

Solving $A\vec{x} = \vec{b}$:

Ch. 2: Lec. 2

- We (people + computers) solve systems of linear equations by a systematic method of Elimination followed by Back substitution
- Due to our man Gauss, hence Gaussian elimination.
- Our first example:



Solving $A\vec{x} = \vec{b}$:

Ch. 2: Lec. 2

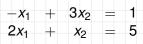
- We (people + computers) solve systems of linear equations by a systematic method of Elimination followed by Back substitution
- Due to our man Gauss, hence Gaussian elimination.
- Our first example:



Solving $A\vec{x} = \vec{b}$:

Ch. 2: Lec. 2

- We (people + computers) solve systems of linear equations by a systematic method of Elimination followed by Back substitution
- Due to our man Gauss, hence Gaussian elimination.
- Our first example:





Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

Basic elimination rules (roughly):

- 1. Strategically, mechanically remove unwanted entries by subtracting a multiple of a row from another.
- Swap rows if needed to create an 'upper triangular form'



Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

Basic elimination rules (roughly):

- 1. Strategically, mechanically remove unwanted entries by subtracting a multiple of a row from another.
- 2. Swap rows if needed to create an 'upper triangular form'



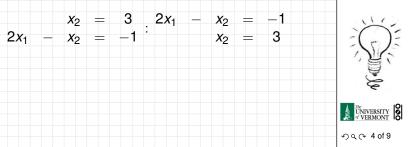
Ch. 2: Lec. 2

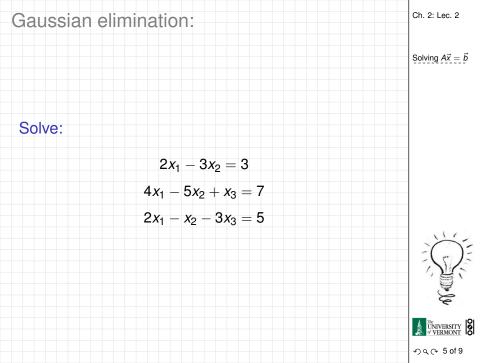
Solving $A\vec{x} = \vec{b}$

Basic elimination rules (roughly):

- 1. Strategically, mechanically remove unwanted entries by subtracting a multiple of a row from another.
- Swap rows if needed to create an 'upper triangular form'







Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

Summary:

Using row operations, we turned this problem:

$$A\vec{x} = \vec{b}: \begin{bmatrix} 2 & -3 & 0 \\ 4 & -5 & 1 \\ 2 & -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}$$

into this problem:

$$U\vec{x} = \vec{d} : \begin{bmatrix} 2 & -3 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

and the latter is easy to solve using back substitution.



Gaussian elimination: Defn: Solving $A\vec{x} = \vec{b}$ The entries along U's main diagonal are the pivots of A. (The pivots are hidden—elimination finds them.) 2 2 C 2 of 9

Defn:

The entries along *U*'s main diagonal are the pivots of *A*. (The pivots are hidden—elimination finds them.)

Defn:

A matrix with only zeros below the main diagonal is called upper triangular. A matrix with only zeros above the main diagonal is called lower triangular. We get from *A* to *U* and the latter is always upper triangular.

Singular means a system has no unique solution.

it may have no solutions or infinitely many spipilo
 Simming a solution of solution of solutions of solutions.

Truth:

f at least one pivot is zero, the matrix will be singular. but the reverse is not necessarily true).



Ch. 2: Lec. 2

Defn:

The entries along *U*'s main diagonal are the pivots of *A*. (The pivots are hidden—elimination finds them.)

Defn:

A matrix with only zeros below the main diagonal is called upper triangular. A matrix with only zeros above the main diagonal is called lower triangular. We get from *A* to *U* and the latter is always upper triangular.

Defn:

Singular means a system has no unique solution.

- It may have no solutions or infinitely many solutions.
- Singular = archaic way of saying 'messed up.'

Fruth:

f at least one pivot is zero, the matrix will be singular. but the reverse is not necessarily true).



Solving $A\vec{x} = \vec{b}$

Ch 2. Lec 2

Defn:

The entries along *U*'s main diagonal are the pivots of *A*. (The pivots are hidden—elimination finds them.)

Defn:

A matrix with only zeros below the main diagonal is called upper triangular. A matrix with only zeros above the main diagonal is called lower triangular. We get from *A* to *U* and the latter is always upper triangular.

Defn:

Singular means a system has no unique solution.

- It may have no solutions or infinitely many solutions.
- Singular = archaic way of saying 'messed up.'

Fruth:

f at least one pivot is zero, the matrix will be singular.



Solving $A\vec{x} = \vec{b}$

Defn:

The entries along *U*'s main diagonal are the pivots of *A*. (The pivots are hidden—elimination finds them.)

Defn:

A matrix with only zeros below the main diagonal is called upper triangular. A matrix with only zeros above the main diagonal is called lower triangular. We get from A to U and the latter is always upper triangular.

Defn:

Singular means a system has no unique solution.

- It may have no solutions or infinitely many solutions.
- Singular = archaic way of saying 'messed up.'

Fruth:

at least one pivot is zero, the matrix will be singular.



, ****

Solving $A\vec{x} = \vec{b}$

Defn:

The entries along *U*'s main diagonal are the pivots of *A*. (The pivots are hidden—elimination finds them.)

Defn:

A matrix with only zeros below the main diagonal is called upper triangular. A matrix with only zeros above the main diagonal is called lower triangular. We get from A to U and the latter is always upper triangular.

Defn:

Singular means a system has no unique solution.

- It may have no solutions or infinitely many solutions.
- Singular = archaic way of saying 'messed up.'

Truth:

If at least one pivot is zero, the matrix will be singular. (but the reverse is not necessarily true).



Solving $A\vec{x} = \vec{b}$

Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

The one true method:

- We simplify A using elimination in the same way every time.
- Eliminate entries one column at a time, moving left to right, and down each column.





Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

The one true method:

- We simplify A using elimination in the same way every time.
- Eliminate entries one column at a time, moving left to right, and down each column.





Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

The one true method:

- We simplify A using elimination in the same way every time.
- Eliminate entries one column at a time, moving left to right, and down each column.

$$X + X + X + X = X$$

$$1 \downarrow + X + X + X = X$$

$$2 \downarrow + 4 \downarrow + X + X = X$$

$$3 \checkmark + 5 \rightarrow + 6 + X = X$$



Ch. 2: Lec. 2

Solving $A\vec{x} = \vec{b}$

To eliminate entry in row i of jth column, subtract a multiple l_{ii} of the jth row from i.





- Note: we cannot find l₃₂ etc., until we are finished with row 1. Pivots are hidden!
- Note: the denominator of each l_{ij} multiplier is the pivot in the jth column.



Ch. 2: Lec. 2

- To eliminate entry in row i of jth column, subtract a multiple l_{ij} of the jth row from i.
- For example:

$$\ell_{21} = 1/2, \, \ell_{31} = -1/2, \, \ell_{41} = ?.$$

- Note: we cannot find l₃₂ etc., until we are finished with row 1. Pivots are hidden!
- Note: the denominator of each l_{ij} multiplier is the pivot in the jth column.



Ch. 2: Lec. 2

- To eliminate entry in row i of jth column, subtract a multiple l_{ij} of the jth row from i.
- For example:

$$\ell_{21} = 1/2, \, \ell_{31} = -1/2, \, \ell_{41} = ?.$$

- Note: we cannot find l₃₂ etc., until we are finished with row 1. Pivots are hidden!
- Note: the denominator of each l_{ij} multiplier is the pivot in the *j*th column.



Ch. 2: Lec. 2

- To eliminate entry in row i of jth column, subtract a multiple l_{ij} of the jth row from i.
- For example:

$$\ell_{21} = 1/2, \, \ell_{31} = -1/2, \, \ell_{41} = ?.$$

- Note: we cannot find l₃₂ etc., until we are finished with row 1. Pivots are hidden!
- Note: the denominator of each lij multiplier is the pivot in the jth column.

