#### Lecture 7 (Chapter 2): Review

Linear Algebra MATH 124, Fall, 2010

#### Prof. Peter Dodds

Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont



















Ch 2: Lec 7



# Outline Review for Exam 1 Review for Exam 1



Ch. 2: Lec. 7





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- ► Chapter 1 and Chapter 2 (Sections 2.1–2.7)
- Chapter 2 is our focus
  - Knowledge of Chapter 1 as needed for Chapter 2 = solving  $A\vec{x} = \vec{b}$
- Want 'understanding' and 'doing' abilities.



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- What dimensions of A mean

  - $\blacktriangleright$   $n = \text{number of unknowns } (x_1, x_2, ...)$
- How to draw the row and column pictures.
- Be able to identify row picture
- (e.g., as representing 2 planes in 3-d)
- How to convert between the three pictures.





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### Solving $A\vec{x} = \vec{b}$ by elimination

### Review for Exam 1

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- Solve four equivalent ways:
- 2. Row operations on augmented r
  - Systematically transform  $A\vec{x} = \vec{b}$  into  $U\vec{x} = \vec{c}$
  - Solve by back substitution
- 3. Row operations with E<sub>ij</sub> and P<sub>ij</sub> matrices
- 4. Factor A as A = LU







### Solving $A\vec{x} = \vec{b}$ by elimination

### Review for Exam 1

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Review for Exam 1

#### Solve four equivalent ways:

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  - substitution\_\_\_\_\_
  - First  $L\vec{c} = \vec{b}$  then  $U\vec{x} = \vec{c}$ .
  - More generally, PA = LU.

#### Understand number of solutions business



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#### Understand number of solutions business:





- Be able to find the pivots of A (they live in U)
- Understand how elimination matrices (E<sub>ii</sub>'s) are
- Understand how L is made up of inverses of • e.g.:  $L = E_{21}^{-1} E_{21}^{-1} E_{22}^{-1} A$
- Understand how L is made up of the Iii multipliers.
- Understand how inverses of elimination matrices are



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- ▶ Be able to find the pivots of *A* (they live in *U*)
- Understand how elimination matrices (E<sub>ij</sub>'s) are constructed from multipliers (I<sub>ij</sub>'s)
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   ▶ e.g.: L = E<sub>21</sub> E<sub>31</sub> E<sub>32</sub> A
- e.g.: 4 = E<sub>21</sub> E<sub>31</sub> E<sub>32</sub> A
- Understand how L is made up of the I<sub>ij</sub> multipliers.
- Understand how inverses of elimination matrices are simply related to elimination matrices.





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 Understand basic matrix algebra Understand matrix multiplication Understand multiplication order matters Understand AB = BA is rarely true

Stuff to know:

Matrix algebra

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- Understand basic matrix algebra
- Understand matrix multiplication
- Understand multiplication order matters
- ► Understand *AB* = *BA* is rarely true

- Understand identity matrix I
- Understand AA 1 = A 1A = I
- Find A with Gauss-Jordan elimination
- ► Perform row reduction on augmented matrix [A|/].
- Understand that that finding  $A^{-1}$  solves  $A\vec{x} = \vec{b}$  but is
- $(AB)^{-1} B^{-1}A^{-1}$





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Review for Exam 1

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- Understand matrix multiplication
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- Understand identity matrix I
- ► Understand  $AA^{-1} = A^{-1}A = I$
- Find  $A^{-1}$  with Gauss-Jordan elimination
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### Stuff to know:

- Transposes
  - Definition: flip entries across main diagonal
  - $A = A^{T}$ : A is symmetric
  - Important property: (AB)<sup>T</sup> = B<sup>T</sup>A<sup>T</sup>

### Extra piece



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#### Transposes

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- If  $A\vec{x} = 0$  has a non-zero solution, A has no inverse
- If  $A\vec{x} = \vec{0}$  has a non-zero solution, then  $A\vec{x} = \vec{b}$  always has infinitely many solutions.
- $(A^{-1})^{\mathrm{T}} = (A^{\mathrm{T}})^{-1}$





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