

The Amusing Law of Benford

Principles of Complex Systems
CSYS/MATH 300, Fall, 2010

Prof. Peter Dodds

Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont



The
UNIVERSITY
of VERMONT



COMPLEX SYSTEMS CENTER



Outline

Benford's law

Benford's Law

References

Benford's Law

References



The law of first digits

Benford's Law: (田)

- ▶
$$P(\text{first digit} = d) \propto \log_b(1 + 1/d)$$
for certain sets of 'naturally' occurring numbers in base b
- ▶ Around 30.1% of first digits are '1', compared to only 4.6% for '9'.
- ▶ First observed by Simon [Newcomb](#)^[2] in 1881
"Note on the Frequency of Use of the Different Digits in Natural Numbers"
- ▶ Independently discovered in 1938 by [Frank Benford](#) (田).
- ▶ Newcomb almost always noted but Benford gets the stamp.



Benford's Law—The Law of First Digits

Benford's law

Benford's Law

References

Observed for

- ▶ Fundamental constants (electron mass, charge, etc.)
- ▶ Utility bills
- ▶ Numbers on tax returns (ha!)
- ▶ Death rates
- ▶ Street addresses
- ▶ Numbers in newspapers

- ▶ Cited as evidence of fraud (⊕) in the 2009 Iranian elections.



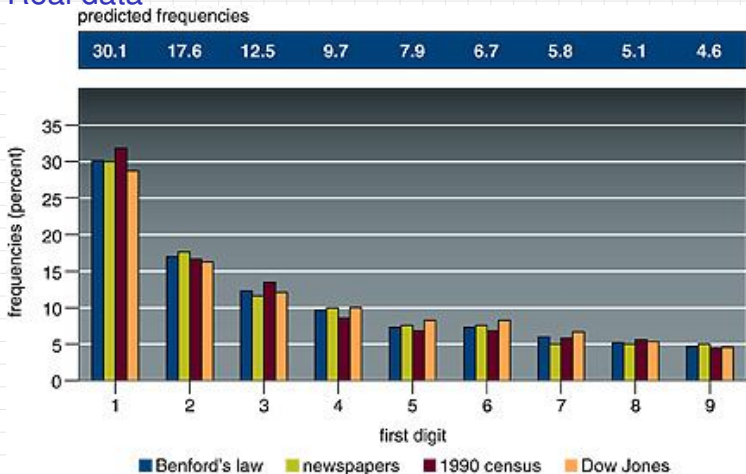
Benford's Law

Benford's law

Real data

Benford's Law

References

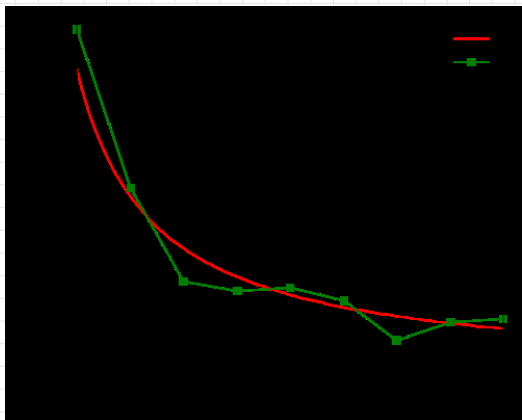


From 'The First-Digit Phenomenon' by T. P. Hill (1998)^[1]



Benford's Law

Physical constants of the universe:



Taken from [here](#) (田).

Benford's law

Benford's Law

References



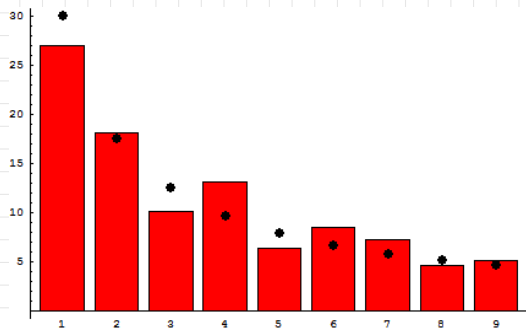
Benford's Law

Benford's law

Population of countries:

Benford's Law

References



Taken from [here](#) (⊞).



Essential story



$$P(\text{first digit} = d) \propto \log_b(1 + 1/d)$$

$$\propto \log_b\left(\frac{d+1}{d}\right)$$

$$\propto \log_b(d+1) - \log_b(d)$$

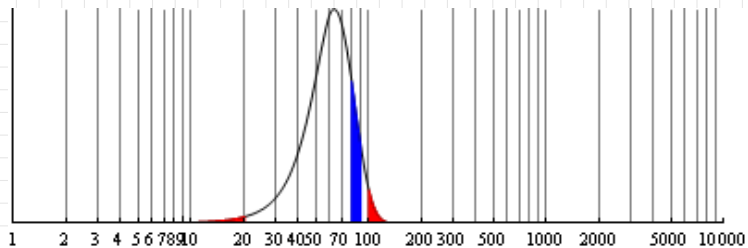
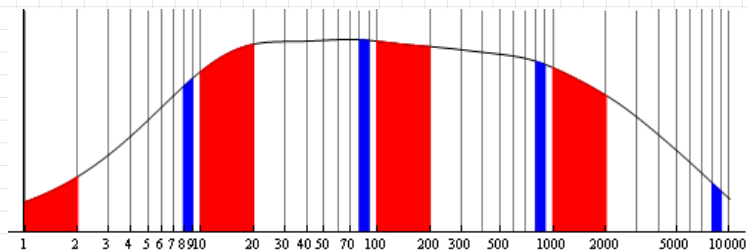
- ▶ Observe this distribution if numbers are distributed uniformly in log-space:

$$P(\ln x) d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} dx$$

- ▶ Power law distributions at work again...
- ▶ Extreme case of $\gamma \simeq 1$.



Benford's law



Taken from [here](#) (田).

Benford's law

Benford's Law

References



References I

- [1] T. P. Hill.
The first-digit phenomenon.
[American Scientist](#), 86:358–, 1998.
- [2] S. Newcomb.
Note on the frequency of use of the different digits in
natural numbers.
[American Journal of Mathematics](#), 4:39–40, 1881.
[pdf](#) (田)

