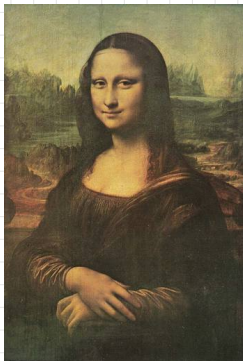


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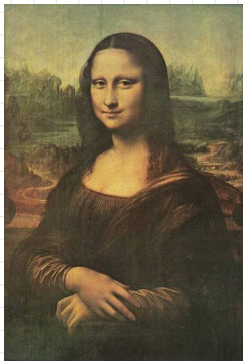
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The completely unpredicted fall of Eastern Europe



Timur Kuran: ^[14, 15] “Now Out of Never: The Element of Surprise in the East European Revolution of 1989”

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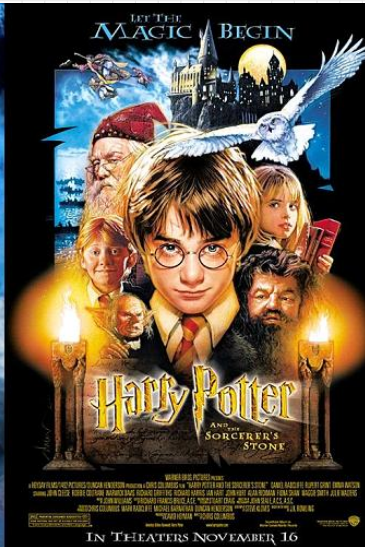
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The dismal predictive powers of editors...

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Messaging with social connections

- ▶ Ads based on message content
- ▶ Buzz media
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Getting others to do things for you

A very good book: **'Influence'** by Robert Cialdini [7]

Six modes of influence

1. *Reciprocation: The Old Give and Take... and Take*
2. *Commitment and Consistency: Hobgoblins of the Mind*
3. *Social Proof: Truths Are Us*
4. *Liking: The Friendly Thief*
5. *Authority: Directed Deference*
6. *Scarcity: The Rule of the Few*



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- ▶ **Reciprocation**: Free samples, Hare Krishnas
- ▶ **Commitment and Consistency**: Hazing
- ▶ **Social Proof**: Catherine Genovese, Jonestown
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- ▶ Useful but can be leveraged...

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Other acts of influence

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Some important models

- ▶ Tipping models—Schelling (1971) [16, 17, 18]
 - ▶ Simulation on checker boards
 - ▶ Idea of thresholds
 - ▶ Explore the Netlogo (田) implementation [21]
- ▶ Threshold models—Granovetter (1978) [10]
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Thresholds

- ▶ Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- ▶ 'Others' may be everyone in a population, an individual's close friends, any reference group.
- ▶ Response can be probabilistic or deterministic.
- ▶ Individual thresholds can vary
- ▶ Assumption: order of others' adoption does not matter...
- ▶ Assumption: level of influence per person is uniform



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- ▶ **Desire to coordinate**, to conform.
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- ▶ ϕ^* = threshold of an individual.
- ▶ $f(\phi_*)$ = distribution of thresholds in a population.
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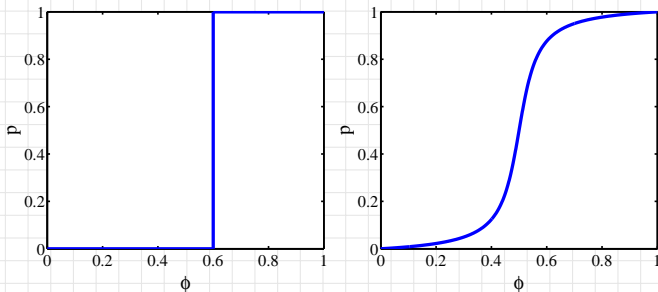


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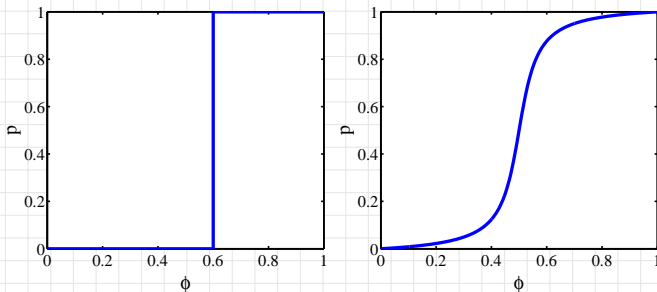
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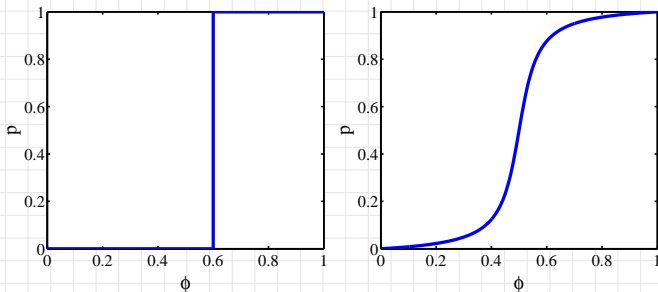
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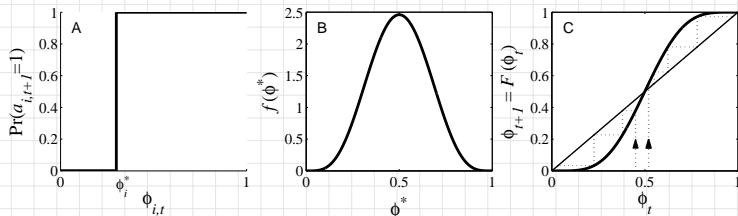
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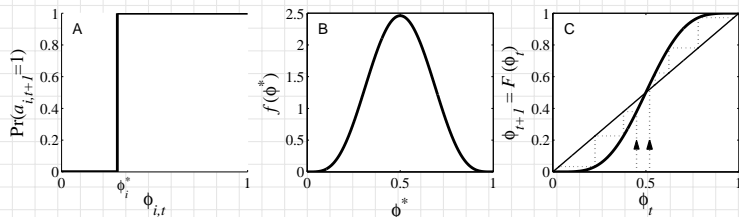


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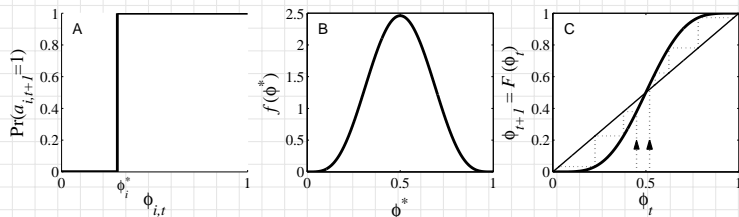


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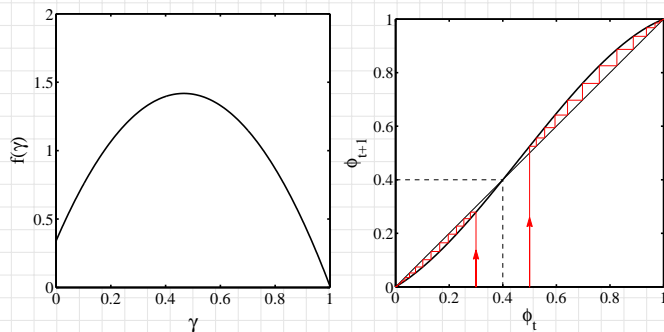
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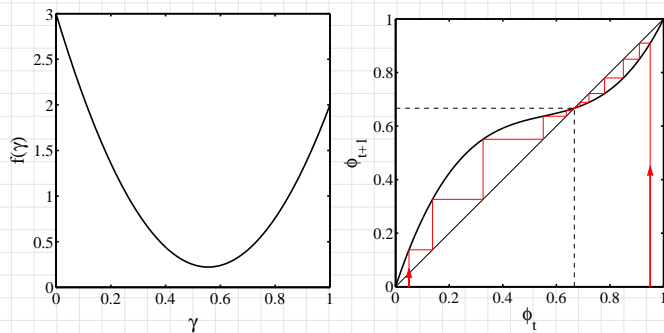
Threshold models



► Another example of critical mass model...



Threshold models



- ▶ Example of single stable state model



Threshold models

Implications for collective action theory:

1. Collective uniformity \nRightarrow individual uniformity
2. Small individual changes \Rightarrow large global changes



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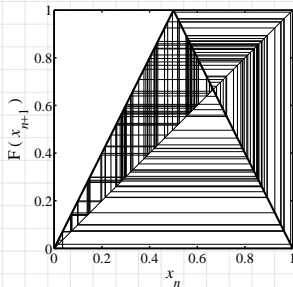
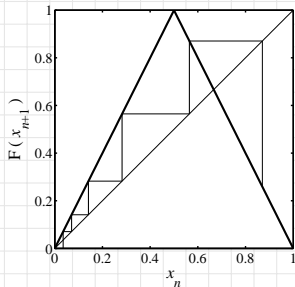
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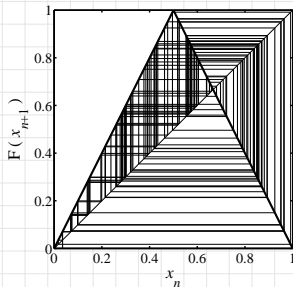
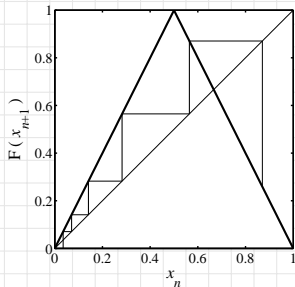


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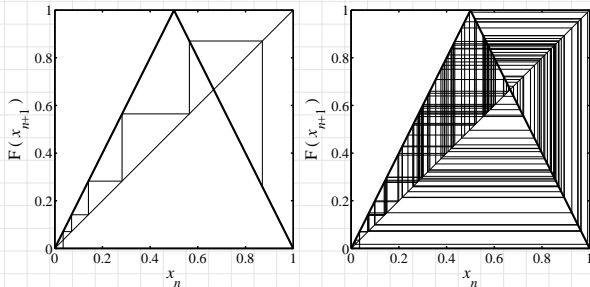


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Many years after Granovetter and Soong's work:

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- ▶ Mean field model → network model
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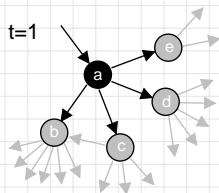


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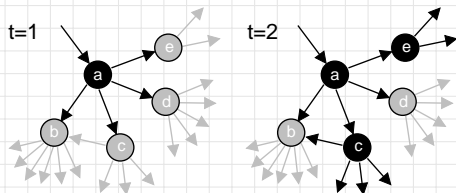
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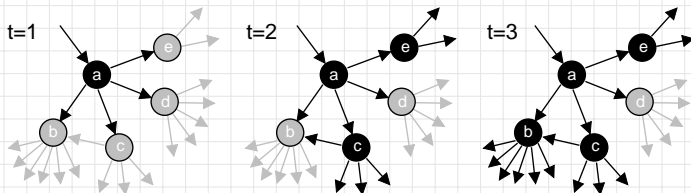
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1. If one individual is initially activated, what is the probability that an activation will spread over a network?
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First study random networks:

- ▶ Start with N nodes with a degree distribution p_k
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The most gullible

Vulnerables:

- ▶ We call individuals who can be activated by just one contact being active **vulnerables**
- ▶ The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

- ▶ Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- ▶ For global cascades on random networks, must have a *global cluster of vulnerables* [20]
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Cascade condition

Back to following a link:

- ▶ A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- ▶ Follows from there being k ways to connect to a node with degree k .
- ▶ Normalization:

$$\sum_{k=0}^{\infty} kP_k \equiv \langle k \rangle$$

- ▶ So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



Cascade condition

Back to following a link:

- ▶ A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.

- ▶ Follows from there being k ways to connect to a node with degree k .
- ▶ Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- ▶ So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



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Next: Vulnerability of linked node

- ▶ Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_* = 0}^{1/k} f(\phi'_*) d\phi'_*$$

- ▶ If linked node is **vulnerable**, it produces $k - 1$ **new** outgoing active links
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Cascade condition

Putting things together:

- ▶ Expected number of active edges produced by an active edge:

$$R = \sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} +$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$



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Cascade condition

So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- ▶ β_k = probability a degree k node is vulnerable.
- ▶ P_k = probability a node has degree k .



Cascade condition

Two special cases:

- ▶ (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- ▶ (2) Giant component exists: $\beta = 1$

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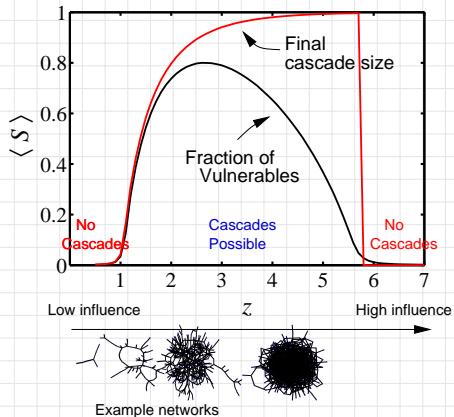
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Cascades on random networks



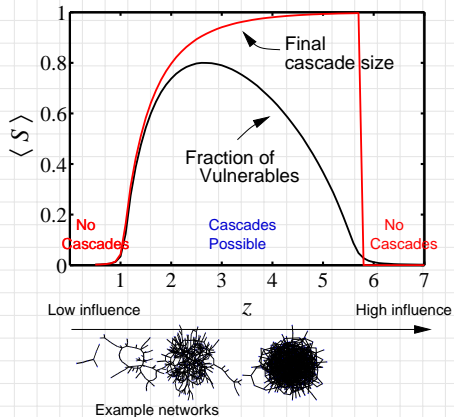
► Cascades occur only if size of max vulnerable cluster > 0 .

► System may be 'robust-yet-fragile'.

► 'Ignorance' facilitates spreading.



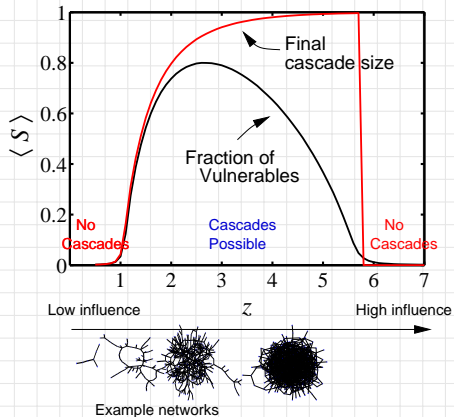
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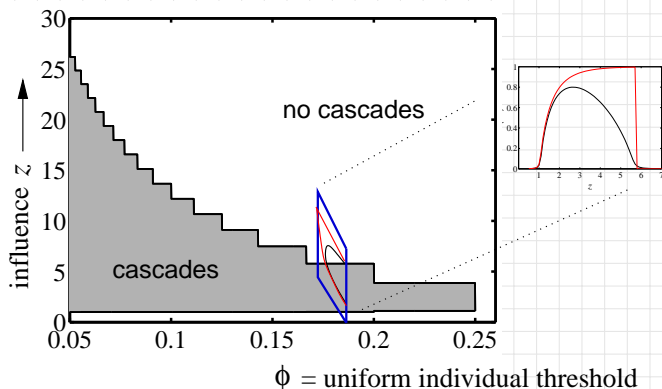
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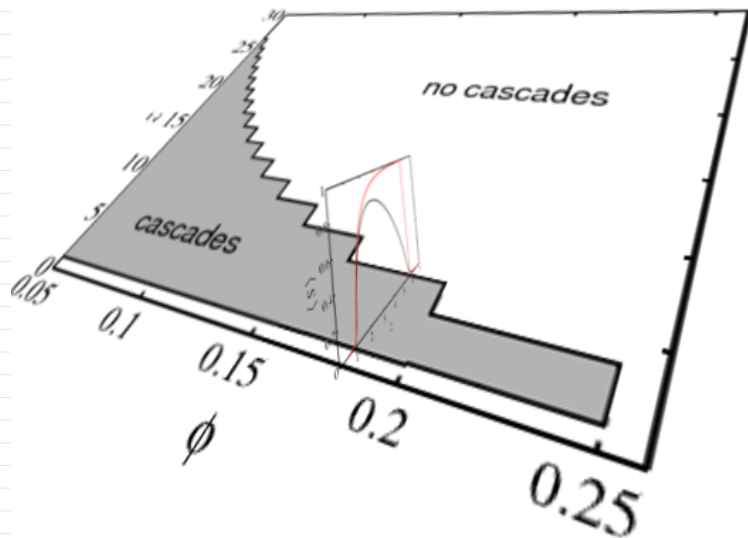
Cascade window for random networks



- ▶ 'Cascade window' widens as threshold ϕ decreases.
- ▶ Lower thresholds enable spreading.



Cascade window for random networks



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Cascade window—summary

For our simple model of a uniform threshold:

1. **Low $\langle k \rangle$** : No cascades in poorly connected networks. No global clusters of any kind.
2. **High $\langle k \rangle$** : Giant component exists but not enough vulnerables.
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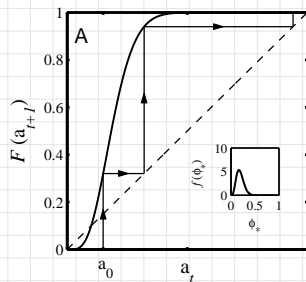
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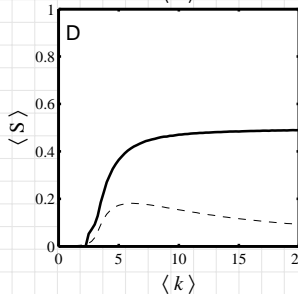
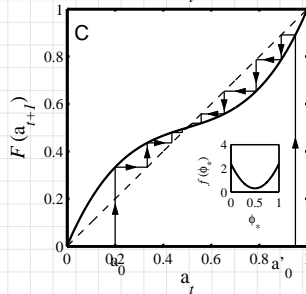
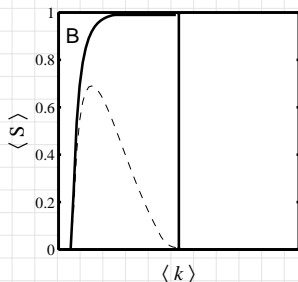
All-to-all versus random networks



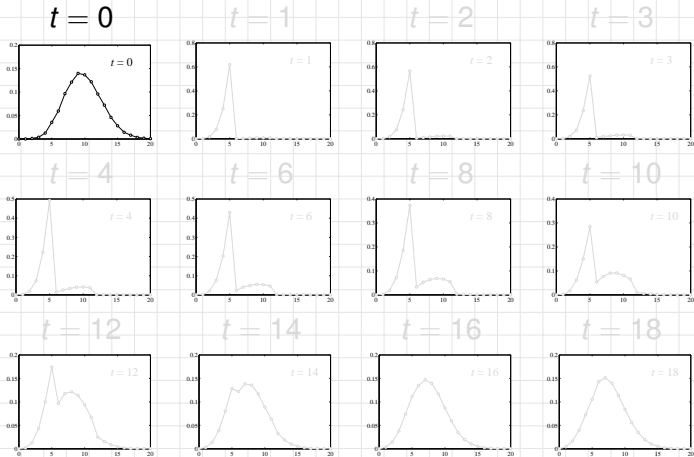
all-to-all networks



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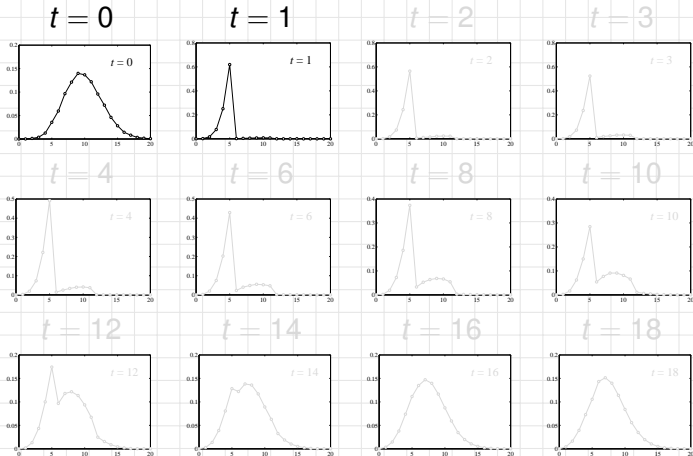
Early adopters—degree distributions



$P_{k,t}$ versus k



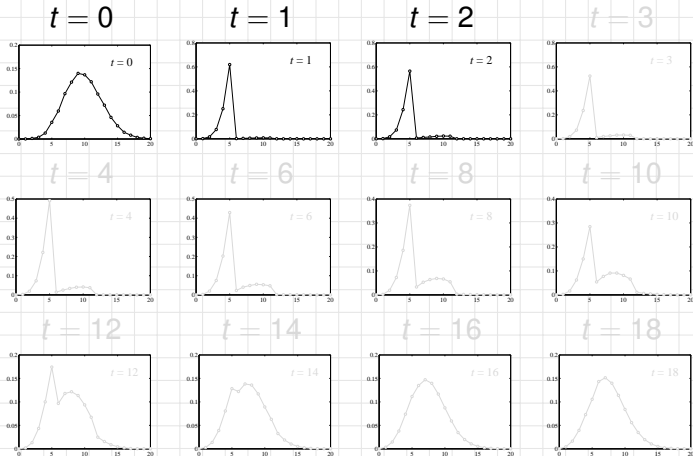
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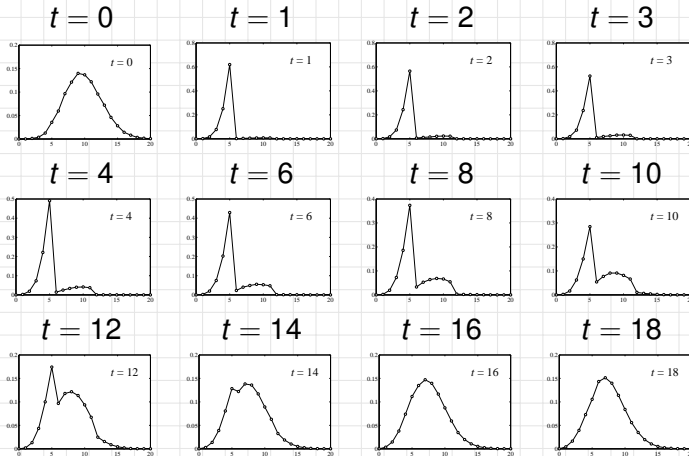
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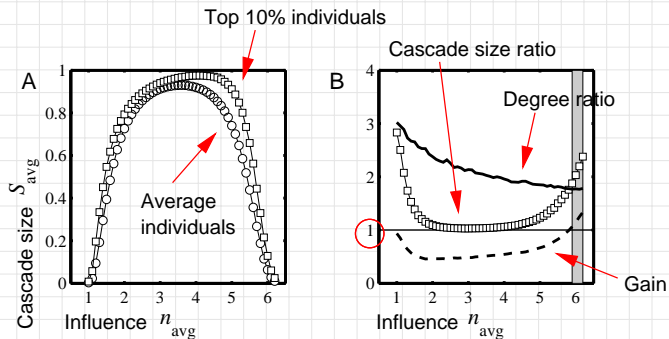
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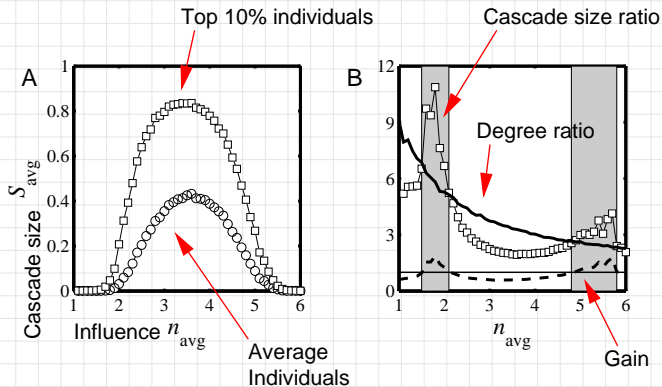
The multiplier effect:



- ▶ Fairly uniform levels of individual influence.
- ▶ Multiplier effect is mostly below 1.



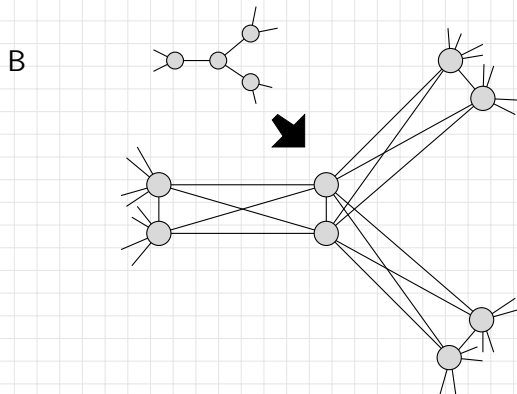
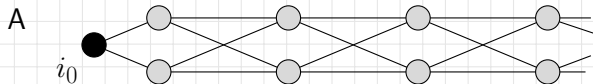
The multiplier effect:



- ▶ Skewed influence distribution example.



Special subnetworks can act as triggers



► $\phi = 1/3$ for all nodes



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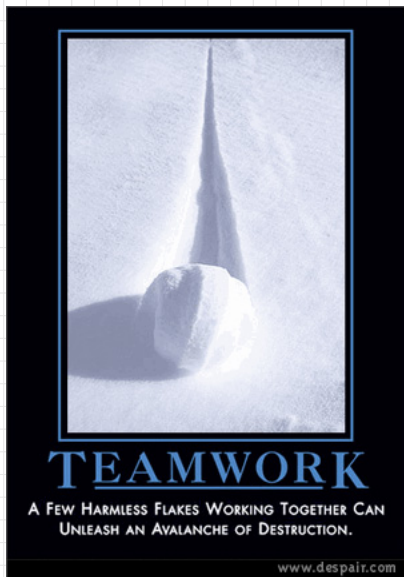
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The power of groups...



despair.com

“A few harmless flakes working together can unleash an avalanche of destruction.”

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Extensions

- ▶ Assumption of sparse interactions is good
- ▶ Degree distribution is (generally) key to a network's function
- ▶ Still, random networks don't represent all networks
- ▶ Major element missing: **group structure**



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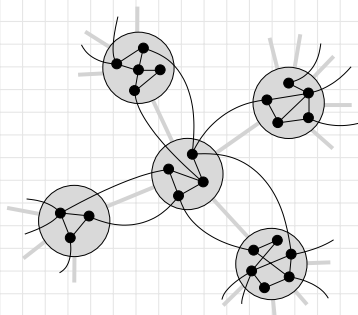


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Group structure—Ramified random networks



p = intergroup connection probability
 q = intragroup connection probability.

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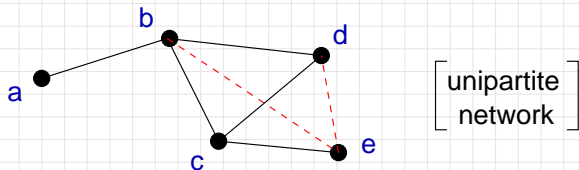
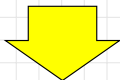
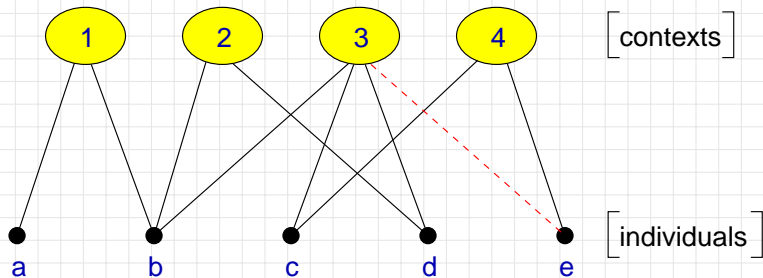
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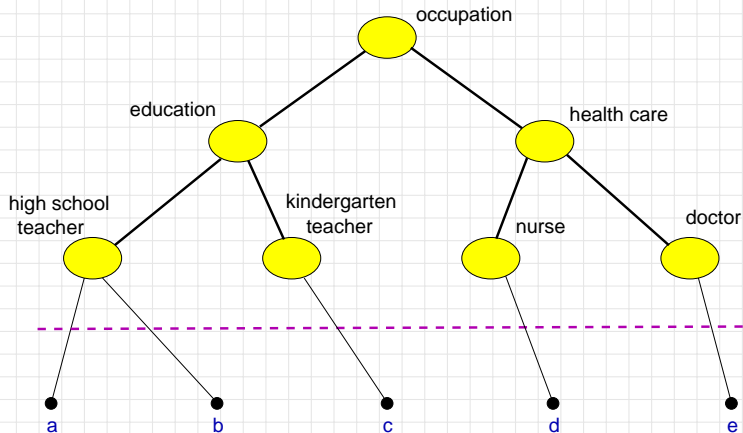
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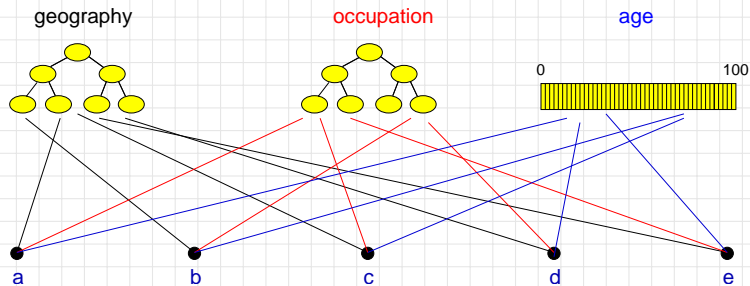
Bipartite networks



Context distance



Generalized affiliation model



(Blau & Schwartz, Simmel, Breiger)



Generalized affiliation model networks with triadic closure

- ▶ Connect nodes with probability $\propto \exp^{-\alpha d}$
where
 α = homophily parameter
and
 d = distance between nodes (height of lowest common ancestor)
- ▶ τ_1 = intergroup probability of friend-of-friend connection
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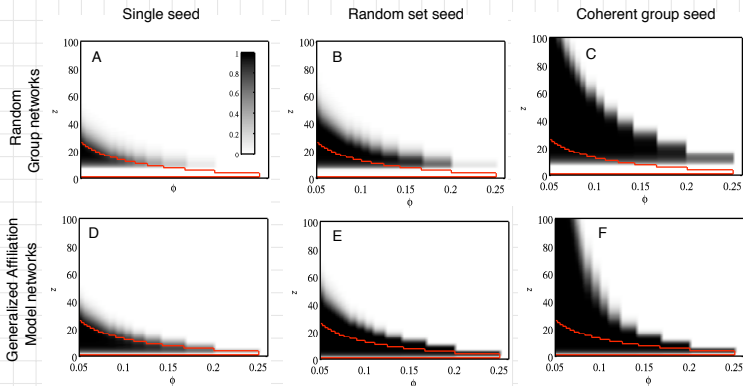


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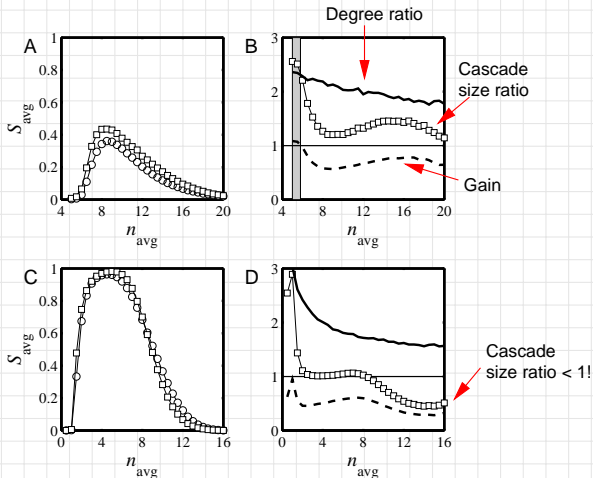
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Cascade windows for group-based networks



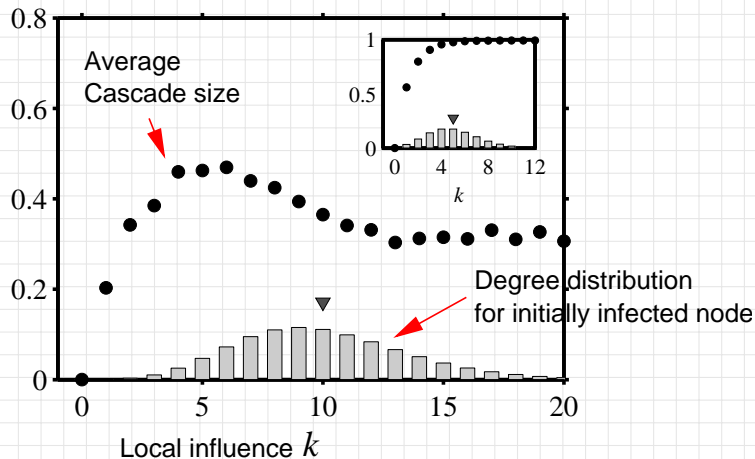
Multiplier effect for group-based networks:



► Multiplier almost always below 1.



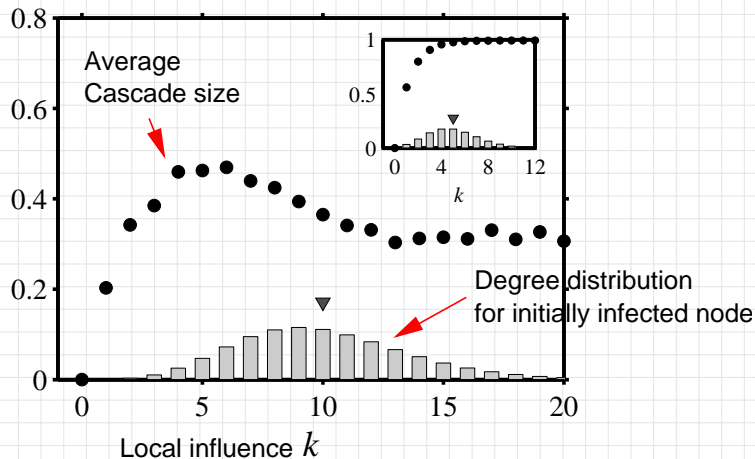
Assortativity in group-based networks



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Summary

- ▶ **'Influential vulnerables'** are key to spread.
- ▶ Early adopters are mostly vulnerables.
- ▶ Vulnerable nodes important but not necessary.
- ▶ Groups may greatly facilitate spread.
- ▶ Seems that cascade condition is a global one.
- ▶ Most extreme/unexpected cascades occur in highly connected networks
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Implications

- ▶ Focus on **the influential vulnerables**.
- ▶ Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- ▶ Only **simple ideas** can spread by word-of-mouth.
(Idea of opinion leaders spreads well...)
- ▶ Want enough individuals who will adopt and display.
- ▶ Displaying can be **passive** = free (yo-yo's, fashion), or **active** = harder to achieve (political messages).
- ▶ Entities can be novel or designed to combine with others, e.g. block another one.

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(Idea of opinion leaders spreads well...)
- ▶ Want enough individuals who will adopt and display.
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Implications

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Social Contagion Models

Background
Granovetter's model
Network version
Groups
Chaos

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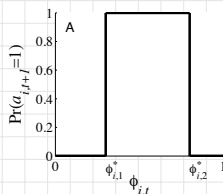
Chaotic contagion:

- ▶ What if individual response functions are not monotonic?
- ▶ Consider a simple deterministic version:
- ▶ Node i has an 'activation threshold' $\phi_{i,1}$
... and a 'de-activation threshold' $\phi_{i,2}$
- ▶ Nodes like to imitate but only up to a limit—they don't want to be like everyone else.



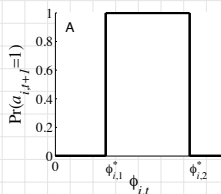
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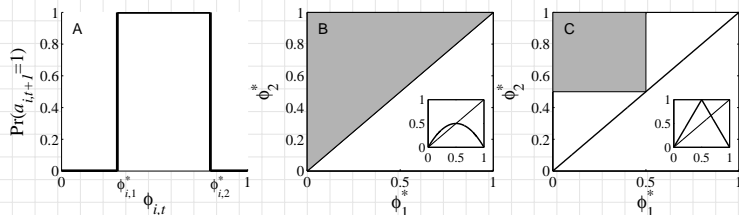


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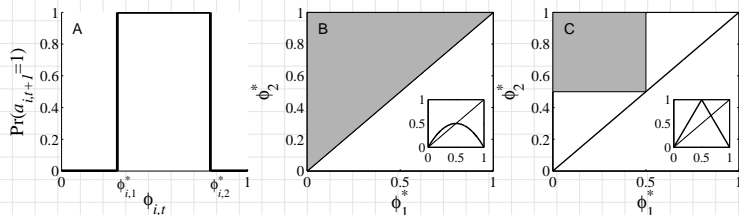
Two population examples:



- ▶ Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- ▶ Insets show composite response function averaged over population.
- ▶ We'll consider plot C's example: [the tent map](#).



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Definition of the tent map:

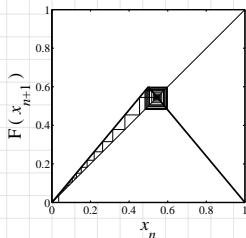
$$F(x) = \begin{cases} rx & \text{for } 0 \leq x \leq \frac{1}{2}, \\ r(1-x) & \text{for } \frac{1}{2} \leq x \leq 1. \end{cases}$$

- ▶ The usual business: look at how F iteratively maps the unit interval $[0, 1]$.



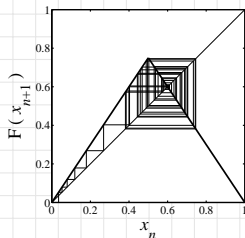
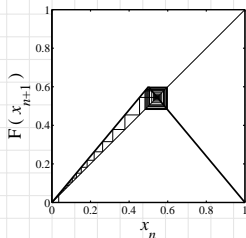
The tent map

Effect of increasing r from 1 to 2.



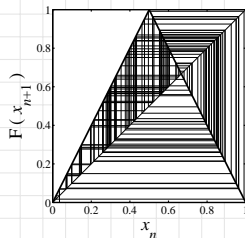
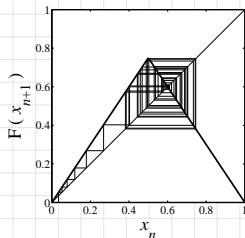
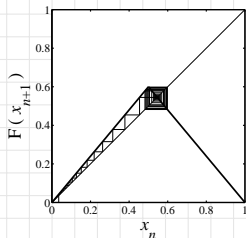
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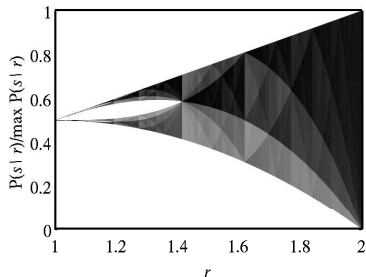
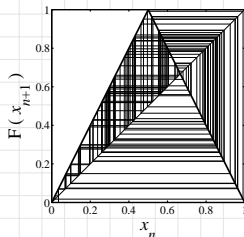
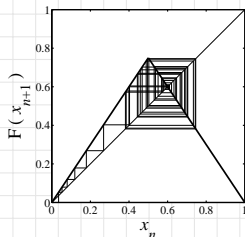
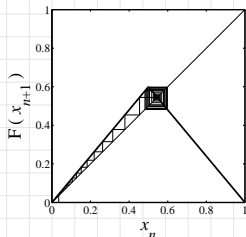


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The tent map

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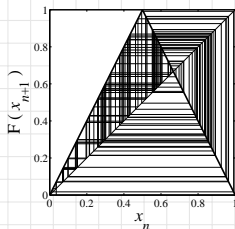
Orbit diagram:

Chaotic behavior increases as map slope r is increased.



Chaotic behavior

Take $r = 2$ case:

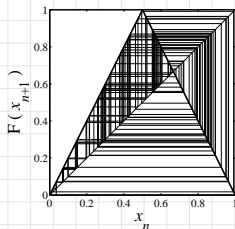


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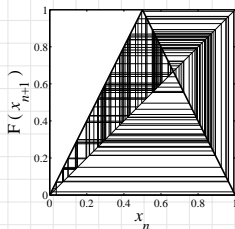


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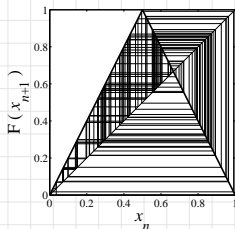


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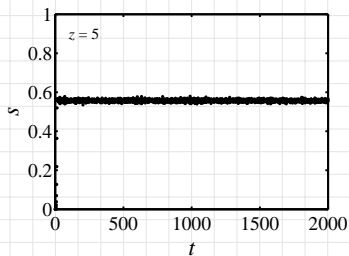
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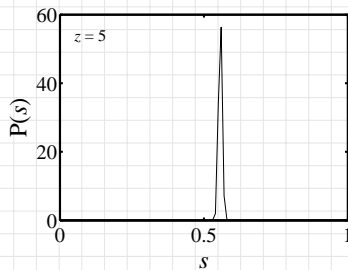
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Invariant densities—stochastic response functions



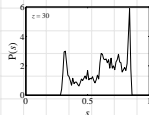
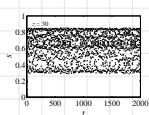
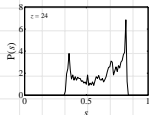
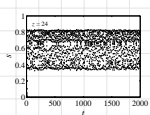
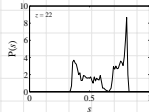
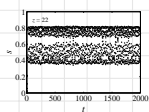
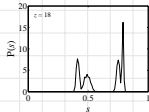
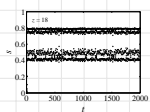
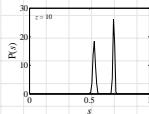
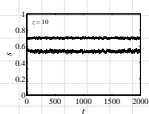
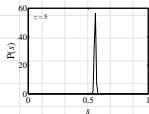
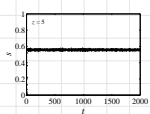
activation time series



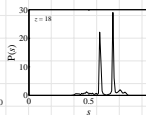
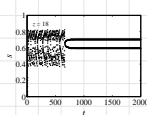
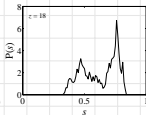
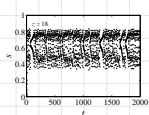
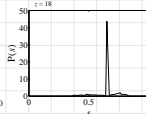
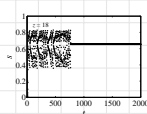
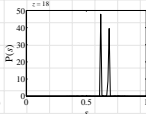
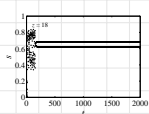
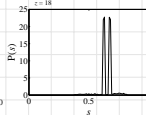
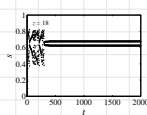
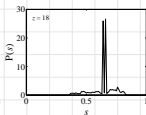
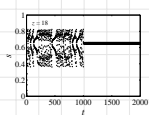
activation density



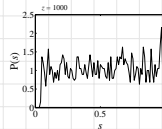
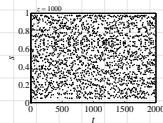
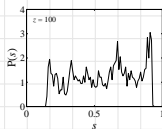
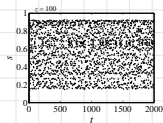
Invariant densities—stochastic response functions



Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



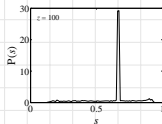
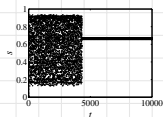
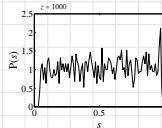
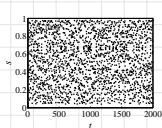
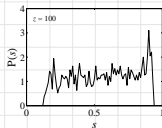
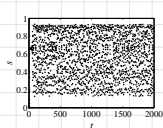
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Trying out higher values of $\langle k \rangle$...



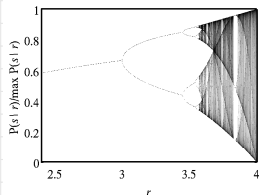
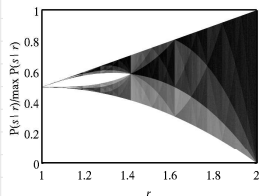
Invariant densities—deterministic response functions



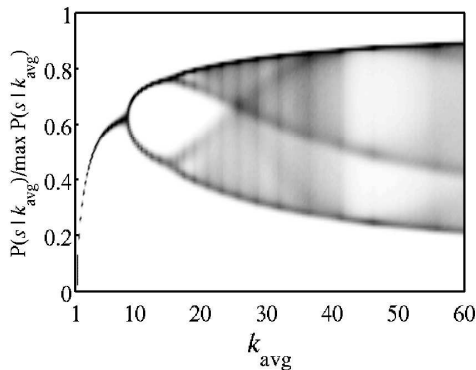
Trying out higher values of $\langle k \rangle$...



Connectivity leads to chaos:



Stochastic response functions:



Chaotic behavior in coupled systems

Coupled maps are well explored
(Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

► \mathcal{N}_i = neighborhood of node i

1. Node states are **continuous**
2. Increase δ and neighborhood size $|\mathcal{N}_i|$
 \Rightarrow **synchronization**

But for contagion model:

1. Node states are **binary**
2. **Asynchrony** remains as connectivity increases



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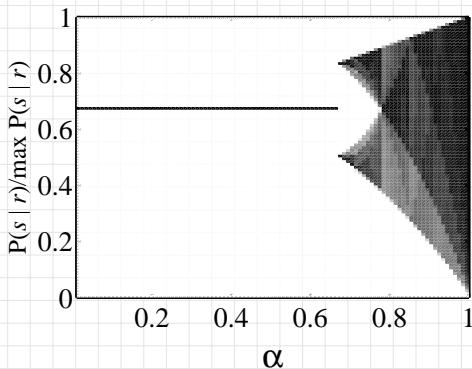
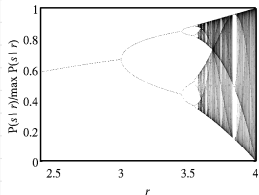
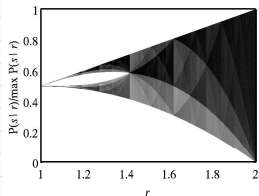
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Bifurcation diagram: Asynchronous updating



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