Random Networks

Complex Networks, CSYS/MATH 303, Spring, 2010

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Random networks

Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

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Random networks

Some features:

- Number of possible edges:

$$0 \leq m \leq \binom{N}{2} = \frac{N(N-1)}{2}$$

- Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.
- Limit of $m = 0$: empty graph.
- Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Real world: links are usually costly so real networks are almost always sparse.

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Random networks

How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)

1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
   - Useful for theoretical work.
2. Take N nodes and add exactly m links by selecting edges without replacement.
   - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
   - Best for adding relatively small numbers of links (most cases).
   - 1 and 2 are effectively equivalent for large N.

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A few more things:

- For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p\frac{1}{2}N(N-1)$$

- So the expected or average degree is

$$\langle k \rangle = \frac{2\,\langle m \rangle}{N}$$

$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{\cancel{2}}{\cancel{N}}p\frac{1}{\cancel{2}}\cancel{N}(N-1) = p(N-1).$$

- Which is what it should be...
- If we keep $\langle k \rangle$ constant then $p \propto 1/N \to 0$ as $N \to \infty$.

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Random networks: examples

Next slides:

Example realizations of random networks

- $N = 500$
- Vary m, the number of edges from 100 to 1000.
- Average degree $\langle k \rangle$ runs from 0.4 to 4.
- Look at full network plus the largest component.

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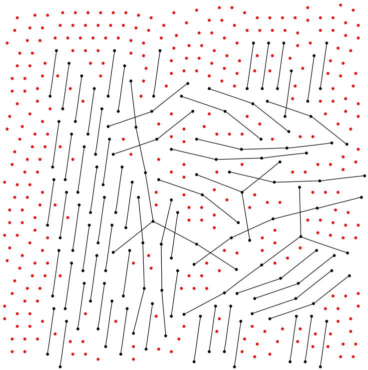
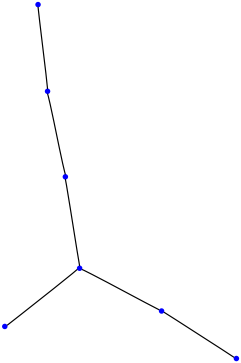


largest component:



$N = 500$, number of edges $m = 100$
average degree $\langle k \rangle = 0.4$

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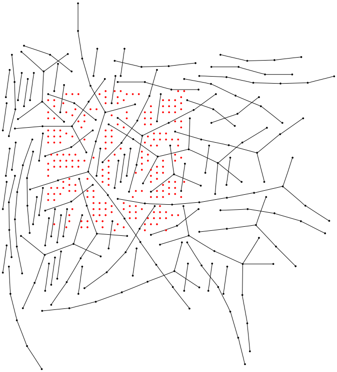
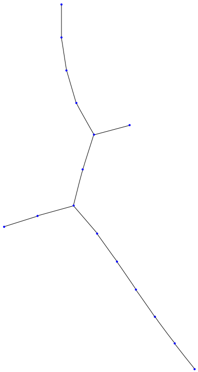
entire network:



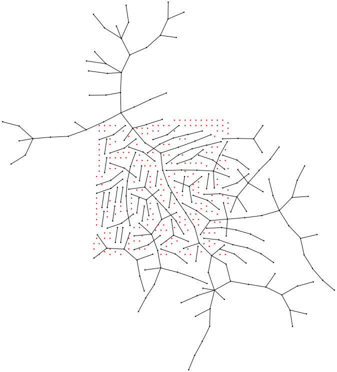
largest component:



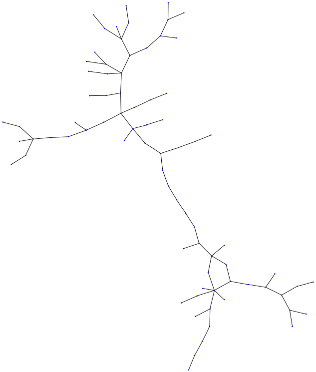
$N = 500$, number of edges $m = 200$
average degree $\langle k \rangle = 0.8$

Random networks: examples

entire network:



largest component:



$N = 500$, number of edges $m = 230$
average degree $\langle k \rangle = 0.92$

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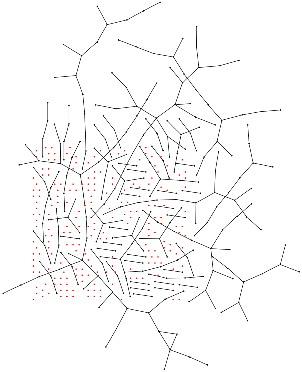
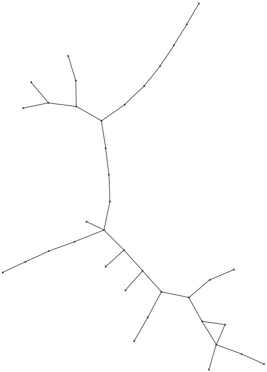
entire network:



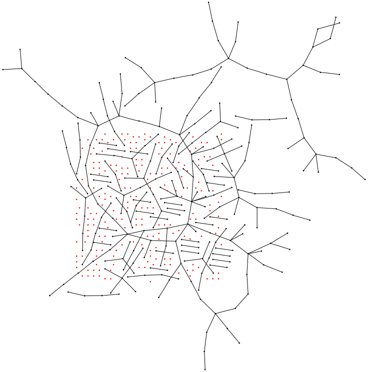
largest component:



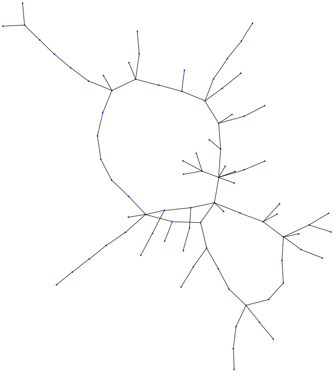
$N = 500$, number of edges $m = 240$
average degree $\langle k \rangle = 0.96$

Random networks: examples

entire network:



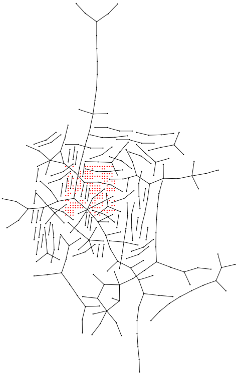
largest component:



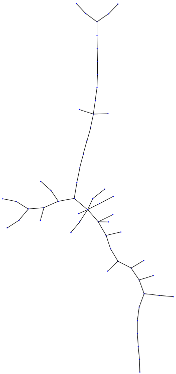
$N = 500$, number of edges $m = 250$
average degree $\langle k \rangle = 1$

Random networks: examples

entire network:



largest component:



$N = 500$, number of edges $m = 260$
average degree $\langle k \rangle = 1.04$

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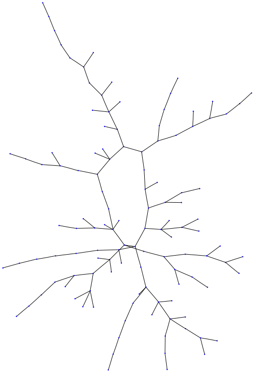
entire network:

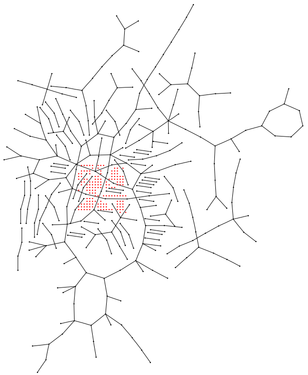
largest component:



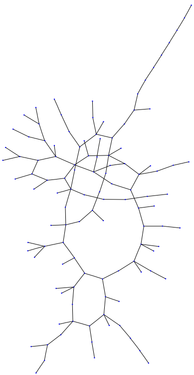
$N = 500$, number of edges $m = 280$
average degree $\langle k \rangle = 1.12$

Random networks: examples

entire network:



largest component:



$N = 500$, number of edges $m = 300$
average degree $\langle k \rangle = 1.2$

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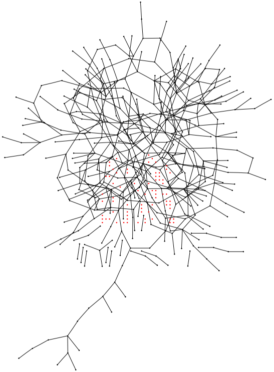
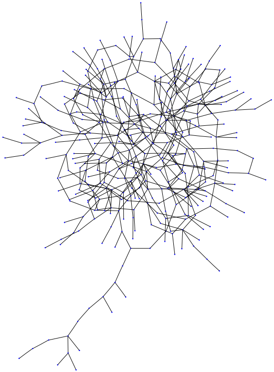
entire network:



largest component:



N = 500, number of edges m = 500
average degree $\langle k \rangle$ = 2

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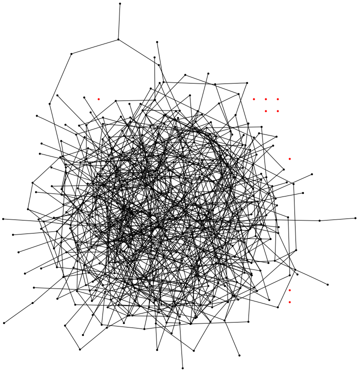
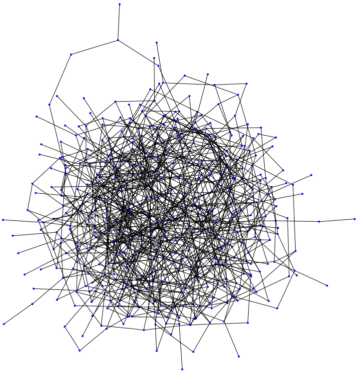
Random networks: examples

entire network:



largest component:



$N = 500$, number of edges $m = 1000$
average degree $\langle k \rangle = 4$

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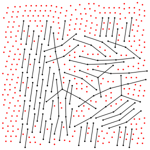
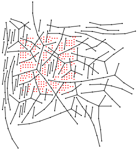
Random networks: examples for N=500

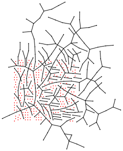


$m = 100$
$\langle k \rangle = 0.4$



$m = 200$
$\langle k \rangle = 0.8$

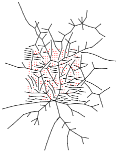
$m = 230$
$\langle k \rangle = 0.92$



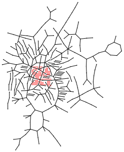
$m = 240$
$\langle k \rangle = 0.96$

$m = 250$
$\langle k \rangle = 1$

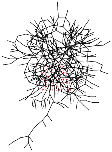
$m = 260$
$\langle k \rangle = 1.04$



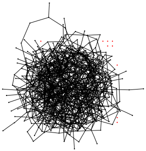
$m = 280$
$\langle k \rangle = 1.12$



$m = 300$
$\langle k \rangle = 1.2$



$m = 500$
$\langle k \rangle = 2$



$m = 1000$
$\langle k \rangle = 4$

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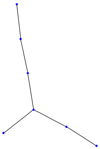
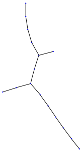
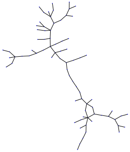


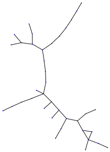
$m = 100$
$\langle k \rangle = 0.4$



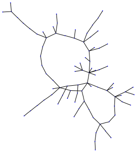
$m = 200$
$\langle k \rangle = 0.8$



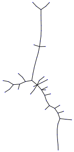
$m = 230$
$\langle k \rangle = 0.92$



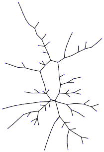
$m = 240$
$\langle k \rangle = 0.96$



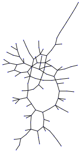
$m = 250$
$\langle k \rangle = 1$



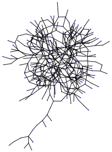
$m = 260$
$\langle k \rangle = 1.04$



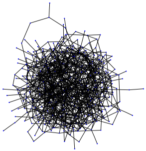
$m = 280$
$\langle k \rangle = 1.12$



$m = 300$
$\langle k \rangle = 1.2$



$m = 500$
$\langle k \rangle = 2$



$m = 1000$
$\langle k \rangle = 4$

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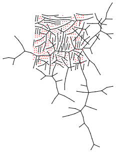
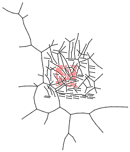
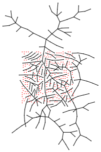


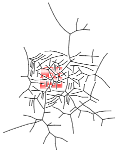
$m = 250$
$\langle k \rangle = 1$



$m = 250$
$\langle k \rangle = 1$



$m = 250$
$\langle k \rangle = 1$



$m = 250$
$\langle k \rangle = 1$

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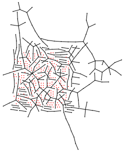
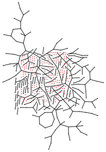
$\langle k \rangle = 1$

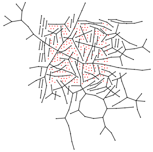


$m = 250$
$\langle k \rangle = 1$

$m = 250$
$\langle k \rangle = 1$



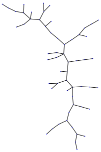
$m = 250$
$\langle k \rangle = 1$



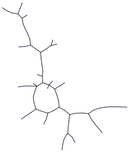
$m = 250$
$\langle k \rangle = 1$

$m = 250$
$\langle k \rangle = 1$

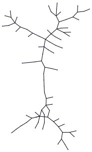
Random networks: largest components



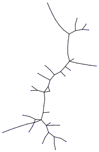
$m = 250$
$\langle k \rangle = 1$



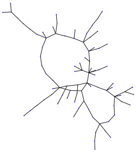
$m = 250$
$\langle k \rangle = 1$



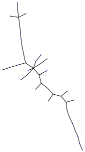
$m = 250$
$\langle k \rangle = 1$



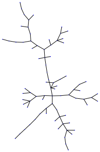
$m = 250$
$\langle k \rangle = 1$



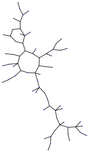
$m = 250$
$\langle k \rangle = 1$



$m = 250$
$\langle k \rangle = 1$



$m = 250$
$\langle k \rangle = 1$



$m = 250$
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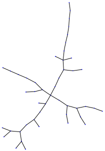
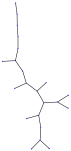


$m = 250$
$\langle k \rangle = 1$



$m = 250$
$\langle k \rangle = 1$

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Random networks

Clustering:

- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient (Newman [1]):

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

- Recall: C_2 = probability that two nodes are connected given they have a friend in common.
- For standard random networks, we have simply that

$$C_2 = p.$$

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Random networks

Clustering:

- So for large random networks ($N \to \infty$), clustering drops to zero.
- Key structural feature of random networks is that they locally look like branching networks (no loops).

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Random networks

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- So for large random networks ($N \to \infty$), clustering drops to zero.
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Random networks

Degree distribution:

- Recall p_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are '$N-1$ choose k' ways the node can be connected to k of the other $N-1$ nodes.
- Each connection occurs with probability p, each non-connection with probability $(1-p)$.
- Therefore have a binomial distribution:

$$P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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Limiting form of $P(k; p, N)$:

- Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$.
- What happens as $N \to \infty$?
- We must end up with the normal distribution right?
- If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \to \infty$.
- But we want to keep $\langle k \rangle$ fixed...
- So examine limit of $P(k; p, N)$ when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1) = \text{constant}$.

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$$= \frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k \rangle^k}{(N-1)^k} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k}$$

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Limiting form of $P(k; p, N)$:

- We are now here:

$$P(k; p, N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

- Now use the excellent result:

$$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x.$$

(Use l'Hôpital's rule to prove.)

- Identifying $n = N - 1$ and $x = -\langle k \rangle$:

$$P(k; \langle k \rangle) \simeq \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{-k} \to \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

- This is a Poisson distribution (⊞) with mean $\langle k \rangle$.

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- Generalize to arbitrary degree distribution P_k.
- Also known as the configuration model [1].
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P_w and form links with probability

$$P(\text{link between } i \text{ and } j) \propto w_i w_j.$$

- But we'll be more interested in
  1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
  2. Examining mechanisms that lead to networks with certain degree distributions.

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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

- $N = 1000$.
- $P_k \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_0 = 0$ (no isolated nodes).
- Vary exponent γ between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

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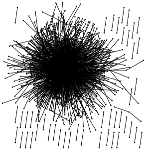
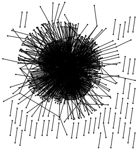
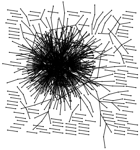
Random networks: examples for N=1000



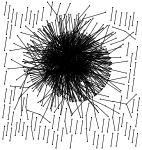
γ = 2.1
$\langle k \rangle$ = 3.448



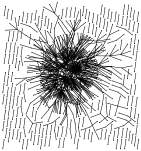
γ = 2.19
$\langle k \rangle$ = 2.986



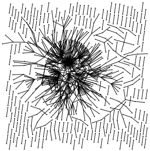
γ = 2.28
$\langle k \rangle$ = 2.306



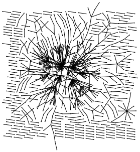
γ = 2.37
$\langle k \rangle$ = 2.504



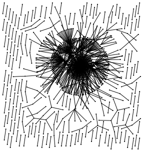
γ = 2.46
$\langle k \rangle$ = 1.856



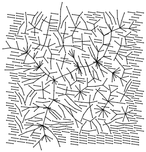
γ = 2.55
$\langle k \rangle$ = 1.712



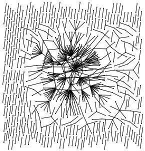
γ = 2.64
$\langle k \rangle$ = 1.6



γ = 2.73
$\langle k \rangle$ = 1.862



γ = 2.82
$\langle k \rangle$ = 1.386



γ = 2.91
$\langle k \rangle$ = 1.49

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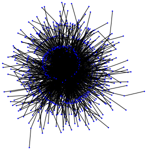

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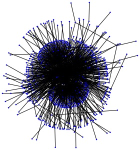

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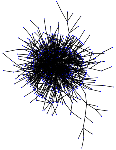

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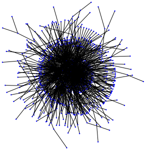

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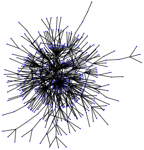
$\langle k \rangle = 2.504$



$\gamma = 2.46$
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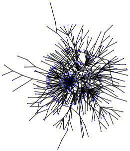

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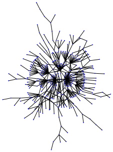

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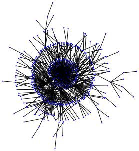

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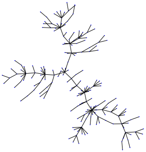

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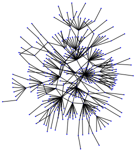

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Poisson basics:

- Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

- Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!}$$

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Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding $\langle k \rangle$ to find the second moment:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

- Variance is then

$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle.$$

- So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.
- Note: This is a special property of Poisson distribution and can trip us up...

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$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle.$$

- So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.
- Note: This is a special property of Poisson distribution and can trip us up...

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Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding $\langle k \rangle$ to find the second moment:

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- Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

$$\boxed{Q_k \propto kP_k}$$

- Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k' P_{k'}} = \frac{kP_k}{\langle k \rangle}.$$

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$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

- Equivalent to friend having degree $k+1$.
- Natural question: what's the expected number of other friends that one friend has?

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- Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right)$, is true for all random networks, independent of degree distribution.
- For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

- Therefore:

$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

- Again, neatness of results is a special property of the Poisson distribution.
- So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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Two reasons why this matters

Reason #1:

- Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

- Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.
- Three peculiarities:
  1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.
  2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big. (e.g., in the case of a power-law distribution)
  3. Your friends are different to you...

Two reasons why this matters

Reason #1:

- Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

- Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.
- Three peculiarities:
  1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.
  2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big. (e.g., in the case of a power-law distribution)
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More on peculiarity #3:

- A node's average # of friends: $\langle k \rangle$
- Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left(1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \geq \langle k$$

- So only if everyone has the same degree (variance$= \sigma^2 = 0$) can a node be the same as its friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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Two reasons why this matters

(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As $N \to \infty$, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that $\exists$ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Giant component = component that comprises a non-zero fraction of a network as $N \to \infty$.
- Note: Component = Cluster

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Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

- Again, see that the second moment is an essential part of the story.
- Equivalent statement: $\langle k^2 \rangle > 2\langle k \rangle$

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Giant component

Standard random networks:

- Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.
- Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.
- When $\langle k \rangle < 1$, all components are finite.
- Fine example of a continuous phase transition (⊞).
- We say $\langle k \rangle = 1$ marks the critical point of the system.

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Giant component

Standard random networks:

- Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.
- Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.
- When $\langle k \rangle < 1$, all components are finite.
- Fine example of a continuous phase transition (⊞).
- We say $\langle k \rangle = 1$ marks the critical point of the system.

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Random networks with skewed P_k:

- e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$ then

$$\langle k^2 \rangle = c \sum_{k=0}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=0}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma} \Big|_{x=0}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

- So giant component always exists for these kinds of networks.
- Cutoff scaling is k^{-3}: if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

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And how big is the largest component?

- Define S_1 as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k \rangle$.
- Let's find S_1 with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 - S_1$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.
- So

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

- Substitute in Poisson distribution...

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- Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1-\delta)}.$$

- Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}.$$

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- First, we can write $\langle k \rangle$ in terms of S_1:

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

- As $\langle k \rangle \to 0$, $S_1 \to 0$.
- As $\langle k \rangle \to \infty$, $S_1 \to 1$.
- Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.
- Only solvable for $S > 0$ when $\langle k \rangle > 1$.
- Really a transcritical bifurcation [2].

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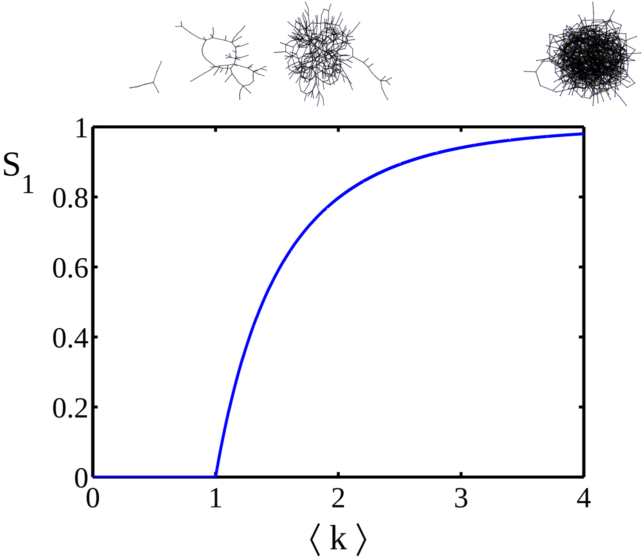

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Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- We need a separate probability δ' for the chance that a node at the end of a random edge is part of the largest component.
- We can do this but we need to enhance our toolkit with Generatingfunctionology... [3]
- (Well, not really but it's fun and we get all sorts of other things...)

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Generating functions

- Idea: Given a sequence $a_0, a_1, a_2, \ldots,$ associate each element with a distinct function or other mathematical object.
- Well-chosen functions allow us to manipulate sequences and retrieve sequence elements.

Definition:

- The generating function (g.f.) for a sequence $\{a_n\}$ is

$$F(x) = \sum_{n=0}^{\infty} a_n x^n.$$

- Roughly: transforms a vector in R^∞ into a function defined on R^1.
- Related to Fourier, Laplace, Mellin, ...

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Simple example

Rolling dice:

- $p_k^{(\square)} = \mathbf{Pr}(\text{throwing a } k) = 1/6$ where $k = 1, 2, \ldots, 6$.
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$$F^{(\square)}(x) = \sum_{k=1}^{6} p_k x^k = \frac{1}{6}(x + x^2 + x^3 + x^4 + x^5 + x^6).$$

- We'll come back to this simple example as we derive various delicious properties of generating functions.

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Example

- Take a degree distribution with exponential decay:

$$P_k = ce^{-\lambda k}$$

where $c = 1 - e^{-\lambda}$.

- The generating function for this distribution is

$$F(x) = \sum_{k=0}^{\infty} P_k x^k = \sum_{k=0}^{\infty} ce^{-\lambda k} x^k = \frac{c}{1 - xe^{-\lambda}}.$$

- Notice that $F(1) = c/(1 - e^{-\lambda}) = 1$.
- For probability distributions, we must always have $F(1) = 1$ since

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Properties of generating functions

- Average degree:

$$\langle k \rangle = \sum_{k=0}^{\infty} k P_k = \sum_{k=0}^{\infty} k P_k x^{k-1} \bigg|_{x=1}$$

$$= \frac{\mathrm{d}}{\mathrm{d}x} F(x) \bigg|_{x=1} = F'(1)$$

- In general, many calculations become simple, if a little abstract.
- For our exponential example:

$$F'(x) = \frac{(1-e^{-\lambda})e^{-\lambda}}{(1-xe^{-\lambda})^2}.$$

- So:

$$\langle k \rangle = F'(1) = \frac{e^{-\lambda}}{(1-e^{-\lambda})}.$$

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Useful pieces for probability distributions:

- Normalization:

$$F(1) = 1$$

- First moment:

$$\langle k \rangle = F'(1)$$

- Higher moments:

$$\langle k^n \rangle = \left(x \frac{\mathrm{d}}{\mathrm{d}x} \right)^n F(x) \Big|_{x=1}$$

- kth element of sequence (general):

$$P_k = \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}x^k} F(x) \Big|_{x=0}$$

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- Higher moments:

$$\langle k^n \rangle = \left(x \frac{\mathrm{d}}{\mathrm{d}x} \right)^n F(x) \bigg|_{x=1}$$

- kth element of sequence (general):

$$P_k = \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}x^k} F(x) \bigg|_{x=0}$$

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Properties of generating functions

Useful pieces for probability distributions:

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Edge-degree distribution

- Recall our condition for a giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1.$$

- Let's rëexpress our condition in terms of generating functions.
- We first need the g.f. for R_k.
- We'll now use this notation:

$F_P(x)$ is the g.f. for P_k.
$F_R(x)$ is the g.f. for R_k.

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$$\langle k \rangle_R = F_R'(1) > 1.$$

- Now find how F_R is related to F_P...

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- Recall giant component condition is $\langle k \rangle_R = F_R'(1) > 1$.
- Since we have $F_R(x) = F_P'(x)/F_P'(1)$,

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- Setting $x = 1$, our condition becomes

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Size distributions

To figure out the size of the largest component (S_1), we need more resolution on component sizes.

Definitions:

- π_n = probability that a random node belongs to a finite component of size $n < \infty$.
- ρ_n = probability a random link leads to a finite subcomponent of size $n < \infty$.

Local-global connection:

$$P_k, R_k \Leftrightarrow \pi_n, \rho_n$$

$$\text{neighbors} \Leftrightarrow \text{components}$$

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Size distributions

G.f.'s for component size distributions:

- $$F_\pi(x) = \sum_{n=0}^{\infty} \pi_n x^n \text{ and } F_\rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$$

The largest component:

- Subtle key: $F_\pi(1)$ is the probability that a node belongs to a finite component.
- Therefore: $S_1 = 1 - F_\pi(1)$.

Our mission, which we accept:

- Find the four generating functions

$$F_P, F_R, F_\pi, \text{ and } F_\rho.$$

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Size distributions

G.f.'s for component size distributions:

- $$F_\pi(x) = \sum_{n=0}^{\infty} \pi_n x^n \text{ and } F_\rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$$

The largest component:

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Useful results we'll need for g.f.'s

Sneaky Result 1:

- Consider two random variables U and V whose values may be $0, 1, 2, \ldots$
- Write probability distributions as U_k and V_k and g.f.'s as F_U and F_V.
- SR1: If a third random variable is defined as

$$W = \sum_{i=1}^{U} V^{(i)} \text{ with each } V^{(i)} \stackrel{d}{=} V$$

then

$$\boxed{F_W(x) = F_U(F_V(x))}$$

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Proof of SR1:

With some concentration, observe:

$$F_W(x) = \sum_{j=0}^{\infty} U_j \sum_{k=0}^{\infty} \underbrace{\sum_{\substack{\{i_1,i_2,\ldots,i_j\} \mid \\ i_1+i_2+\ldots+i_j=k}} V_{i_1} x^{i_1} V_{i_2} x^{i_2} \cdots V_{i_j} x^{i_j}}_{x^k \text{ piece of } \left(\sum_{i'=0}^{\infty} V_{i'} x^{i'}\right)^j}$$

$$= \sum_{j=0}^{\infty} U_j \left(F_V(x)\right)^j$$

$$= F_U\left(F_V(x)\right) \checkmark$$

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Sneaky Result 2:

- Start with a random variable U with distribution U_k $(k = 0, 1, 2, \ldots)$
- SR2: If a second random variable is defined as

$$V = U + 1 \quad \text{then} \quad \boxed{F_V(x) = xF_U(x)}$$

- Reason: $V_k = U_{k-1}$ for $k \geq 1$ and $V_0 = 0$.
- $$\therefore F_V(x) = \sum_{k=0}^{\infty} V_k x^k = \sum_{k=1}^{\infty} U_{k-1} x^k$$

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- Start with a random variable U with distribution U_k $(k = 0, 1, 2, \ldots)$
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$$V = U + 1 \text{ then } \boxed{F_V(x) = xF_U(x)}$$

- Reason: $V_k = U_{k-1}$ for $k \geq 1$ and $V_0 = 0$.
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$$\therefore F_V(x) = \sum_{k=0}^{\infty} V_k x^k = \sum_{k=1}^{\infty} U_{k-1} x^k$$

$$= x \sum_{j=0}^{\infty} U_j x^j = xF_U(x). \checkmark$$

Useful results we'll need for g.f.'s

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Generalization of SR2:

- (1) If $V = U + i$ then

$$F_V(x) = x^i F_U(x).$$

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Connecting generating functions

- Goal: figure out forms of the component generating functions, F_π and F_ρ.
- π_n = probability that a random node belongs to a finite component of size n

$$= \sum_{k=0}^{\infty} P_k \times \Pr\left(\begin{array}{c}\text{sum of sizes of subcomponents} \\ \text{at end of } k \text{ random links} = n-1\end{array}\right)$$

- Therefore: $\boxed{F_\pi(x) = \underbrace{x}_{\text{SR2}} \underbrace{F_P\left(F_\rho(x)\right)}_{\text{SR1}}}$
- Extra factor of x accounts for random node itself.

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Connecting generating functions

- We now have two functional equations connecting our generating functions:

$$F_\pi(x) = xF_P\left(F_\rho(x)\right) \text{ and } F_\rho(x) = xF_R\left(F_\rho(x)\right)$$

- Taking stock: We know $F_P(x)$ and $F_R(x) = F_P'(x)/F_P'(1)$.
- We first untangle the second equation to find F_ρ
- We can do this because it only involves F_ρ and F_R.
- The first equation then immediately gives us F_π in terms of F_ρ and F_R.

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Component sizes

- Remembering vaguely what we are doing:

  Finding F_π to obtain the fractional size of the largest component $S_1 = 1 - F_\pi(1)$.

- Set $x = 1$ in our two equations:

$$F_\pi(1) = F_P\left(F_\rho(1)\right) \text{ and } F_\rho(1) = F_R\left(F_\rho(1)\right)$$

- Solve second equation numerically for $F_\rho(1)$.
- Plug $F_\rho(1)$ into first equation to obtain $F_\pi(1)$.

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Example: Standard random graphs.

- We can show $F_P(x) = e^{-\langle k \rangle (1-x)}$

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- We're first after $S_1 = 1 - F_\pi(1)$ so set $x = 1$ and replace $F_\pi(1)$ by $1 - S_1$:

$$1 - S_1 = e^{-\langle k \rangle S_1}$$

Or: $\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}$

- Just as we found with our dirty trick ...
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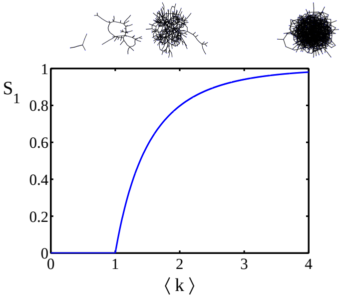

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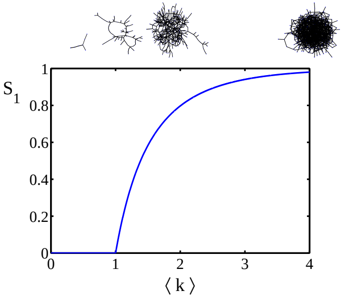

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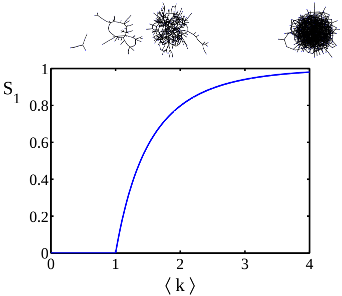

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Average component size

- Next: find average size of finite components $\langle n \rangle$.
- Using standard G.F. result: $\langle n \rangle = F'_\pi(1)$.
- Try to avoid finding $F_\pi(x)$...
- Starting from $F_\pi(x) = xF_P(F_\rho(x))$, we differentiate:

$$F'_\pi(x) = F_P(F_\rho(x)) + xF'_\rho(x)F'_P(F_\rho(x))$$

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- Now set $x = 1$ in both equations.
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Example: Standard random graphs.

- Use fact that $F_P = F_R$ and $F_\pi = F_\rho$.
- Two differentiated equations reduce to only one:

$$F'_\pi(x) = F_P\left(F_\pi(x)\right) + xF'_\pi(x)F'_P\left(F_\pi(x)\right)$$

$$\text{Rearrange:} \quad F'_\pi(x) = \frac{F_P\left(F_\pi(x)\right)}{1 - xF'_P\left(F_\pi(x)\right)}$$

- Simplify denominator using $F'_P(x) = \langle k \rangle F_P(x)$
- Replace $F_P(F_\pi(x))$ using $F_\pi(x) = xF_P(F_\pi(x))$.
- Set $x = 1$ and replace $F_\pi(1)$ with $1 - S_1$.

$$\text{End result:} \quad \langle n \rangle = F'_\pi(1) = \frac{(1 - S_1)}{1 - \langle k \rangle (1 - S_1)}$$

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$$\text{Rearrange:} \quad F'_\pi(x) = \frac{F_P\left(F_\pi(x)\right)}{1 - xF'_P\left(F_\pi(x)\right)}$$

- Simplify denominator using $F'_P(x) = \langle k \rangle F_P(x)$
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- Our result for standard random networks:

$$\langle n \rangle = F'_\pi(1) = \frac{(1 - S_1)}{1 - \langle k \rangle (1 - S_1)}$$

- Recall that $\langle k \rangle = 1$ is the critical value of average degree for standard random networks.
- Look at what happens when we increase $\langle k \rangle$ to 1 from below.
- We have $S_1 = 0$ for all $\langle k \rangle < 1$ so

$$\langle n \rangle = \frac{1}{1 - \langle k \rangle}$$

- This blows up as $\langle k \rangle \to 1$.
- Reason: we have a power law distribution of component sizes at $\langle k \rangle = 1$.
- Typical critical point behavior....

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- Limits of $\langle k \rangle = 0$ and ∞ make sense for

$$\langle n \rangle = F'_{\pi}(1) = \frac{(1 - S_1)}{1 - \langle k \rangle (1 - S_1)}$$

- As $\langle k \rangle \to 0$, $S_1 = 0$, and $\langle n \rangle \to 1$.
- All nodes are isolated.
- As $\langle k \rangle \to \infty$, $S_1 \to 1$ and $\langle n \rangle \to 0$.
- No nodes are outside of the giant component.

Extra on largest component size:

- For $\langle k \rangle = 1$, $S_1 \sim N^{2/3}$.
- For $\langle k \rangle < 1$, $S_1 \sim \log N$.

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