Principles of Complex Systems, CSYS/MATH 300—Assignment 2
University of Vermont, Fall 2010

Dispersed: Monday, October 4, 2010.
Due: By start of lecture, 1:00 pm, Thursday, October 14, 2010.
Some useful reminders:
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Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2010-08UVM-300`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. Consider a modified version of the Barabàsi-Albert (BA) model [1] where two possible mechanisms are now in play. As in the original model, start with m_0 nodes at time $t = 0$. Let's make these initial guys connected such that each has degree 1. The two mechanisms are:

   M1: With probability p, a new node of degree 1 is added to the network. At time $t + 1$, a node connects to an existing node j with probability

   $$P(\text{connect to node } j) = \frac{k_j}{\sum_{i=1}^{N(t)} k_i} \tag{1}$$

   where k_j is the degree of node j and $N(t)$ is the number of nodes in the system at time t.

   M2: With probability $q = 1 - p$, a randomly chosen node adds a new edge, connecting to node j with the same preferential attachment probability as above.

   Note that in the limit $q = 0$, we retrieve the original BA model (with the difference that we are adding one link at a time rather than m here).

   In the long time limit $t \to \infty$, what is the expected form of the degree distribution P_k?

   Do we move out of the original model's universality class?

2. Now take the Barabàsi-Albert model with an attachment kernel $A_k = k^{1/2}$. Take newly arriving nodes as adding m links ($m = 1$ for the preceding question).

   Use the same approach as in class (which is a modified version of the original derivation in [1]), to determine the long-time limiting form of the degree distribution P_k.

   A catch and a hint: to normalize the attachment kernel at each point in time t, we have to divide by the sum of all degrees in the network (as per Eq. 1 above). Recall that for the original model, the sum of all degrees nicely simplified to $2mt + m_0$ (check over this). But now we have the sum of $k_i^{1/2}$, and its form is not obvious. Here's the help: assume that

$$\sum_{i=1}^{N(t)} k_i^{1/2} = \lambda t$$

   where λ is to be determined later. In other words, assume that the normalization factor grows linearly with t, as it did for the original model. If this is indeed true, then you will be able to justify it once you have found P_k.

References

[1] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–511, 1999.