

Social Contagion

Principles of Complex Systems CSYS/MATH 300, Fall, 2010

Prof. Peter Dodds

Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont



The
UNIVERSITY
of VERMONT



COMPLEX SYSTEMS CENTER



Social Contagion
Models

Background
Granovetter's model
Network version
Groups
Chaos

References

Outline

Social Contagion Models

Background

Granovetter's model

Network version

Groups

Chaos

References

Social Contagion

Social Contagion
Models

Background

Granovetter's model

Network version

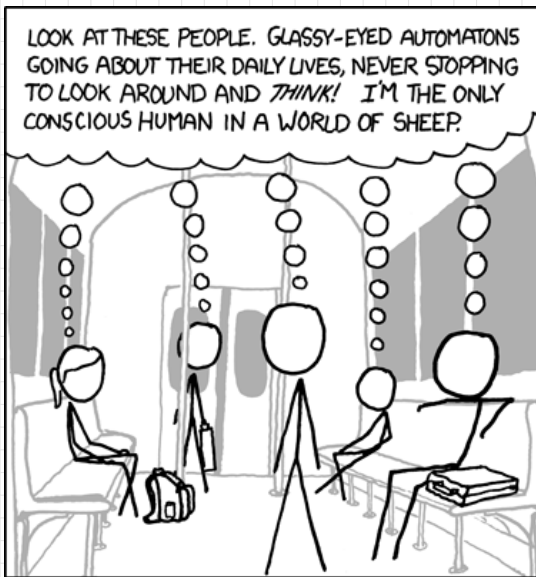
Groups

Chaos

References



Social Contagion



<http://xkcd.com/610/> (田)



Social Contagion



Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

Groups

Chaos

References



Social Contagion



Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

Groups


Chaos

References



Social Contagion

Examples abound

- ▶ fashion
- ▶ striking
- ▶ smoking (田) [6]
- ▶ residential segregation [16]
- ▶ ipods
- ▶ obesity (田) [5]
- ▶ Harry Potter
- ▶ voting
- ▶ gossip
- ▶ Rubik's cube 
- ▶ religious beliefs
- ▶ **leaving lectures**

SIR and SIR**S** contagion possible

- ▶ Classes of behavior versus specific behavior: **dieting**



Framingham heart study:

Evolving network stories (Christakis and Fowler):

- ▶ The spread of quitting smoking (田) [6]
- ▶ The spread of spreading (田) [5]
- ▶ Also: happiness (田) [8], loneliness, ...
- ▶ The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (田)

Controversy:

- ▶ Are your friends making you fat? (田) (Clive Thomson, NY Times, September 10, 2009).
- ▶ Everything is contagious (田)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).



Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

Groups

Chaos

References

Two focuses for us

- ▶ Widespread media influence
- ▶ Word-of-mouth influence

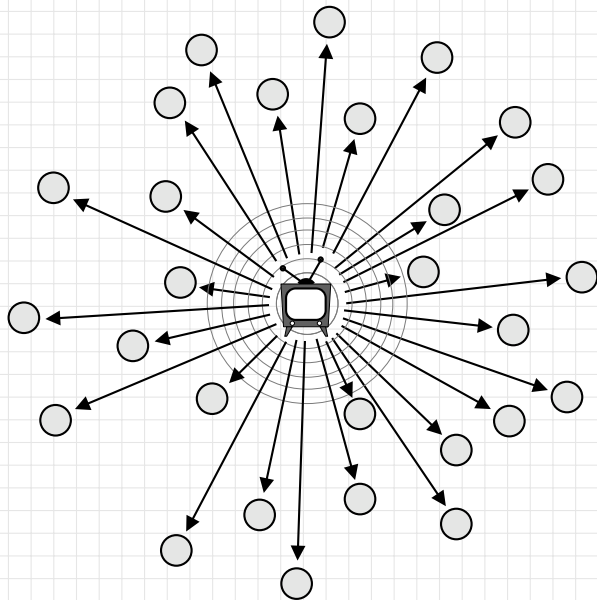


We need to understand influence

- ▶ Who influences whom? Very hard to measure...
- ▶ What kinds of influence response functions are there?
- ▶ Are some individuals super influencers?
Highly popularized by Gladwell^[9] as 'connectors'
- ▶ The infectious idea of opinion leaders (Katz and Lazarsfeld)^[13]



The hypodermic model of influence



Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

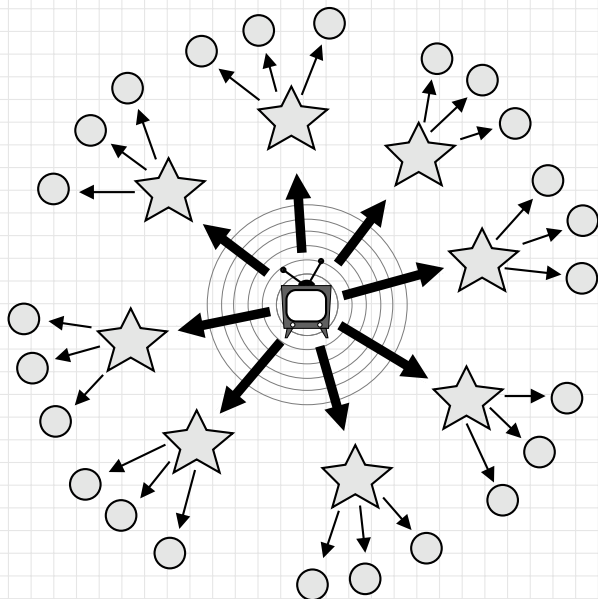
Groups

Chaos

References



The two step model of influence [13]



Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

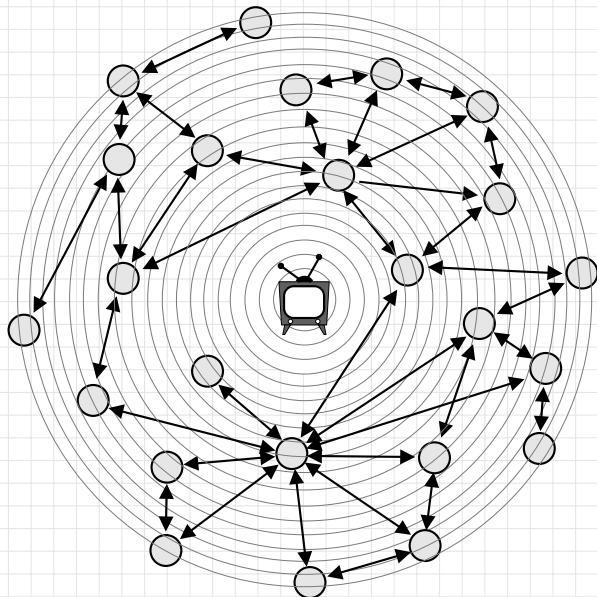
Groups

Chaos

References



The general model of influence



Social Contagion

Social Contagion
Models

Background

Granovetter's model

Network version

Groups

Chaos

References

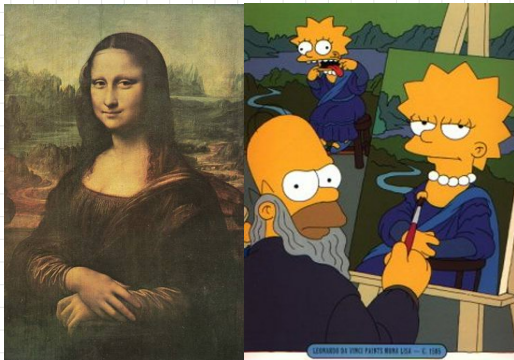


Why do things spread?

- ▶ Because of properties of special individuals?
- ▶ Or system level properties?
- ▶ Is the match that lights the fire important?
- ▶ Yes. But only because we are narrative-making machines...
- ▶ We like to think things happened for reasons...
- ▶ Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- ▶ System/group properties harder to understand
- ▶ Always good to examine what is said before and after the fact...



The Mona Lisa



- ▶ “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- ▶ Not the world’s greatest painting from the start...
- ▶ Escalation through theft, vandalism, **parody**, ...

Background

Granovetter's model

Network version

Groups

Chaos



The completely unpredicted fall of Eastern Europe



Timur Kuran: ^[14, 15] “Now Out of Never: The Element of Surprise in the East European Revolution of 1989”

Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

Groups

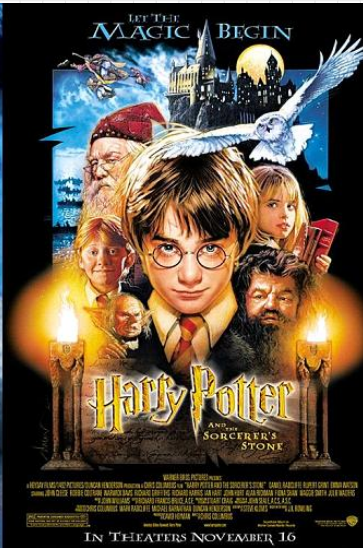
Chaos

References



The dismal predictive powers of editors...

Social Contagion



Social Contagion Models

Background

Granovetter's model

Network version

Groups

Chaos

References



Messaging with social connections

- ▶ Ads based on message content (e.g., Google and email)
- ▶ Buzz media
- ▶ Facebook's advertising: Beacon (田)



Getting others to do things for you

A very good book: **'Influence'** by Robert Cialdini ^[7]

Six modes of influence

1. **Reciprocation**: *The Old Give and Take... and Take*
2. **Commitment and Consistency**: *Hobgoblins of the Mind*
3. **Social Proof**: *Truths Are Us*
4. **Liking**: *The Friendly Thief*
5. **Authority**: *Directed Deference*
6. **Scarcity**: *The Rule of the Few*



Examples

- ▶ **Reciprocation**: Free samples, Hare Krishnas
- ▶ **Commitment and Consistency**: Hazing
- ▶ **Social Proof**: Catherine Genovese, Jonestown
- ▶ **Liking**: Separation into groups is enough to cause problems.
- ▶ **Authority**: Milgram's obedience to authority experiment.
- ▶ **Scarcity**: Prohibition.



Getting others to do things for you

- ▶ Cialdini's modes are heuristics that help up us get through life.
- ▶ Useful but can be leveraged...

Social Contagion

Social Contagion
Models

Background

Granovetter's model

Network version

Groups

Chaos

References



Other acts of influence

- ▶ Conspicuous Consumption (Veblen, 1912)
- ▶ Conspicuous Destruction (Potlatch)



Some important models

- ▶ Tipping models—Schelling (1971) [16, 17, 18]
 - ▶ Simulation on checker boards
 - ▶ Idea of thresholds
 - ▶ Explore the Netlogo (田) implementation [21]
- ▶ Threshold models—Granovetter (1978) [10]
- ▶ Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
 - ▶ Social learning theory, Informational cascades,...



Thresholds

- ▶ Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- ▶ 'Others' may be everyone in a population, an individual's close friends, any reference group.
- ▶ Response can be probabilistic or deterministic.
- ▶ Individual thresholds can vary
- ▶ Assumption: order of others' adoption does not matter... **(unrealistic)**.
- ▶ Assumption: level of influence per person is uniform **(unrealistic)**.



Some possible origins of thresholds:

- ▶ **Desire to coordinate**, to conform.
- ▶ **Lack of information**: impute the worth of a good or behavior based on degree of adoption (social proof)
- ▶ Economics: **Network effects** or **network externalities**
- ▶ Externalities = Effects on others not directly involved in a transaction
- ▶ Examples: telephones, fax machine, Facebook, operating systems
- ▶ An individual's utility increases with the adoption level among peers and the population in general

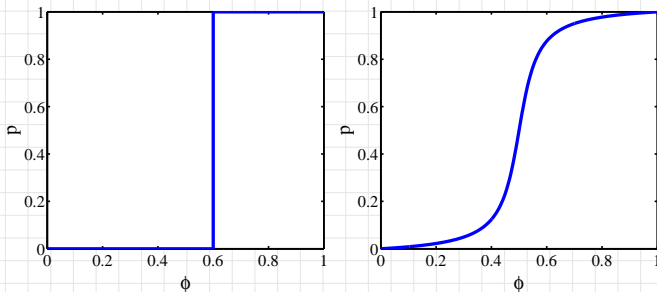


Granovetter's Threshold model—definitions

- ▶ ϕ^* = threshold of an individual.
- ▶ $f(\phi_*)$ = distribution of thresholds in a population.
- ▶ $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)d\phi'_*$
- ▶ ϕ_t = fraction of people 'rioting' at time step t .



Threshold models



- ▶ Example threshold influence response functions:
deterministic and **stochastic**
- ▶ ϕ = fraction of contacts 'on' (e.g., rioting)
- ▶ Two states: S and I.



Threshold models

- ▶ At time $t + 1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.



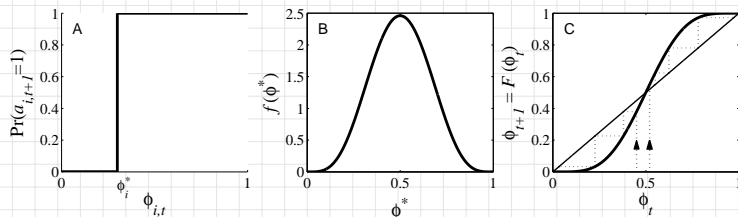
$$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*) d\phi_* = F(\phi_*) \Big|_0^{\phi_t} = F(\phi_t)$$

- ▶ \Rightarrow Iterative maps of the unit interval $[0, 1]$.



Threshold models

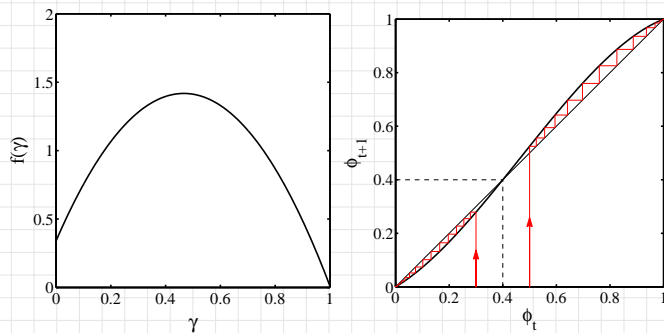
Action based on perceived behavior of others.



- ▶ Two states: S and I.
- ▶ ϕ = fraction of contacts 'on' (e.g., rioting)
- ▶ Discrete time update (strong assumption!)
- ▶ This is a **Critical mass model**



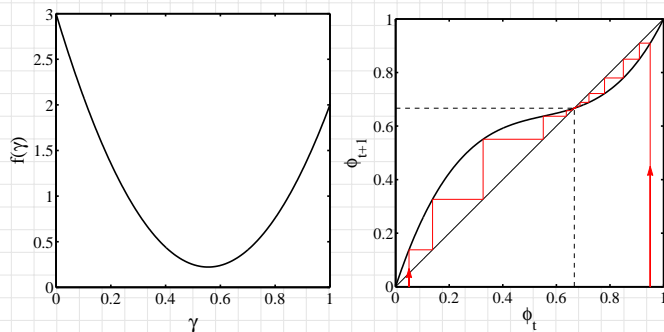
Threshold models



- ▶ Another example of critical mass model...



Threshold models



- ▶ Example of single stable state model



Threshold models

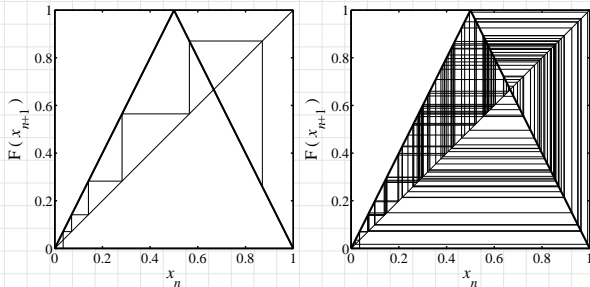
Implications for collective action theory:

1. Collective uniformity \nRightarrow individual uniformity
2. Small individual changes \Rightarrow large global changes



Threshold models

Chaotic behavior possible [12, 11]



- ▶ Period doubling arises as map amplitude r is increased.
- ▶ Synchronous update assumption is crucial



Threshold model on a network

Many years after Granovetter and Soong's work:

“A simple model of global cascades on random networks”

D. J. Watts. Proc. Natl. Acad. Sci., 2002 ^[20]

- ▶ Mean field model \rightarrow network model
- ▶ Individuals now have a limited view of the world

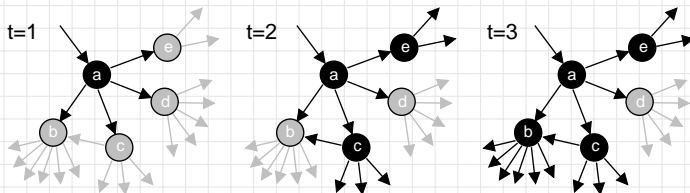


Threshold model on a network

- ▶ Interactions between individuals now represented by a network
- ▶ Network is **sparse**
- ▶ Individual i has k_i contacts
- ▶ Influence on each link is **reciprocal** and of **unit weight**
- ▶ Each individual i has a fixed threshold ϕ_i
- ▶ Individuals repeatedly poll contacts on network
- ▶ Synchronous, discrete time updating
- ▶ Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- ▶ Individuals remain active when switched (no recovery = SI model)



Threshold model on a network



- ▶ All nodes have threshold $\phi = 0.2$.



The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?



First study random networks:

- ▶ Start with N nodes with a degree distribution p_k
- ▶ Nodes are randomly connected (carefully so)
- ▶ Aim: Figure out when activation will propagate
- ▶ Determine a **cascade condition**



Follow active links

- ▶ An active link is a link connected to an activated node.
- ▶ If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- ▶ We need to understand which nodes can be activated when only one of their neighbors becomes active.



The most gullible

Vulnerables:

- ▶ We call individuals who can be activated by just one contact being active **vulnerables**
- ▶ The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

- ▶ Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- ▶ For global cascades on random networks, must have a *global cluster of vulnerables* ^[20]
- ▶ **Cluster of vulnerables = critical mass**
- ▶ Network story: 1 node \rightarrow critical mass \rightarrow everyone.



Cascade condition

Back to following a link:

- ▶ A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- ▶ Follows from there being k ways to connect to a node with degree k .
- ▶ Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- ▶ So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



Cascade condition

Next: Vulnerability of linked node

- ▶ Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) d\phi'_*$$

- ▶ If linked node is **vulnerable**, it produces $k - 1$ **new** outgoing active links
- ▶ If linked node is **not vulnerable**, it produces **no** active links.



Cascade condition

Putting things together:

- ▶ Expected number of active edges produced by an active edge:

$$R = \sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1 - \beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}}$$
$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$



Cascade condition

So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- ▶ β_k = probability a degree k node is vulnerable.
- ▶ P_k = probability a node has degree k .



Cascade condition

Two special cases:

- ▶ (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

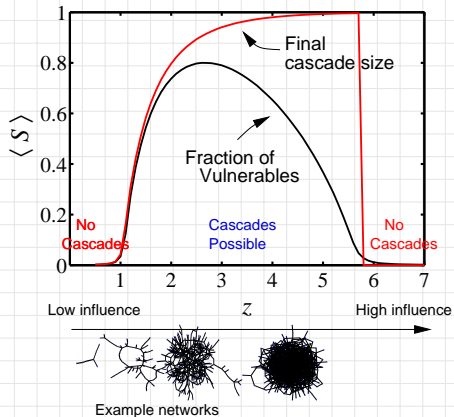
$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- ▶ (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$



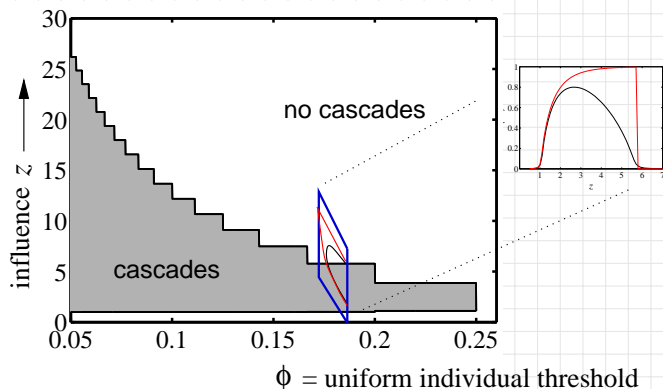
Cascades on random networks



- ▶ Cascades occur only if size of max vulnerable cluster > 0 .
- ▶ System may be 'robust-yet-fragile'.
- ▶ 'Ignorance' facilitates spreading.



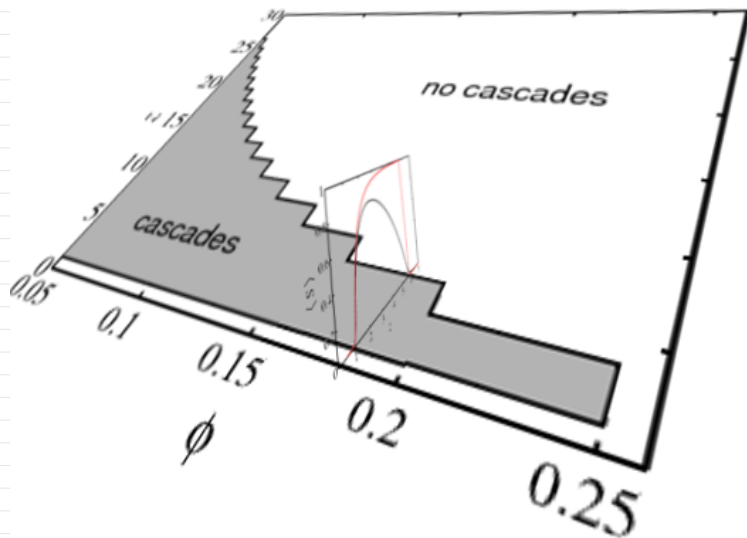
Cascade window for random networks



- ▶ **'Cascade window'** widens as threshold ϕ decreases.
- ▶ Lower thresholds enable spreading.



Cascade window for random networks



Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

Groups

Chaos

References



Cascade window—summary

For our simple model of a uniform threshold:

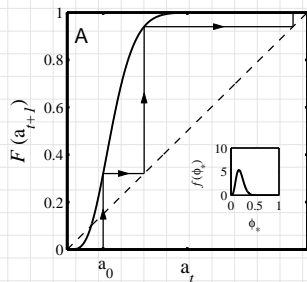
1. **Low $\langle k \rangle$** : No cascades in poorly connected networks. No global clusters of any kind.
2. **High $\langle k \rangle$** : Giant component exists but not enough vulnerables.
3. **Intermediate $\langle k \rangle$** : Global cluster of vulnerables exists. Cascades are possible in **“Cascade window.”**



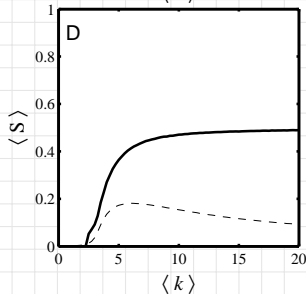
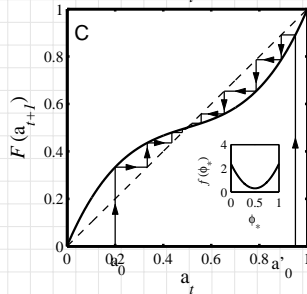
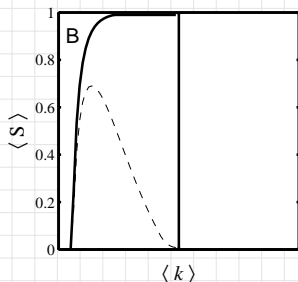
All-to-all versus random networks



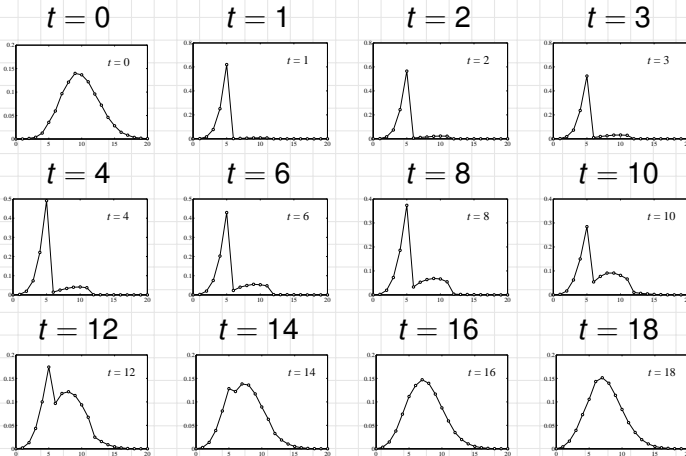
all-to-all networks



random networks



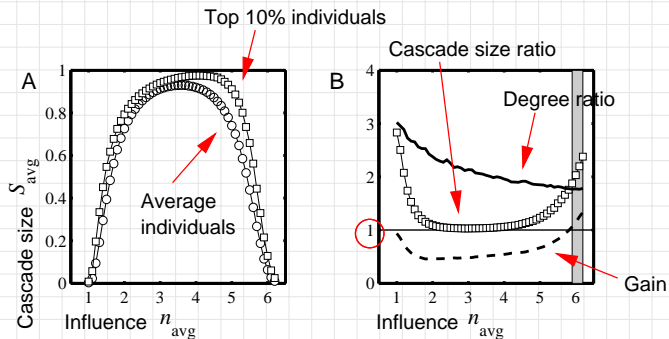
Early adopters—degree distributions



$P_{k,t}$ versus k



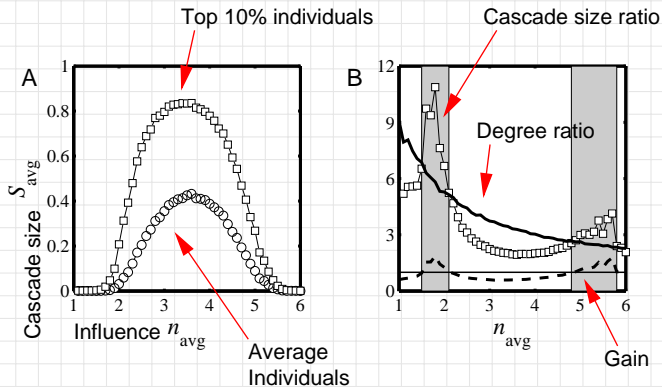
The multiplier effect:



- ▶ Fairly uniform levels of individual influence.
- ▶ Multiplier effect is mostly below 1.



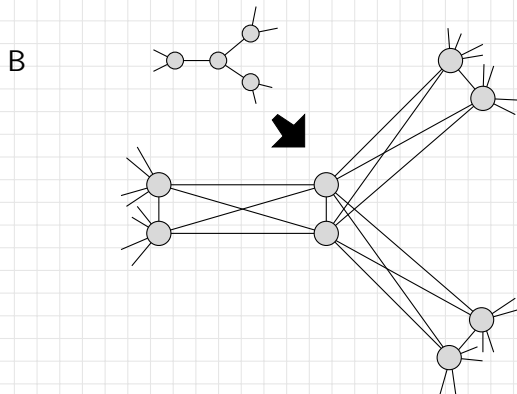
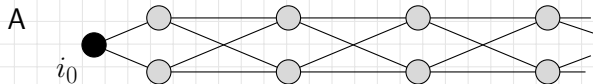
The multiplier effect:



- ▶ Skewed influence distribution example.



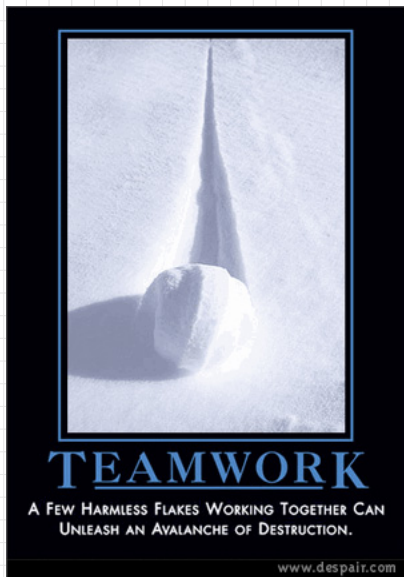
Special subnetworks can act as triggers



► $\phi = 1/3$ for all nodes



The power of groups...



despair.com

“A few harmless flakes working together can unleash an avalanche of destruction.”

Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

Groups

Chaos

References

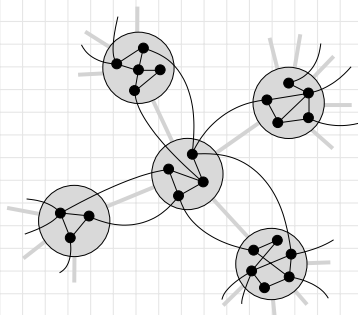


Extensions

- ▶ Assumption of sparse interactions is good
- ▶ Degree distribution is (generally) key to a network's function
- ▶ Still, random networks don't represent all networks
- ▶ Major element missing: **group structure**



Group structure—Ramified random networks



p = intergroup connection probability
 q = intragroup connection probability.

Social Contagion

Social Contagion Models

Background

Granovetter's model

Network version

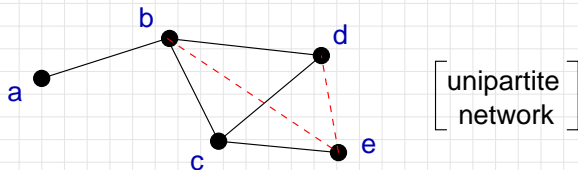
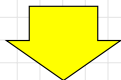
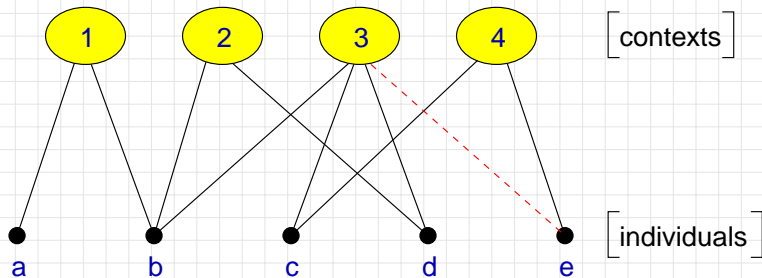
Groups

Chaos

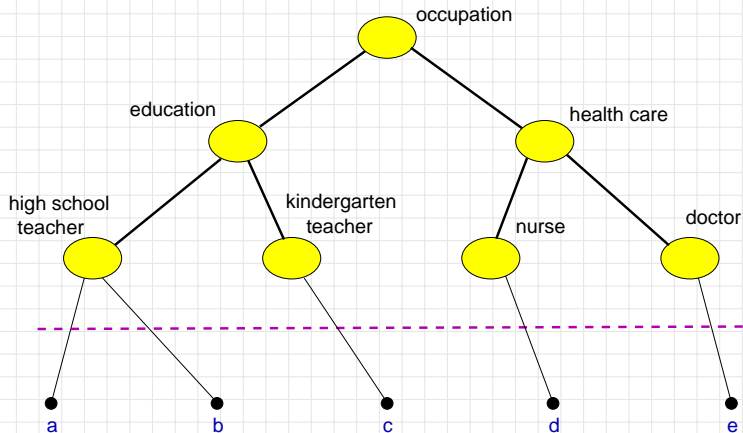
References



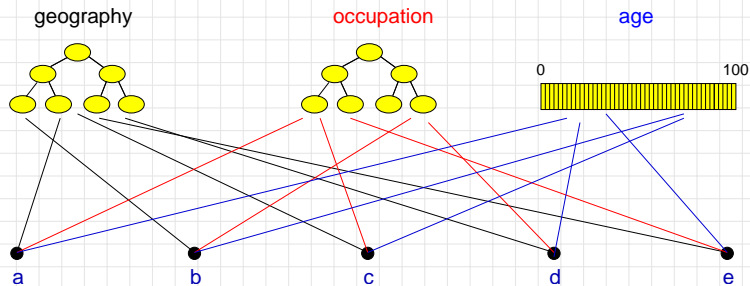
Bipartite networks



Context distance



Generalized affiliation model



(Blau & Schwartz, Simmel, Breiger)

Social Contagion

Social Contagion Models

Background
Granovetter's model
Network version
Groups
Chaos

References

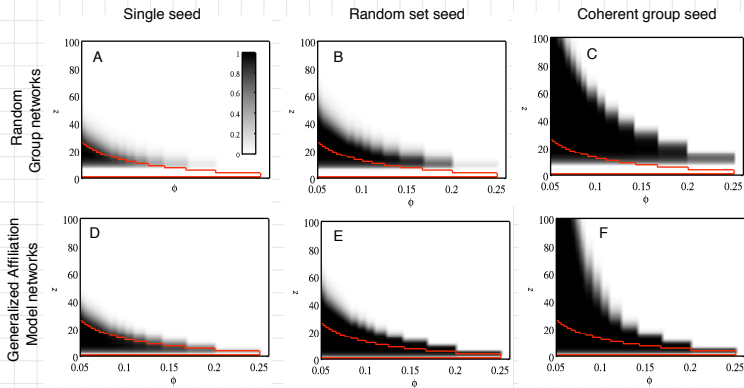


Generalized affiliation model networks with triadic closure

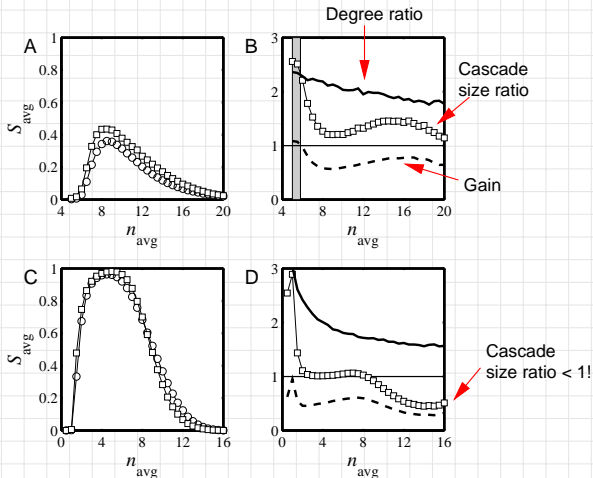
- ▶ Connect nodes with probability $\propto \exp^{-\alpha d}$
where
 α = homophily parameter
and
 d = distance between nodes (height of lowest common ancestor)
- ▶ τ_1 = intergroup probability of friend-of-friend connection
- ▶ τ_2 = intragroup probability of friend-of-friend connection



Cascade windows for group-based networks



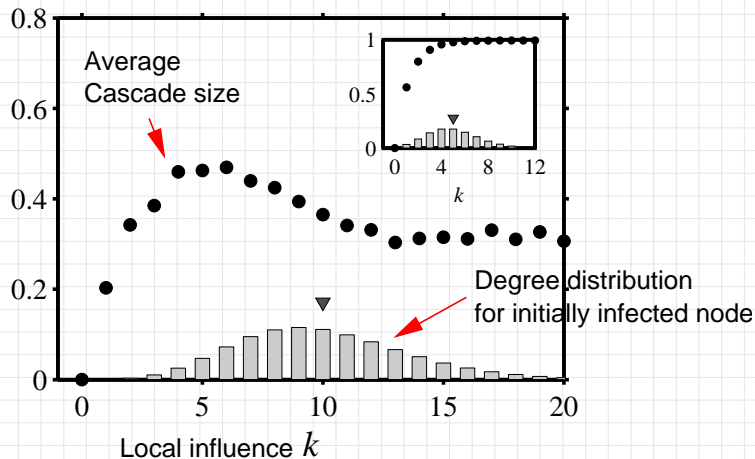
Multiplier effect for group-based networks:



► Multiplier almost always below 1.



Assortativity in group-based networks



- ▶ The most connected nodes aren't always the most 'influential.'
- ▶ **Degree assortativity** is the reason.



Summary

- ▶ 'Influential vulnerables' are key to spread.
- ▶ Early adopters are mostly vulnerables.
- ▶ Vulnerable nodes important but not necessary.
- ▶ Groups may greatly facilitate spread.
- ▶ Seems that cascade condition is a global one.
- ▶ Most extreme/unexpected cascades occur in highly connected networks
- ▶ 'Influentials' are posterior constructs.
- ▶ Many potential influentials exist.



Implications

- ▶ Focus on **the influential vulnerables**.
- ▶ Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- ▶ Only **simple ideas** can spread by word-of-mouth.
(Idea of opinion leaders spreads well...)
- ▶ Want enough individuals who will adopt and display.
- ▶ Displaying can be **passive** = free (yo-yo's, fashion), or **active** = harder to achieve (political messages).
- ▶ Entities can be novel or designed to combine with others, e.g. block another one.

Social Contagion Models

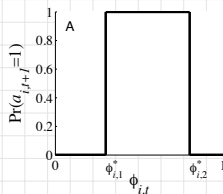
Background
Granovetter's model
Network version
Groups
Chaos

References

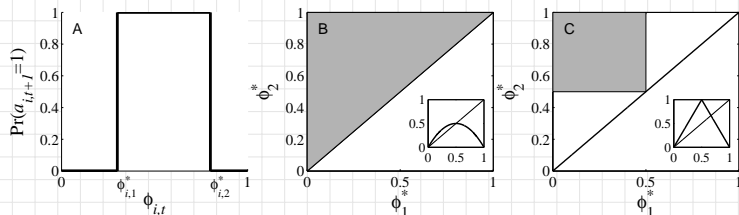


Chaotic contagion:

- ▶ What if individual response functions are not monotonic?
- ▶ Consider a simple deterministic version:
- ▶ Node i has an 'activation threshold' $\phi_{i,1}$
... and a 'de-activation threshold' $\phi_{i,2}$
- ▶ Nodes like to imitate but only up to a limit—they don't want to be like everyone else.



Two population examples:



- ▶ Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- ▶ Insets show composite response function averaged over population.
- ▶ We'll consider plot C's example: [the tent map](#).



Definition of the tent map:

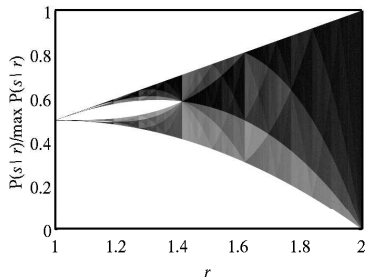
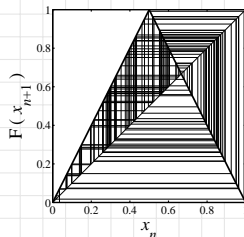
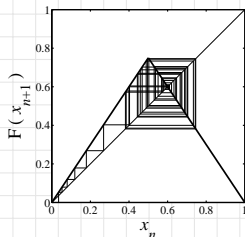
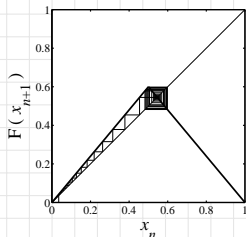
$$F(x) = \begin{cases} rx & \text{for } 0 \leq x \leq \frac{1}{2}, \\ r(1-x) & \text{for } \frac{1}{2} \leq x \leq 1. \end{cases}$$

- ▶ The usual business: look at how F iteratively maps the unit interval $[0, 1]$.



The tent map

Effect of increasing r from 1 to 2.



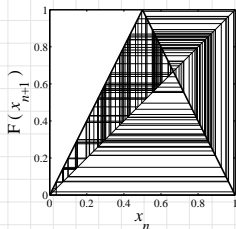
Orbit diagram:

Chaotic behavior increases as map slope r is increased.



Chaotic behavior

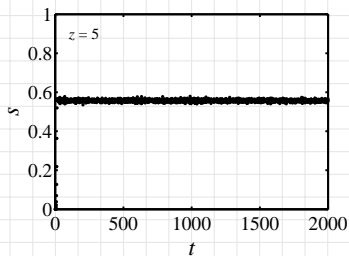
Take $r = 2$ case:



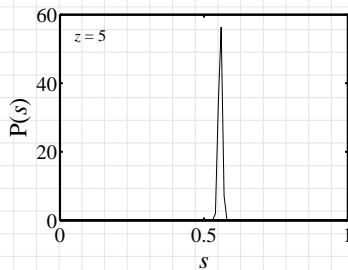
- ▶ What happens if nodes have limited information?
- ▶ As before, allow interactions to take place on a sparse random network.
- ▶ Vary average degree $z = \langle k \rangle$, a measure of information



Invariant densities—stochastic response functions



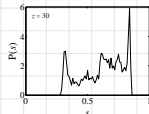
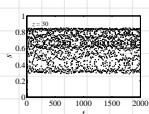
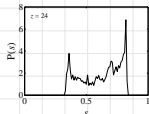
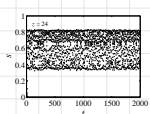
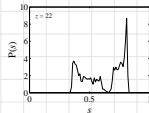
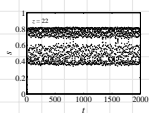
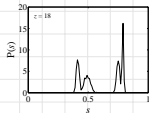
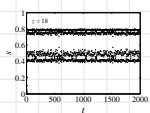
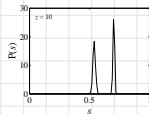
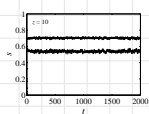
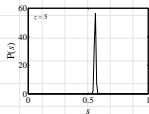
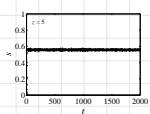
activation time series



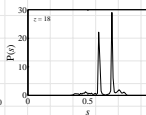
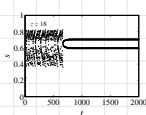
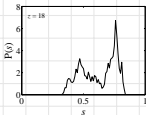
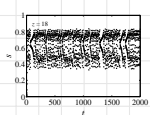
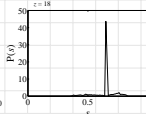
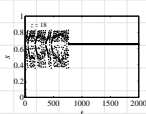
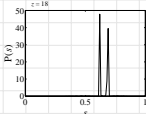
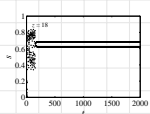
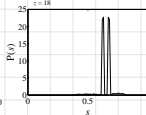
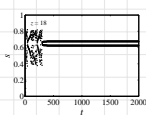
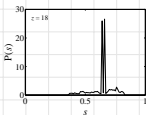
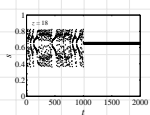
activation density



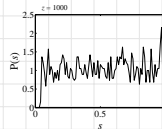
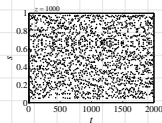
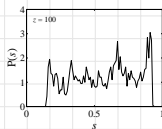
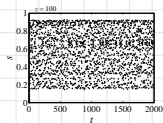
Invariant densities—stochastic response functions



Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



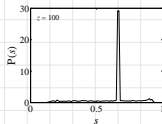
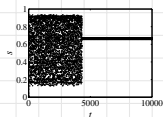
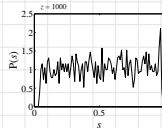
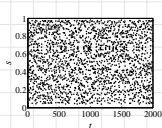
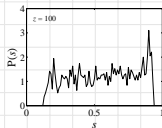
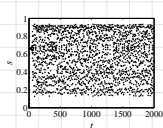
Invariant densities—stochastic response functions



Trying out higher values of $\langle k \rangle$...



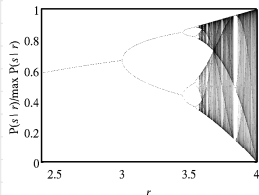
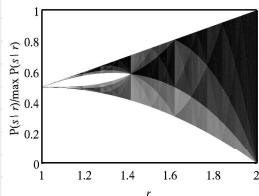
Invariant densities—deterministic response functions



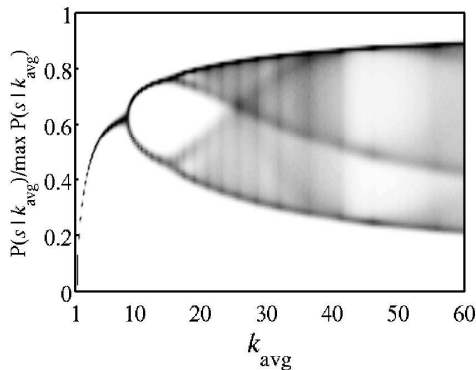
Trying out higher values of $\langle k \rangle$...



Connectivity leads to chaos:



Stochastic response functions:



Chaotic behavior in coupled systems

Coupled maps are well explored
(Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

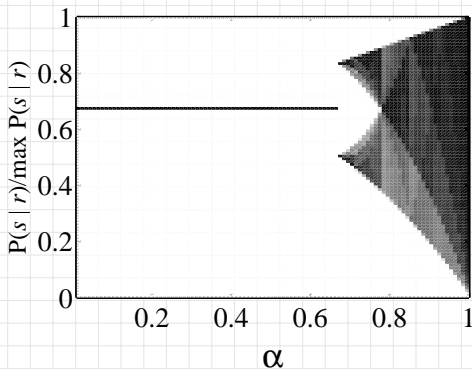
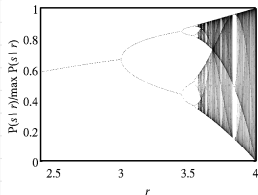
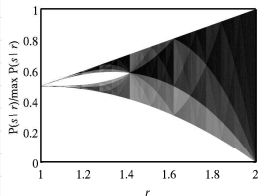
- ▶ \mathcal{N}_i = neighborhood of node i
- 1. Node states are **continuous**
- 2. Increase δ and neighborhood size $|\mathcal{N}|$
 \Rightarrow **synchronization**

But for contagion model:

1. Node states are **binary**
2. **Asynchrony remains** as connectivity increases



Bifurcation diagram: Asynchronous updating



Social Contagion

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Groups
- Chaos**
-

References



References I

- [1] S. Bikhchandani, D. Hirshleifer, and I. Welch.
A theory of fads, fashion, custom, and cultural change as informational cascades.
[J. Polit. Econ.](#), 100:992–1026, 1992.
- [2] S. Bikhchandani, D. Hirshleifer, and I. Welch.
Learning from the behavior of others: Conformity, fads, and informational cascades.
[J. Econ. Perspect.](#), 12(3):151–170, 1998. pdf (田)
- [3] J. M. Carlson and J. Doyle.
Highly optimized tolerance: A mechanism for power laws in design systems.
[Phys. Rev. E](#), 60(2):1412–1427, 1999. pdf (田)



References II

- [4] J. M. Carlson and J. Doyle.
Highly optimized tolerance: Robustness and design
in complex systems.
[Phys. Rev. Lett., 84\(11\):2529–2532, 2000.](#) pdf (田)
- [5] N. A. Christakis and J. H. Fowler.
The spread of obesity in a large social network over
32 years.
[New England Journal of Medicine, 357:370–379,
2007.](#) pdf (田)
- [6] N. A. Christakis and J. H. Fowler.
The collective dynamics of smoking in a large social
network.
[New England Journal of Medicine, 358:2249–2258,
2008.](#) pdf (田)



References III

- [7] R. B. Cialdini.
Influence: Science and Practice.
Allyn and Bacon, Boston, MA, 4th edition, 2000.
- [8] J. H. Fowler and N. A. Christakis.
Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.
[BMJ, 337:article #2338, 2008.](#) pdf (田)
- [9] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.
- [10] M. Granovetter.
Threshold models of collective behavior.
[Am. J. Sociol., 83\(6\):1420–1443, 1978.](#) pdf (田)



References IV

- [11] M. Granovetter and R. Soong.
Threshold models of diversity: Chinese restaurants,
residential segregation, and the spiral of silence.
[Sociological Methodology, 18:69–104, 1988. pdf \(田\)](#)
- [12] M. S. Granovetter and R. Soong.
Threshold models of interpersonal effects in
consumer demand.
[Journal of Economic Behavior & Organization, 7:83–99, 1986.](#)
Formulates threshold as function of price, and
introduces exogenous supply curve. [pdf \(田\)](#)
- [13] E. Katz and P. F. Lazarsfeld.
[Personal Influence.](#)
The Free Press, New York, 1955.



References V

- [14] T. Kuran.
Now out of never: The element of surprise in the
east european revolution of 1989.
[World Politics](#), 44:7–48, 1991. pdf (田)
- [15] T. Kuran.
Private Truths, Public Lies: The Social
Consequences of Preference Falsification.
[Harvard University Press, Cambridge, MA, Reprint
edition, 1997.](#)
- [16] T. Schelling.
Dynamic models of segregation.
[J. Math. Sociol.](#), 1:143–186, 1971.



References VI

- [17] T. C. Schelling.
Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities.
[J. Conflict Resolut.](#), 17:381–428, 1973. pdf (田)
- [18] T. C. Schelling.
Micromotives and Macrobehavior.
Norton, New York, 1978.
- [19] D. Sornette.
Critical Phenomena in Natural Sciences.
Springer-Verlag, Berlin, 2nd edition, 2003.
- [20] D. J. Watts.
A simple model of global cascades on random networks.
[Proc. Natl. Acad. Sci.](#), 99(9):5766–5771, 2002.
pdf (田)



[21] U. Wilensky.

Netlogo segregation model.

<http://ccl.northwestern.edu/netlogo/models/Segregation>. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL., 1998.

