# Random Networks <br> Complex Networks，CSYS／MATH 303，Spring， 2010 

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## Random networks

Pure，abstract random networks：
－Consider set of all networks with $N$ labelled nodes and $m$ edges．
－Standard random network＝randomly chosen network from this set．
－To be clear：each network is equally probable．
－Sometimes equiprobability is a good assumption，but it is always an assumption．
－Known as Erdős－Rényi random networks or ER graphs．

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## Random networks

Pure, abstract random networks:

- Consider set of all networks with $N$ labelled nodes and $m$ edges.
- Standard random network = randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.


## Random networks

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## Random networks

Basics

## Some features:

- Number of possible edges:

$$
0 \leq m \leq\binom{ N}{2}=\frac{N(N-1)}{2}
$$

- Given $m$ edges, there are $\left(\begin{array}{c}\binom{N}{2}\end{array}\right)$ different possible networks.
- Crazy factorial explosion for $1 \ll m \ll\binom{N}{2}$.
- Limit of $m=0$ : empty graph.
- Limit of $m=\binom{N}{2}$ : complete or fully-connected graph.
- Real world: links are usually costly so real networks are almost always sparse.


## Random networks

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Some features：
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－Given $m$ edges，there are $\binom{N}{\substack{N \\ m}}$ different possible networks．
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## Random networks

How to build standard random networks：
－Given $N$ and $m$ ．
－Two probablistic methods
1．Connect each of the $\binom{N}{2}$ pairs with appropriate probability $p$ ．

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## Random networks

How to build standard random networks：
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－Two probablistic methods（we＇ll see a third later on）
Connect each of the $\binom{N}{2}$ pairs with appropriate probability $p$ ．

2．Take $N$ nodes and add exactly $m$ links by selecting edges without replacement．

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## Random networks

How to build standard random networks:

- Given $N$ and $m$.
- Two probablistic methods (we'll see a third later on)

1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability $p$.
2. Take $N$ nodes and add exactly $m$ links by selecting edges without replacement.


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- Given $N$ and $m$.
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1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability $p$.

- Useful for theoretical work.

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－Algorithm：Randomly choose a pair of nodes $i$ and $j$ ， $i \neq j$ ，and connect if unconnected；repeat until all $m$

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－Best for adding relatively small numbers of links
（most cases）．
-1 and 2 are effectively equivalent for large $N$ ．

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A few more things：
－For method 1，\＃links is probablistic：

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－Which is what it should be．．．
－If we keep $\langle k\rangle$ constant then $p \propto 1 / N \rightarrow 0$ as
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## Random networks

A few more things：
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$$
\langle m\rangle=p\binom{N}{2}=p \frac{1}{2} N(N-1)
$$

－So the expected or average degree is

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\langle m\rangle=p\binom{N}{2}=p \frac{1}{2} N(N-1)
$$

－So the expected or average degree is

$$
\langle k\rangle=\frac{2\langle m\rangle}{N}
$$

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\langle m\rangle=p\binom{N}{2}=p \frac{1}{2} N(N-1)
$$

－So the expected or average degree is

$$
\begin{gathered}
\langle k\rangle=\frac{2\langle m\rangle}{N} \\
=\frac{2}{N} p \frac{1}{2} N(N-1)=\frac{2}{N} p^{1} \frac{1}{2} N(N-1)=p(N-1) .
\end{gathered}
$$

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\langle m\rangle=p\binom{N}{2}=p \frac{1}{2} N(N-1)
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\begin{gathered}
\langle k\rangle=\frac{2\langle m\rangle}{N} \\
=\frac{2}{N} p \frac{1}{2} N(N-1)=\frac{2}{X} p \frac{1}{2} N(N-1)=p(N-1) .
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## Random networks

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\langle m\rangle=p\binom{N}{2}=p \frac{1}{2} N(N-1)
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- Which is what it should be...
- If we keep $\langle k\rangle$ constant then $p \propto 1 / N \rightarrow 0$ as

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## Random networks

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－Which is what it should be．．．
－If we keep $\langle k\rangle$ constant then $p \propto 1 / N \rightarrow 0$ as $N \rightarrow \infty$ ．

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## Random networks：examples

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Next slides：
Example realizations of random networks
－$N=500$
－Vary $m$ ，the number of edges from 100 to 1000.
－Average degree $\langle k\rangle$ runs from 0.4 to 4 ．
－Look at full network plus the largest component．

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entire network：

$N=500$ ，number of edges $m=100$ average degree $\langle k\rangle=0.4$

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largest component：
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$N=500$ ，number of edges $m=200$ average degree $\langle k\rangle=0.8$

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 entire network:largest component:

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$N=500$, number of edges $m=230$ average degree $\langle k\rangle=0.92$

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largest component：
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$N=500$ ，number of edges $m=240$ average degree $\langle k\rangle=0.96$

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## Random networks：examples

largest component：


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$N=500$ ，number of edges $m=250$ average degree $\langle k\rangle=1$

## Random networks：examples

 entire network：largest component：

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$N=500$ ，number of edges $m=260$ average degree $\langle k\rangle=1.04$

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$N=500$ ，number of edges $m=280$ average degree $\langle k\rangle=1.12$

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## Random networks：examples

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$N=500$ ，number of edges $m=300$ average degree $\langle k\rangle=1.2$

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$N=500$ ，number of edges $m=500$ average degree $\langle k\rangle=2$

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$N=500$ ，number of edges $m=1000$ average degree $\langle k\rangle=4$

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$$

## Random networks：examples for $N=500$

## Basics


$m=230$
$\langle k\rangle=0.92$

$$
\begin{aligned}
& m=240 \\
& \langle k\rangle=0.96
\end{aligned}
$$

$m=250$
$\langle k\rangle=1$
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$m=260$
$\langle k\rangle=1.04$

$$
m=280
$$

$\langle k\rangle=1.12$
$m=300$
$\langle k\rangle=12$
$m=500$
$m=1000$
$\langle k\rangle=2$
$\langle k\rangle=4$

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## Random networks: largest components

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## Random networks：examples for $N=500$



$$
m=250
$$

$\langle k\rangle=1$

$$
m=250
$$

$\langle k\rangle=1$

$$
\begin{aligned}
& m=250 \\
& \langle k\rangle=1
\end{aligned}
$$

$\langle k\rangle=1$
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$$
\langle k\rangle=1
$$

$$
m=250
$$

$$
m=250
$$

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$m=250$
$m=250$
$m=250$
$\langle k\rangle=1$
$\langle k\rangle=1$
$m=250$
$m=250$
$\langle k\rangle=1$
$\langle k\rangle=1$

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## Random networks

## Clustering：

－For method 1 ，what is the clustering coefficient for a finite network？
－Consider triangle／triple clustering coefficient
（Newman ${ }^{[11) \text { ：}}$

－Recall：$C_{2}=$ probability that two nodes are connected given they have a friend in common．
－For standard random networks，we have simply that

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## Random networks

## Clustering：

－For method 1 ，what is the clustering coefficient for a finite network？
－Consider triangle／triple clustering coefficient （Newman ${ }^{[1]}$ ）：

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

－Recall：$C_{2}=$ probability that two nodes are connected given they have a friend in common．
－For standard random networks，we have simply that

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## Random networks

## Clustering：

－For method 1 ，what is the clustering coefficient for a finite network？
－Consider triangle／triple clustering coefficient （Newman ${ }^{[1]}$ ）：

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

－Recall：$C_{2}=$ probability that two nodes are connected given they have a friend in common．
－For standard random networks，we have simply that


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## Random networks

## Clustering：

－For method 1 ，what is the clustering coefficient for a finite network？
－Consider triangle／triple clustering coefficient （Newman ${ }^{[1]}$ ）：

$$
C_{2}=\frac{3 \times \# \text { triangles }}{\# \text { triples }}
$$

－Recall：$C_{2}=$ probability that two nodes are connected given they have a friend in common．
－For standard random networks，we have simply that

$$
C_{2}=p .
$$

## Random networks

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## Clustering：

－So for large random networks $(N \rightarrow \infty)$ ，clustering drops to zero．
－Key structural feature of random networks is that they locally look like branching networks（no loops）．

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## Clustering：

－So for large random networks $(N \rightarrow \infty)$ ，clustering drops to zero．
－Key structural feature of random networks is that they locally look like branching networks（no loops）．

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## Random networks

Degree distribution:

- Recall $p_{k}=$ probability that a randomly selected node has degree $k$.
- Consider method 1 for constructing random networks: each possible link is realized with probability $p$.
- Now consider one node: there are ' $N-1$ choose $k$ ' ways the node can be connected to $k$ of the other N-1 nodes.
- Each connection occurs with probability p, each non-connection with probability $(1-p)$.

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- Therefore have a binomial distribution:



## Random networks

Degree distribution：
－Recall $p_{k}=$ probability that a randomly selected node has degree $k$ ．
－Consider method 1 for constructing random networks：each possible link is realized with probability $p$ ．
－Now consider one node：there are＇$N$－ 1 choose $k$＇ ways the node can be connected to $k$ of the other $N-1$ nodes．

Each connection occurs with probability p，each non－connection with probability $(1-p)$ ．

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## Random networks

Degree distribution：
－Recall $p_{k}=$ probability that a randomly selected node has degree $k$ ．
－Consider method 1 for constructing random networks：each possible link is realized with probability $p$ ．
－Now consider one node：there are＇$N-1$ choose $k$＇ ways the node can be connected to $k$ of the other $N-1$ nodes．
－Each connection occurs with probability p，each non－connection with probability $(1-p)$ ．

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## Random networks

Degree distribution：
－Recall $p_{k}=$ probability that a randomly selected node has degree $k$ ．
－Consider method 1 for constructing random networks：each possible link is realized with probability $p$ ．
－Now consider one node：there are＇$N-1$ choose $k$＇ ways the node can be connected to $k$ of the other $N-1$ nodes．
－Each connection occurs with probability $p$ ，each non－connection with probability $(1-p)$ ．

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## Random networks

Degree distribution：
－Recall $p_{k}=$ probability that a randomly selected node has degree $k$ ．
－Consider method 1 for constructing random networks：each possible link is realized with probability $p$ ．
－Now consider one node：there are＇$N-1$ choose $k$＇ ways the node can be connected to $k$ of the other $N-1$ nodes．
－Each connection occurs with probability $p$ ，each non－connection with probability（ $1-p$ ）．

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－Therefore have a binomial distribution：

$$
P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k} .
$$

## Random networks

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Limiting form of $P(k ; p, N)$ :

- Our degree distribution:
$P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$.
- What happens as $N \rightarrow \infty$ ?
- We must end up with the normal distribution right?
- If $p$ is fixed, then we would end up with a Gaussian with average degree $\langle k\rangle \simeq p N \rightarrow \infty$.
- But we want to keep $\langle k\rangle$ fixed...
- So examine limit of $P(k ; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k\rangle=p(N-1)=$ constant.

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## Random networks

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Limiting form of $P(k ; p, N)$ ：
－Our degree distribution： $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$.
－What happens as $N \rightarrow \infty$ ？
－We must end up with the normal distribution right？
－If $p$ is fixed，then we would end up with a Gaussian with average degree $\langle k\rangle \simeq p N \rightarrow \infty$ ．
－But we want to keep $\langle k\rangle$ fixed．．．
－So examine limit of $P(k ; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k\rangle=p(N-1)=$ constant．

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## Random networks

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Limiting form of $P(k ; p, N)$ ：
－Our degree distribution： $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$ ．
－What happens as $N \rightarrow \infty$ ？
－We must end up with the normal distribution right？
－If $p$ is fixed，then we would end up with a Gaussian with average degree $\langle k\rangle \simeq p N \rightarrow \infty$ ．
－But we want to keep $\langle k\rangle$ fixed．．．
－So examine limit of $P(k ; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k\rangle=p(N-1)=$ constant．

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## Random networks

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－Our degree distribution： $P(k ; p, N)=\binom{N-1}{k} p^{k}(1-p)^{N-1-k}$ ．
－What happens as $N \rightarrow \infty$ ？
－We must end up with the normal distribution right？


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－So examine limit of $P(k ; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k\rangle=p(N-1)=$ constant．

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－So examine limit of $P(k ; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k\rangle=p(N-1)=$ constant．

## Limiting form of $P(k ; p, N)$ ：

－Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed：

$$
P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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## Limiting form of $P(k ; p, N)$ ：

－Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed：

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
& \quad=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ :

## Basics

- Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed:

$$
\begin{gathered}
P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
=\frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{gathered}
$$

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$$

## Limiting form of $P(k ; p, N)$ ：

－Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed：

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{N^{k}\left(1-\frac{1}{N}\right) \cdots\left(1-\frac{k}{N}\right)}{k!N^{k}} \frac{\langle k\rangle^{k}}{\left(1-\frac{1}{N}\right)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ ：

－Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed：

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{A^{k}\left(1-\frac{1}{N}\right) \cdots\left(1-\frac{k}{N}\right)}{k!A^{k}} \frac{\langle k\rangle^{k}}{\left(1-\frac{1}{N}\right)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ ：

－Substitute $p=\frac{\langle k\rangle}{N-1}$ into $P(k ; p, N)$ and hold $k$ fixed：

$$
\begin{aligned}
& P(k ; p, N)=\binom{N-1}{k}\left(\frac{\langle k\rangle}{N-1}\right)^{k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&=\frac{(N-1)!}{k!(N-1-k)!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
&= \frac{(N-1)(N-2) \cdots(N-k)}{k!} \frac{\langle k\rangle^{k}}{(N-1)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k} \\
& \simeq \frac{A^{k}\left(1-\frac{1}{N}\right) \cdots\left(1-\frac{k}{N}\right)}{k!A^{k}} \frac{\langle k\rangle^{k}}{\left(1-\frac{1}{N}\right)^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
\end{aligned}
$$

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## Limiting form of $P(k ; p, N)$ ：

－We are now here：

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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## Limiting form of $P(k ; p, N)$ ：

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P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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## Limiting form of $P(k ; p, N)$ ：

－We are now here：

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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$$
P(k ;\langle k\rangle) \simeq \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}\left(1-\frac{\langle k\rangle}{N-1}\right)^{-k}
$$

－This is a Poisson distribution（ $\boxplus$ ）with mean $\langle k\rangle$ ．
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## Limiting form of $P(k ; p, N)$ ：

－We are now here：

$$
P(k ; p, N) \simeq \frac{\langle k\rangle^{k}}{k!}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}
$$

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$$
P(k ;\langle k\rangle) \simeq \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}\left(1-\frac{\langle k\rangle}{N-1}\right)^{-k} \rightarrow \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
$$

－This is a Poisson distribution（ $\boxplus$ ）with mean $\langle k\rangle$ ．

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## General random networks

－So．．．standard random networks have a Poisson degree distribution

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$$
P(\text { link between } i \text { and } j) \propto w_{i} w_{j} .
$$

－But we＇ll be more interested in

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## General random networks

－So．．．standard random networks have a Poisson degree distribution
－Generalize to arbitrary degree distribution $P_{k}$ ．
－Also known as the configuration model
－Can generalize construction method from ER random networks．
－Assign each node a weight $w$ from some distribution $P_{w}$ and form links with probability

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－But we＇ll be more interested in

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## General random networks

－So．．．standard random networks have a Poisson degree distribution
－Generalize to arbitrary degree distribution $P_{k}$ ．
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$$

－But we＇ll be more interested in
1．Randomly wiring up（and rewiring）already existing nodes with fixed degrees．
2．Examining mechanisms that lead to networks with certain degree distributions．

## General random networks

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1．Randomly wiring up（and rewiring）already existing nodes with fixed degrees．
2．Examining mechanisms that lead to networks with certain degree distributions．

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2．Examining man

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## Random networks：examples

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Example realizations of random networks with power law degree distributions：

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## Random networks：examples

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Coming up：
Example realizations of random networks with power law degree distributions：
－$N=1000$ ．
－Set $P_{0}=0$（no isolated nodes）．
－Vary exponent $\gamma$ between 2.10 and 2．91．
－Again，look at full network plus the largest component．
－Apart from degree distribution，wiring is random．

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## Random networks：examples

Coming up：
Example realizations of random networks with power law degree distributions：
－$N=1000$ ．
－$P_{k} \propto k^{-\gamma}$ for $k \geq 1$ ．
－Set $P_{0}=0$（no isolated nodes）．
－Vary exponent $\gamma$ between 2.10 and 2．91．
－Again，look at full network plus the largest component．

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－Apart from degree distribution，wiring is random．

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## Random networks：examples

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Example realizations of random networks with power law degree distributions：
－$N=1000$ ．
－$P_{k} \propto k^{-\gamma}$ for $k \geq 1$ ．
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－Apart from degree distribution，wiring is random．

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－Apart from dearee distribution，wiring is random．

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－Apart from degree distribution，wiring is random．

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－Apart from degree distribution，wiring is random．

## Random networks: examples for $N=1000$



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$$
\text { 官 } \quad \text { Q }
$$

## Random networks: largest components

## Basics


$\gamma=2.28$
$\langle k\rangle=2.306$

$$
\begin{aligned}
& \gamma=2.37 \\
& \langle k\rangle=2.504
\end{aligned}
$$

$\langle k\rangle=1.856$
$\gamma=2.55$
$\langle k\rangle=1.712$
$\gamma=2.64$
$\gamma=2.73$
$\langle k\rangle=1.6$

$\begin{array}{ll}\gamma=2.82 & \gamma=2.91 \\ \langle k\rangle=1.386 & \langle k\rangle=1.49\end{array}$
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$$
\gamma=2.46
$$

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$$
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$$

## Poisson basics：

－Normalization：we must have

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

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## Poisson basics：

－Normalization：we must have

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
$$

－Checking：

$$
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
$$

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\begin{gathered}
\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \\
=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!}
\end{gathered}
$$

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## Poisson basics:

- Normalization: we must have

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\sum_{k=0}^{\infty} P(k ;\langle k\rangle)=1
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## Poisson basics：

－Mean degree：we must have

$$
\langle k\rangle=\sum_{k=0}^{\infty} k P(k ;\langle k\rangle)
$$

## Checking：



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\sum_{k=0}^{\infty} k P(k ;\langle k\rangle)=\sum_{k=0}^{\infty} k \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle}
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－We＇ll get to a better way of doing this．．．
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=e^{-\langle k\rangle} \sum_{k=1}^{\infty} \frac{\langle k\rangle^{k}}{(k-1)!}
\end{gathered}
$$

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- We'll get to a better way of doing this...


## Poisson basics：

－The variance of degree distributions for random networks turns out to be very important．
－Use calculation similar to one for finding $\langle k\rangle$ to find the second moment：

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－So standard deviation $\sigma$ is equal to $\sqrt{\langle k\rangle}$ ．
－Note：This is a special property of Poisson distribution and can trip us up．．．

## Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding $\langle k\rangle$ to find the second moment:

$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
$$

- Variance is then

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－The variance of degree distributions for random networks turns out to be very important．
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$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
$$

－Variance is then

$$
\sigma^{2}=\left\langle k^{2}\right\rangle-\langle k\rangle^{2}
$$

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－The variance of degree distributions for random networks turns out to be very important．
－Use calculation similar to one for finding $\langle k\rangle$ to find the second moment：

$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
$$

－Variance is then

$$
\sigma^{2}=\left\langle k^{2}\right\rangle-\langle k\rangle^{2}=\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle^{2}
$$

－So standard deviation $\sigma$ is equal to $\sqrt{\langle k\rangle}$ ．
－Note：This is a special property of Poisson distribution and can trip us up．．．

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## Poisson basics：

－The variance of degree distributions for random networks turns out to be very important．
－Use calculation similar to one for finding $\langle k\rangle$ to find the second moment：

$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
$$

－Variance is then

$$
\sigma^{2}=\left\langle k^{2}\right\rangle-\langle k\rangle^{2}=\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle^{2}=\langle k\rangle .
$$

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－So standard deviation $\sigma$ is equal to $\sqrt{\langle k\rangle}$ ．
－Note：This is a special property of Poisson distribution and can trip us up．．．

## Poisson basics：

－The variance of degree distributions for random networks turns out to be very important．
－Use calculation similar to one for finding $\langle k\rangle$ to find the second moment：

$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
$$

－Variance is then

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## The edge－degree distribution：

－The degree distribution $P_{k}$ is fundamental for our description of many complex networks

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－Again：$P_{k}$ is the degree of randomly chosen node．
－A second very important distribution arises from choosing randomly on edges rather than on nodes．
－Define $Q_{k}$ to be the probability the node at a random end of a randomly chosen edge has degree $k$ ．
－Now choosing nodes based on their degree（i．e．， size）：


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－Normalized form：

$$
Q_{k}=\frac{k P_{k}}{\sum_{k^{\prime}=0}^{\infty} k^{\prime} P_{k^{\prime}}}
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## The edge－degree distribution：

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Q_{k} \propto k P_{k}
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－Normalized form：

$$
Q_{k}=\frac{k P_{k}}{\sum_{k^{\prime}=0}^{\infty} k^{\prime} P_{k^{\prime}}}=\frac{k P_{k}}{\langle k\rangle} .
$$

## The edge－degree distribution：

－For random networks，$Q_{k}$ is also the probability that a friend（neighbor）of a random node has $k$ friends． Useful variant on $Q_{k}$


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－Equivalent to friend having degree $k+1$ ．
－Natural question：what＇s the expected number of other friends that one friend has？

## The edge－degree distribution：

－For random networks，$Q_{k}$ is also the probability that a friend（neighbor）of a random node has $k$ friends．
－Useful variant on $Q_{k}$ ：
$R_{k}=$ probability that a friend of a random node has $k$ other friends．

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$R_{k}=$ probability that a friend of a random node has $k$ other friends．

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R_{k}=\frac{(k+1) P_{k+1}}{\sum_{k^{\prime}=0}\left(k^{\prime}+1\right) P_{k^{\prime}+1}}=\frac{(k+1) P_{k+1}}{\langle k\rangle}
$$

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## The edge－degree distribution：

－Given $R_{k}$ is the probability that a friend has $k$ other
friends，then the average number of friends＇other friends is

$$
\langle k\rangle_{R}=\sum_{k=0}^{\infty} k R_{k}=\sum_{k=0}^{\infty} k \frac{(k+1) P_{k+1}}{\langle k\rangle}
$$

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（where we have sneakily matched up indices）


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－Given $R_{k}$ is the probability that a friend has $k$ other friends，then the average number of friends＇other friends is

$$
\begin{aligned}
\langle k\rangle_{R} & =\sum_{k=0}^{\infty} k R_{k}=\sum_{k=0}^{\infty} k \frac{(k+1) P_{k+1}}{\langle k\rangle} \\
& =\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}
\end{aligned}
$$

（where we have sneakily matched up indices）

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&\langle k\rangle_{R}=\sum_{k=0}^{\infty} k R_{k}=\sum_{k=0}^{\infty} k \frac{(k+1) P_{k+1}}{\langle k\rangle} \\
&=\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1} \\
&=\frac{1}{\langle k\rangle} \sum_{k=1}^{\infty}\left((k+1)^{2}-(k+1)\right) P_{k+1}
\end{aligned}
$$

（where we have sneakily matched up indices）

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（where we have sneakily matched up indices）

$$
=\frac{1}{\langle k\rangle} \sum_{j=0}^{\infty}\left(j^{2}-j\right) P_{j} \quad(\text { using } j=k+1)
$$

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\end{aligned}
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（where we have sneakily matched up indices）

$$
\begin{gathered}
\left.=\frac{1}{\langle k\rangle} \sum_{j=0}^{\infty}\left(j^{2}-j\right) P_{j} \quad \text { (using } j=k+1\right) \\
=\frac{1}{\langle k\rangle}\left(\left\langle k^{2}\right\rangle-\langle k\rangle\right)
\end{gathered}
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## The edge－degree distribution：

－Note：our result，$\langle k\rangle_{R}=\frac{1}{\langle k\rangle}\left(\left\langle k^{2}\right\rangle-\langle k\rangle\right)$ ，is true for all random networks，independent of degree distribution．
－For standard random networks，recall

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－Again，neatness of results is a special property of the Poisson distribution．
－So friends on average have $\langle k\rangle$ other friends，and $\langle k\rangle+1$ total friends．．．

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$$
\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle .
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－Therefore：


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## Two reasons why this matters

Reason \#1:

- Average \# friends of friends per node is
- Key: Average depends on the 1st and 2nd moments of $P_{k}$ and not just the 1st moment.
- Three peculiarities:

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## Two reasons why this matters

Reason \＃1：
－Average \＃friends of friends per node is

$$
\left\langle k_{2}\right\rangle=\langle k\rangle \times\langle k\rangle_{R}
$$

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\left\langle k_{2}\right\rangle=\langle k\rangle \times\langle k\rangle_{R}=\langle k\rangle \frac{1}{\langle k\rangle}\left(\left\langle k^{2}\right\rangle-\langle k\rangle\right)=\left\langle k^{2}\right\rangle-\langle k\rangle .
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- Key: Average depends on the 1st and 2nd moments of $P_{k}$ and not just the 1st moment.

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## Two reasons why this matters

Reason \＃1：
－Average \＃friends of friends per node is

$$
\left\langle k_{2}\right\rangle=\langle k\rangle \times\langle k\rangle_{R}=\langle k\rangle \frac{1}{\langle k\rangle}\left(\left\langle k^{2}\right\rangle-\langle k\rangle\right)=\left\langle k^{2}\right\rangle-\langle k\rangle .
$$

－Key：Average depends on the 1st and 2nd moments of $P_{k}$ and not just the 1st moment．
－Three peculiarities：
1．We might guess $\left\langle k_{2}\right\rangle=\langle k\rangle(\langle k\rangle-1)$ but it＇s actually $\langle k(k-1)\rangle$ ．
2．If $P_{k}$ has a large second moment， then $\left\langle k_{2}\right\rangle$ will be big．

3．Your friends are different to you．．．

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## Two reasons why this matters

More on peculiarity \＃3：
－A node＇s average \＃of friends：$\langle k\rangle$

－Comparison：

－So only if everyone has the same degree （variance $=\sigma^{2}=0$ ）can a node be the same as its friends．
－Intuition：for random networks，the more connected a node，the more likely it is to be chosen as a friend．

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## Two reasons why this matters

More on peculiarity \＃3：
－A node＇s average \＃of friends：$\langle k\rangle$
－Friend＇s average \＃of friends：$\frac{\left\langle k^{2}\right\rangle}{\langle k\rangle}$
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\frac{\left\langle k^{2}\right\rangle}{\langle k\rangle}=\langle k\rangle \frac{\left\langle k^{2}\right\rangle}{\langle k\rangle^{2}}=\langle k\rangle \frac{\sigma^{2}+\langle k\rangle^{2}}{\langle k\rangle^{2}}
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－So only if everyone has the same degree （variance $=\sigma^{2}=0$ ）can a node be the same as its friends．
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References friends．
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## Two reasons why this matters

More on peculiarity \#3:

- A node's average \# of friends: $\langle k\rangle$
- Friend's average \# of friends: $\frac{\left\langle k^{2}\right\rangle}{\langle k\rangle}$
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$$
\frac{\left\langle k^{2}\right\rangle}{\langle k\rangle}=\langle k\rangle \frac{\left\langle k^{2}\right\rangle}{\langle k\rangle^{2}}=\langle k\rangle \frac{\sigma^{2}+\langle k\rangle^{2}}{\langle k\rangle^{2}}=\langle k\rangle\left(1+\frac{\sigma^{2}}{\langle k\rangle^{2}}\right) \geq\langle k\rangle
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- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.


## Two reasons why this matters

（Big）Reason \＃2：
－$\langle k\rangle_{R}$ is key to understanding how well random networks are connected together．
＞e．g．，we＇d like to know what＇s the size of the largest component within a network．
$-\Delta s N \rightarrow \infty$ ，does our network have a giant component？
－Defn：Component＝connected subnetwork of nodes such that $\exists$ path between each pair of nodes in the subnetwork，and no node outside of the subnetwork is connected to it．
－Defn：Giant component＝component that comprises a non－zero fraction of a network as $N \rightarrow \infty$ ．
－Note：Component＝Cluster

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## Two reasons why this matters

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－$\langle k\rangle_{R}$ is key to understanding how well random networks are connected together．
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## Structure of random networks

## Giant component：

－A giant component exists if when we follow a random edge，we are likely to hit a node with at least 1 other outgoing edge．
－Equivalently，expect exponential growth in node number as we move out from a random node． All of this is the same as requiring $\langle k\rangle_{B}>1$ Giant component condition（or percolation condition）：


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Again，see that the second moment is an essential part of the story．
＞Equivalent statement：$\left\langle k^{2}\right\rangle>2\langle k\rangle$

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## Structure of random networks

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－Giant component condition（or percolation condition）：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1
$$

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－Equivalent statement：$\left\langle k^{2}\right\rangle>2\langle k\rangle$

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## Giant component

## Standard random networks：

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－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
－When $\langle k\rangle<1$ ，all components are finite．
－Fine example of a continuous phase transition（ $\boxplus$ ）．
－We say $\langle k\rangle=1$ marks the critical point of the system．

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## Giant component

## Standard random networks：

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$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}
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－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
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## Giant component

## Standard random networks:

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$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}
$$

- Therefore when $\langle k\rangle>1$, standard random networks have a giant component.
- When $\langle k\rangle<1$, all components are finite.
- Fine example of a continuous phase transition ( $\boxplus$ ).
- We say $\langle k\rangle=1$ marks the critical point of the system.


## Giant component

## Standard random networks:

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$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}=\langle k\rangle
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- Therefore when $\langle k\rangle>1$, standard random networks have a giant component.
- When $\langle k\rangle<1$, all components are finite.
- Fine example of a continuous phase transition ( $\boxplus$ ).
- We say $\langle k\rangle=1$ marks the critical point of the system.


## Giant component

## Standard random networks：

－Recall $\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle$ ．
－Condition for giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}=\langle k\rangle
$$

－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
－When $\langle k\rangle<1$ ，all components are finite．
－Fine example of a continuous
－We say $\langle k\rangle=1$ marks the critical point of the system．

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## Giant component

## Standard random networks：

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\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}=\langle k\rangle
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－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
－When $\langle k\rangle<1$ ，all components are finite．
－Fine example of a continuous phase transition（ $\square$ ）．
$>$ We say $\langle k\rangle=1$ marks the critical point of the system．

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回 $\quad$ のく

## Giant component

## Standard random networks：

－Recall $\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle$ ．
－Condition for giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}=\langle k\rangle
$$

－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
－When $\langle k\rangle<1$ ，all components are finite．
－Fine example of a continuous phase transition（ $\boxplus$ ）．
－We say $\langle k\rangle=1$ marks the critical point of the system．

## Giant component

## Basics

## Standard random networks：

－Recall $\left\langle k^{2}\right\rangle=\langle k\rangle^{2}+\langle k\rangle$ ．
－Condition for giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}=\frac{\langle k\rangle^{2}+\langle k\rangle-\langle k\rangle}{\langle k\rangle}=\langle k\rangle
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－Therefore when $\langle k\rangle>1$ ，standard random networks have a giant component．
－When $\langle k\rangle<1$ ，all components are finite．
－Fine example of a continuous phase transition（ $\boxplus$ ）．
－We say $\langle k\rangle=1$ marks the critical point of the system．

## Giant component

Random networks with skewed $P_{k}$ ：
－e．g，if $P_{k}=c k^{-\gamma}$ with $2<\gamma<3$ then

$$
\left\langle k^{2}\right\rangle=c \sum_{k=0}^{\infty} k^{2} k^{-\gamma}
$$

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－So giant component always exists for these kinds of networks．
－Cutoff scaling is $k^{-3}$ ：if $\gamma>3$ then we have to look harder at $\langle k\rangle_{R}$ ．

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## Giant component

Random networks with skewed $P_{k}$ ：
－e．g，if $P_{k}=c k^{-\gamma}$ with $2<\gamma<3$ then

$$
\begin{gathered}
\left\langle k^{2}\right\rangle=c \sum_{k=0}^{\infty} k^{2} k^{-\gamma} \\
\sim \int_{x=0}^{\infty} x^{2-\gamma} \mathrm{d} x
\end{gathered}
$$

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－So giant component always exists for these kinds of networks．
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司 つQく

## Giant component

Random networks with skewed $P_{k}$ ：
－e．g，if $P_{k}=c k^{-\gamma}$ with $2<\gamma<3$ then

$$
\begin{array}{r}
\left\langle k^{2}\right\rangle=c \sum_{k=0}^{\infty} k^{2} k^{-\gamma} \\
\sim \int_{x=0}^{\infty} x^{2-\gamma} \mathrm{d} x \\
\left.\propto x^{3-\gamma}\right|_{x=0} ^{\infty}
\end{array}
$$

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\sim \int_{x=0}^{\infty} x^{2-\gamma} \mathrm{d} x \\
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\end{gathered}
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\left.\propto x^{3-\gamma}\right|_{x=0} ^{\infty}=\infty \quad(>\langle k\rangle) .
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－Cutoff scaling is $k^{-3}$ ：if $\gamma>3$ then we have to look harder at $\langle k\rangle_{R}$ ．

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## Giant component

## And how big is the largest component？

－Define $S_{1}$ as the size of the largest component．
－Consider an infinite ER random network with average degree $\langle k\rangle$ ．
－Let＇s find $S_{i}$ with a back－of－the－envelope argument．
－Define $\delta$ as the probability that a randomly chosen node does not belong to the largest component．
－Simple connection：$\delta=1-S_{1}$ ．
－Dirty trick：If a randomly chosen node is not part of the largest component，then none of its neighbors are．

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－So

－Substitute in Poisson distribution．．．

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## Giant component

## And how big is the largest component？

－Define $S_{1}$ as the size of the largest component．
－Consider an infinite ER random network with average degree $\langle k\rangle$ ．
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－Define $\delta$ as the probability that a randomly chosen node does not belong to the largest component．
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－Substitute in Poisson distribution．．．

## Giant component

## And how big is the largest component？

－Define $S_{1}$ as the size of the largest component．
－Consider an infinite ER random network with average degree $\langle k\rangle$ ．
－Let＇s find $S_{1}$ with a back－of－the－envelope argument．
$\rightarrow$ Define $\delta$ as the probability that a randomly chosen node does not belong to the largest component．
－Simple connection：$\delta-1-S_{1}$
－Dirty trick：If a randomly chosen node is not part of the largest component，then none of its neighbors are．

－Substitute in Poisson distribution．．．

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## Giant component

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－Define $S_{1}$ as the size of the largest component．
－Consider an infinite ER random network with average degree $\langle k\rangle$ ．
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－Substitute in Poisson distribution．．．

## Giant component

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－Substitute in Poisson distribution．．

## Giant component

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－Substitute in Poisson distribution．．．

## Giant component

## And how big is the largest component？

－Define $S_{1}$ as the size of the largest component．
－Consider an infinite ER random network with average degree $\langle k\rangle$ ．
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－So

$$
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}
$$

－Substitute in Poisson distribution．

## Giant component

And how big is the largest component？
－Define $S_{1}$ as the size of the largest component．
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－Let＇s find $S_{1}$ with a back－of－the－envelope argument．
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－So

$$
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}
$$

－Substitute in Poisson distribution．．．

## Giant component

- Carrying on:

$$
\delta=\sum_{k=0}^{\infty} \boldsymbol{P}_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k}
$$

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- Now substitute in $\delta=1-S_{1}$ and rearrange to obtain:

$$
S_{1}=1-e^{-\langle k\rangle S_{1}} .
$$

## Giant component

－Carrying on：

$$
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k}
$$

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－Now substitute in $\delta=1-S_{1}$ and rearrange to obtain：


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## Giant component

－Carrying on：

$$
\begin{gathered}
\delta=\sum_{k=0}^{\infty} P_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k} \\
=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{(\langle k\rangle \delta)^{k}}{k!}
\end{gathered}
$$

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－Now substitute in $\delta=1-S_{1}$ and rearrange to obtain：


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## Giant component

－Carrying on：

$$
\begin{aligned}
\delta= & \sum_{k=0}^{\infty} P_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k} \\
& =e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{(\langle k\rangle \delta)^{k}}{k!} \\
= & e^{-\langle k\rangle} e^{\langle k\rangle \delta}=e^{-\langle k\rangle(1-\delta)}
\end{aligned}
$$

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－Now substitute in $\delta=1-S_{1}$ and rearrange to obtain：


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## Giant component

－Carrying on：

$$
\begin{aligned}
& \delta= \sum_{k=0}^{\infty} P_{k} \delta^{k}=\sum_{k=0}^{\infty} \frac{\langle k\rangle^{k}}{k!} e^{-\langle k\rangle} \delta^{k} \\
&=e^{-\langle k\rangle} \sum_{k=0}^{\infty} \frac{(\langle k\rangle \delta)^{k}}{k!} \\
&=e^{-\langle k\rangle} e^{\langle k\rangle \delta}=e^{-\langle k\rangle(1-\delta)}
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## Giant component

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&=e^{-\langle k\rangle} e^{\langle k\rangle \delta}=e^{-\langle k\rangle(1-\delta)}
\end{aligned}
$$

－Now substitute in $\delta=1-S_{1}$ and rearrange to obtain：

$$
S_{1}=1-e^{-\langle k\rangle S_{1}} .
$$

## Giant component

－We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$ ．
First，we can write $\langle k\rangle$ in terms of $S_{1}$ ：

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## Giant component

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－We can figure out some limits and details for $S_{1}=1-e^{-\langle k\rangle S_{1}}$.
－First，we can write $\langle k\rangle$ in terms of $S_{1}$ ：

$$
\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}} .
$$

－Notice that at $\langle k\rangle=1$ ，the critical point，$S_{1}=0$ ．
－Only solvable for $S>0$ when $\langle k\rangle>1$ ．
－Really a transcritical bifurcation

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－As $\langle k\rangle \rightarrow 0, s_{1} \rightarrow 0$.
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## Giant component

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## Giant component

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－Only solvable for $S>0$ when $\langle k\rangle>1$ ．
－Really a transcritical bifurcation ${ }^{[2]}$ ．

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## Giant component

Turns out we were lucky．．．
－Our dirty trick only works for ER random networks．
－The problem：We assumed that neighbors have the same probability $\delta$ of belonging to the largest component．
－But we know our friends are different from us．．．
－Works for ER random networks because $\langle k\rangle=\langle k\rangle_{R}$ ．
－We need a separate probability $\delta^{\prime}$ for the chance that a node at the end of a random edge is part of the largest component．

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－We can do this but we need to enhance our toolkit with Generatingfunctionology．．．
－（Well，not really but it＇s fun and we get all sorts of other things．．．）

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## Giant component

Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability $\delta$ of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because $\langle k\rangle=\langle k\rangle_{R}$.
- We need a separate probability $\delta^{\prime}$ for the chance that a node at the end of a random edge is part of the largest component.
- We can do this but we need to enhance our toolkit with Generatingfunctionology..
- (Well, not really but it's fun and we get all sorts of other things...)

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## Giant component

Turns out we were lucky．．．
－Our dirty trick only works for ER random networks．
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## Giant component

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（Well，not really but it＇s fun and we get all sorts of other things．．．）

## Giant component

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## Generating functions

- Idea: Given a sequence $a_{0}, a_{1}, a_{2}, \ldots$, associate each element with a distinct function or other mathematical object.
- Well-chosen functions allow us to manipulate sequences and retrieve sequence elements.

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## Generating functions

－Idea：Given a sequence $a_{0}, a_{1}, a_{2}, \ldots$ ，associate each element with a distinct function or other mathematical object．
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## Generating functions

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－Well－chosen functions allow us to manipulate sequences and retrieve sequence elements．

Definition：
－The generating function（g．f．）for a sequence $\left\{a_{n}\right\}$ is

$$
F(x)=\sum_{n=0}^{\infty} a_{n} x^{n}
$$

$\rightarrow$ Roughly：transforms a vector in $R^{\infty}$ into a function defined on $R^{1}$ ．
－Related to Fourier，Laplace，Mellin，

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## Simple example

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Rolling dice：
－$p_{k}^{(\square)}=\operatorname{Pr}($ throwing a $k)=1 / 6$ where $k=1,2, \ldots, 6$ ．

－We＇ll come back to this simple example as we derive various delicious properties of generating functions．

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## Simple example

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Rolling dice：
－$p_{k}^{(\square)}=\operatorname{Pr}($ throwing a $k)=1 / 6$ where $k=1,2, \ldots, 6$ ．

$$
F^{(\square)}(x)=\sum_{k=1}^{6} p_{k} x^{k}=\frac{1}{6}\left(x+x^{2}+x^{3}+x^{4}+x^{5}+x^{6}\right) .
$$

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$$
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$$

## Simple example

Rolling dice：
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## Example

－Take a degree distribution with exponential decay：

$$
P_{k}=c e^{-\lambda k}
$$

where $c=1-e^{-\lambda}$ ．
The generating function for this distribution is

－Notice that $F(1)=c /\left(1-e^{-\lambda}\right)=1$ ．
－For probability distributions，we must always have $F(1)=1$ since

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## Properties of generating functions

－Average degree：

$$
\langle k\rangle=\sum_{k=0}^{\infty} k P_{k}=\sum_{k=0}^{\infty} k P_{k} x^{k-1}
$$

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## Properties of generating functions

－Average degree：

$$
\langle k\rangle=\sum_{k=0}^{\infty} k P_{k}=\left.\sum_{k=0}^{\infty} k P_{k} x^{k-1}\right|_{x=1}
$$

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## Properties of generating functions

－Average degree：

$$
\begin{aligned}
\langle k\rangle= & \sum_{k=0}^{\infty} k P_{k}=\left.\sum_{k=0}^{\infty} k P_{k} x^{k-1}\right|_{x=1} \\
& =\left.\frac{\mathrm{d}}{\mathrm{~d} x} F(x)\right|_{x=1}
\end{aligned}
$$

－In general，many calculations become simple，if a little abstract．
－For our exponential example：

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& =\left.\frac{\mathrm{d}}{\mathrm{~d} x} F(x)\right|_{x=1}=F^{\prime}(1)
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## Properties of generating functions

- Average degree:

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- In general, many calculations become simple, if a little abstract.
- For our exponential example:

$$
F^{\prime}(x)=\frac{\left(1-e^{-\lambda}\right) e^{-\lambda}}{\left(1-x e^{-\lambda}\right)^{2}}
$$

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－So：

$$
\langle k\rangle=F^{\prime}(1)=\frac{e^{-\lambda}}{\left(1-e^{-\lambda}\right)}
$$

## Properties of generating functions

## Useful pieces for probability distributions：

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## Properties of generating functions

## Useful pieces for probability distributions：

－Normalization：

$$
F(1)=1
$$

－First moment：
－Higher moments：

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－$k$ th element of sequence（general）：

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## Properties of generating functions

## Useful pieces for probability distributions:

- Normalization:

$$
F(1)=1
$$

- First moment:

$$
\langle k\rangle=F^{\prime}(1)
$$

- Higher moments:

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- $k$ th element of sequence (general):


## Properties of generating functions

## Useful pieces for probability distributions：

－Normalization：

$$
F(1)=1
$$

－First moment：

$$
\langle k\rangle=F^{\prime}(1)
$$

－Higher moments：

$$
\left\langle k^{n}\right\rangle=\left.\left(x \frac{\mathrm{~d}}{\mathrm{~d} x}\right)^{n} F(x)\right|_{x=1}
$$

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－$k$ th element of sequence（general）：

## Properties of generating functions

## Useful pieces for probability distributions：

－Normalization：

$$
F(1)=1
$$

－First moment：

$$
\langle k\rangle=F^{\prime}(1)
$$

－Higher moments：

$$
\left\langle k^{n}\right\rangle=\left.\left(x \frac{\mathrm{~d}}{\mathrm{~d} x}\right)^{n} F(x)\right|_{x=1}
$$

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－$k$ th element of sequence（general）：

$$
P_{k}=\left.\frac{1}{k!} \frac{\mathrm{d}^{k}}{\mathrm{~d} x^{k}} F(x)\right|_{x=0}
$$

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## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

## －Let＇s rëexpress our condition in terms of generating functions． <br> －We first need the g．f．for $R_{k}$ ． <br> －We＇ll now use this notation：

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－Condition in terms of g．f．is：

－Now find how $F_{R}$ is related to $F_{P} \ldots$

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## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$
－We＇ll now use this notation：

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－Condition in terms of g．f．is：

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## Edge－degree distribution

－Recall our condition for a giant component：

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$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$ ．
－We＇ll now use this notation：

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－Condition in terms of g．f．is：

## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$ ．
－We＇ll now use this notation：

－Condition in terms of g．f．is：

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－Now find how $F_{R}$ is related to $F_{P}$ ．

## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$ ．
－We＇ll now use this notation：
$F_{P}(x)$ is the g．f．for $P_{k}$.
－Condition in terms of g．f．is：

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－Now find how $F_{R}$ is related to $F_{P}$ ．

## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$ ．
－We＇ll now use this notation：
$F_{P}(x)$ is the g．f．for $P_{k}$.
$F_{R}(x)$ is the g．f．for $R_{k}$ ．
－Condition in terms of g．f．is：

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－Now find how $F_{R}$ is related to $F_{P}$ ．

## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$ ．
－We＇ll now use this notation：
$F_{P}(x)$ is the g．f．for $P_{k}$.
$F_{R}(x)$ is the g．f．for $R_{k}$ ．
－Condition in terms of g．f．is：

$$
\langle k\rangle_{R}=F_{R}^{\prime}(1)>1
$$

－Now find how $F_{R}$ is related to $F_{P}$ ．．

## Edge－degree distribution

－Recall our condition for a giant component：

$$
\langle k\rangle_{R}=\frac{\left\langle k^{2}\right\rangle-\langle k\rangle}{\langle k\rangle}>1 .
$$

－Let＇s rëexpress our condition in terms of generating functions．
－We first need the g．f．for $R_{k}$ ．
－We＇ll now use this notation：
$F_{P}(x)$ is the g．f．for $P_{k}$.
$F_{R}(x)$ is the g．f．for $R_{k}$ ．
－Condition in terms of g．f．is：

$$
\langle k\rangle_{R}=F_{R}^{\prime}(1)>1 .
$$

－Now find how $F_{R}$ is related to $F_{P} \ldots$

## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{(k\rangle} x^{k}
$$



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Finally，since $\langle k\rangle=F_{P}^{\prime}(1)$ ，


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## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k}
$$



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Finally，since $\langle k\rangle=F_{P}^{\prime}(1)$ ，


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$$

## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ ：


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## Finally，since $\langle k\rangle=F_{P}^{\prime}(1)$ ，



## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ ：

$$
F_{R}(x)=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} j P_{j} x^{j-1}
$$



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Finally，since $\langle k\rangle=F_{P}^{\prime}(1)$ ，


## Edge-degree distribution

- We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ :

$$
F_{R}(x)=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} j P_{j} x^{j-1}=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} P_{j} \frac{\mathrm{~d}}{\mathrm{~d} \chi} x^{j}
$$

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Finally, since $\langle k\rangle=F_{P}^{\prime}(1)$,


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## Edge-degree distribution

- We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ :

$$
\begin{gathered}
F_{R}(x)=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} j P_{j} x^{j-1}=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} P_{j} \frac{\mathrm{~d}}{\mathrm{~d} x} x^{j} \\
=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x} \sum_{j=1}^{\infty} P_{j} x^{j}=\frac{1}{\langle k\rangle} \frac{\mathrm{d} x}{\mathrm{~d} x}\left(F_{P}(x)-P_{0}\right)=\frac{1}{\langle k\rangle} F_{p}^{\prime}(x) .
\end{gathered}
$$

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Finally, since $\langle k\rangle=F_{p}^{\prime}(1)$,


## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ ：

$$
\begin{array}{r}
F_{R}(x)=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} j P_{j} x^{j-1}=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} P_{j} \frac{\mathrm{~d}}{\mathrm{~d} x} x^{j} \\
=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x} \sum_{j=1}^{\infty} P_{j} x^{j}=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x}\left(F_{P}(x)-P_{0}\right)=\frac{1}{\langle k\rangle}
\end{array}
$$

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Finally，since $\langle k\rangle=F_{p}^{\prime}(1)$ ，


## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ ：

$$
\begin{gathered}
F_{R}(x)=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} j P_{j} x^{j-1}=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} P_{j} \frac{\mathrm{~d}}{\mathrm{~d} x} x^{j} \\
=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x} \sum_{j=1}^{\infty} P_{j} x^{j}=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x}\left(F_{P}(x)-P_{0}\right)=\frac{1}{\langle k\rangle} F_{P}^{\prime}(x) .
\end{gathered}
$$

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Finally，since $\langle k\rangle=F_{p}^{\prime}(1)$ ，


## Edge－degree distribution

－We have

$$
F_{R}(x)=\sum_{k=0}^{\infty} R_{k} x^{k}=\sum_{k=0}^{\infty} \frac{(k+1) P_{k+1}}{\langle k\rangle} x^{k} .
$$

Shift index to $j=k+1$ and pull out $\frac{1}{\langle k\rangle}$ ：

$$
\begin{gathered}
F_{R}(x)=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} j P_{j} x^{j-1}=\frac{1}{\langle k\rangle} \sum_{j=1}^{\infty} P_{j} \frac{\mathrm{~d}}{\mathrm{~d} x} x^{j} \\
=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x} \sum_{j=1}^{\infty} P_{j} x^{j}=\frac{1}{\langle k\rangle} \frac{\mathrm{d}}{\mathrm{~d} x}\left(F_{P}(x)-P_{0}\right)=\frac{1}{\langle k\rangle} F_{P}^{\prime}(x) .
\end{gathered}
$$

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Finally，since $\langle k\rangle=F_{P}^{\prime}(1)$ ，

$$
F_{R}(X)=\frac{F^{\prime}(X)}{F_{P}^{\prime}(1)}
$$

## Edge－degree distribution

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Definitions
－Recall giant component condition is $\langle k\rangle_{R}=F_{R}^{\prime}(1)>1$ ．
$\Rightarrow$ Since we have $F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)$ ， Setting $x=1$ ，our condition becomes

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## Edge-degree distribution

- Recall giant component condition is
$\langle k\rangle_{R}=F_{R}^{\prime}(1)>1$.
- Since we have $F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)$,

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## Edge－degree distribution

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$\langle k\rangle_{R}=F_{R}^{\prime}(1)>1$ ．
－Since we have $F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)$ ，

$$
F_{R}^{\prime}(x)=\frac{F_{P}^{\prime \prime}(x)}{F_{P}^{\prime}(1)} .
$$

－Setting $x=1$ ，our condition becomes

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## Edge－degree distribution

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$$
F_{R}^{\prime}(x)=\frac{F_{P}^{\prime \prime}(x)}{F_{P}^{\prime}(1)} .
$$

－Setting $x=1$ ，our condition becomes

$$
\frac{F_{P}^{\prime \prime}(1)}{F_{P}^{\prime}(1)}>1
$$

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## Size distributions

To figure out the size of the largest component $\left(S_{1}\right)$ ，we need more resolution on component sizes．

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## Size distributions

To figure out the size of the largest component $\left(S_{1}\right)$ ，we need more resolution on component sizes．
Definitions：
－$\pi_{n}=$ probability that a random node belongs to a finite component of size $n<\infty$ ．
$\rho_{n}=$ probability a random link leads to a finite
subcomponent of size $n<\infty$ ．

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neighbors $\Leftrightarrow$ components

## Size distributions

To figure out the size of the largest component $\left(S_{1}\right)$ ，we need more resolution on component sizes．
Definitions：
－$\pi_{n}=$ probability that a random node belongs to a finite component of size $n<\infty$ ．
－$\rho_{n}=$ probability a random link leads to a finite subcomponent of size $n<\infty$ ．

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## Size distributions

To figure out the size of the largest component $\left(S_{1}\right)$ ，we need more resolution on component sizes．
Definitions：
－$\pi_{n}=$ probability that a random node belongs to a finite component of size $n<\infty$ ．
－$\rho_{n}=$ probability a random link leads to a finite subcomponent of size $n<\infty$ ．

Local－global connection：

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$$
P_{k}, R_{k} \Leftrightarrow \pi_{n}, \rho_{n}
$$

neighbors $\Leftrightarrow$ components

## Size distributions

G.f.'s for component size distributions:

The largest component:
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Our mission, which we accept:

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## Size distributions

## G.f.'s for component size distributions:

$$
F_{\pi}(x)=\sum_{n=0}^{\infty} \pi_{n} x^{n} \text { and } F_{\rho}(x)=\sum_{n=0}^{\infty} \rho_{n} x^{n}
$$

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## Size distributions

G．f．＇s for component size distributions：

$$
F_{\pi}(x)=\sum_{n=0}^{\infty} \pi_{n} x^{n} \text { and } F_{\rho}(x)=\sum_{n=0}^{\infty} \rho_{n} x^{n}
$$

The largest component：
－Subtle key：$F_{\pi}(1)$ is the probability that a node belongs to a finite component．

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Our mission，which we accept：

## Size distributions

G．f．＇s for component size distributions：

$$
F_{\pi}(x)=\sum_{n=0}^{\infty} \pi_{n} x^{n} \text { and } F_{\rho}(x)=\sum_{n=0}^{\infty} \rho_{n} x^{n}
$$

The largest component：
－Subtle key：$F_{\pi}(1)$ is the probability that a node belongs to a finite component．
－Therefore：$S_{1}=1-F_{\pi}(1)$ ．

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Our mission，which we accept：

## Size distributions

G．f．＇s for component size distributions：

$$
F_{\pi}(x)=\sum_{n=0}^{\infty} \pi_{n} x^{n} \text { and } F_{\rho}(x)=\sum_{n=0}^{\infty} \rho_{n} x^{n}
$$

The largest component：
－Subtle key：$F_{\pi}(1)$ is the probability that a node belongs to a finite component．
－Therefore：$S_{1}=1-F_{\pi}(1)$ ．

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Our mission，which we accept：
－Find the four generating functions

$$
F_{P}, F_{R}, F_{\pi} \text {, and } F_{\rho} .
$$

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 1：

> －Consider two random variables $U$ and $V$ whose values may be $0,1,2$ ，
> －Write probability distributions as $U_{k}$ and $V_{k}$ and g．f．＇s as $F_{U}$ and $F_{V}$ ．
> －SR1：If a third random variable is defined as


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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 1：

－Consider two random variables $U$ and $V$ whose values may be $0,1,2, \ldots$
－Write probability distributions as $U_{k}$ and $V_{k}$ and g．f．＇s as $F_{U}$ and $F_{V}$ ．
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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 1：

－Consider two random variables $U$ and $V$ whose values may be $0,1,2, \ldots$
－Write probability distributions as $U_{k}$ and $V_{k}$ and g．f．＇s as $F_{U}$ and $F_{V}$ ．
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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 1：

－Consider two random variables $U$ and $V$ whose values may be $0,1,2, \ldots$
－Write probability distributions as $U_{k}$ and $V_{k}$ and g．f．＇s as $F_{U}$ and $F_{V}$ ．
－SR1：If a third random variable is defined as

$$
W=\sum_{i=1}^{U} V^{(i)} \text { with each } V^{(i)} \stackrel{d}{=} V
$$

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then


## Useful results we'll need for g.f.'s

## Sneaky Result 1:

- Consider two random variables $U$ and $V$ whose values may be $0,1,2, \ldots$
- Write probability distributions as $U_{k}$ and $V_{k}$ and g.f.'s as $F_{U}$ and $F_{V}$.
- SR1: If a third random variable is defined as

$$
W=\sum_{i=1}^{U} V^{(i)} \text { with each } V^{(i)} \stackrel{d}{=} V
$$

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then

$$
F_{W}(x)=F_{U}\left(F_{V}(x)\right)
$$

## Proof of SR1:

Write probability that variable $W$ has value $k$ as $W_{k}$.


## Proof of SR1：

Write probability that variable $W$ has value $k$ as $W_{k}$ ．

$$
W_{k}=\sum_{j=0}^{\infty} U_{j} \times \operatorname{Pr}(\text { sum of } j \text { draws of variable } V=k)
$$

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## Proof of SR1：

Write probability that variable $W$ has value $k$ as $W_{k}$ ．

$$
\begin{aligned}
W_{k}=\sum_{j=0}^{\infty} & U_{j} \times \operatorname{Pr}(\text { sum of } j \text { draws of variable } V=k) \\
& =\sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left\{i_{i}, \ldots, \ldots, i_{j}\right\} \\
i_{1}+i_{2}+\ldots+i=k}} V_{i_{1}} v_{i_{2}} \cdots V_{i_{j}}
\end{aligned}
$$



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## Proof of SR1：

Write probability that variable $W$ has value $k$ as $W_{k}$ ．

$$
\begin{gathered}
W_{k}=\sum_{j=0}^{\infty} U_{j} \times \operatorname{Pr}(\text { sum of } j \text { draws of variable } V=k) \\
=\sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left\{i_{1}, i_{2}, \ldots, i_{j}\right\} \\
i_{1}+i_{2}+\ldots+j=k}} V_{i_{1}} v_{i_{2}} \cdots v_{i_{j}} \\
\therefore F_{W}(x)=\sum_{k=0}^{\infty} W_{k} x^{k}=\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} U_{j} \sum_{i, 2} v_{i_{1}} v_{i_{2}} \cdots
\end{gathered}
$$

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## Proof of SR1：

Write probability that variable $W$ has value $k$ as $W_{k}$ ．

$$
\begin{gathered}
W_{k}=\sum_{j=0}^{\infty} U_{j} \times \operatorname{Pr}(\text { sum of } j \text { draws of variable } V=k) \\
=\sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left\{i_{1}, i_{2}, \ldots, i_{j}\right\} \\
i_{1}+i_{2}+\ldots+i_{j}=k}} V_{i_{1}} v_{i_{2}} \cdots V_{i_{j}} \\
\therefore F_{W}(x)=\sum_{k=0}^{\infty} W_{k} x^{k}=\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left.i_{1}, i_{2}, \ldots, i_{j} \mid\right\} \\
i_{1}+i_{2}+\ldots+i_{j}=k}} V_{i_{1}} V_{i_{2}} \cdots V_{i_{j} x^{k}}
\end{gathered}
$$

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## Proof of SR1：

Write probability that variable $W$ has value $k$ as $W_{k}$ ．

$$
\begin{aligned}
& W_{k}=\sum_{j=0}^{\infty} U_{j} \times \operatorname{Pr}(\text { sum of } j \text { draws of variable } V=k) \\
& =\sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left\{1_{1}, i_{2}, \ldots, i_{1}\right\} \\
i_{1}+i_{2}+\ldots+j_{j}=k}} V_{i_{1}} v_{i_{2}} \cdots V_{i_{j}} \\
& \therefore F_{W}(x)=\sum_{k=0}^{\infty} W_{k} x^{k}=\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left.i_{1}, i_{2}, \ldots, j_{j}\right\} \mid \\
i_{1}+i_{2}+\ldots+i_{j}=k}} V_{i_{1}} V_{i_{2}} \cdots V_{i_{j} x^{k}} \\
& =\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty}
\end{aligned}
$$

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Write probability that variable $W$ has value $k$ as $W_{k}$ ．

$$
\begin{aligned}
& W_{k}=\sum_{j=0}^{\infty} U_{j} \times \operatorname{Pr}(\text { sum of } j \text { draws of variable } V=k) \\
& =\sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left\{i_{1}, i_{2}, \ldots, i_{2}\right\} \\
i_{1}+i_{2}+\ldots+j_{j}=k}} V_{i_{1}} V_{i_{2}} \cdots V_{i_{j}} \\
& \therefore F_{W}(x)=\sum_{k=0}^{\infty} W_{k} x^{k}=\sum_{k=0}^{\infty} \sum_{j=0}^{\infty} U_{j} \sum_{\substack{\left\{_{1}, i_{2}, \ldots, i_{j}\right\} \mid \\
i_{1}+i_{2}+\ldots+i_{j}=k}} V_{i_{1}} V_{i_{2}} \cdots V_{i_{j} x^{k}} \\
& =\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty} \sum_{\substack{i_{1}, i_{2}, \ldots, i_{i j} \mid \\
i_{1}+i_{2}+\ldots+j_{j}=k}} V_{i_{1}} x^{i_{1}} V_{i_{2}} x^{i_{2}} \ldots V_{i_{j}} x^{j_{j}}
\end{aligned}
$$

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## Proof of SR1：

With some concentration，observe：

$$
F_{W}(x)=\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty} \underbrace{\sum_{\substack{\left\{i_{1}, i_{2}, \ldots, i_{2}\right\} \\ i_{1}+i_{2}+\ldots+i_{j}=k}} V_{i_{1}} x^{i_{1}} V_{i_{2}} x^{i_{2}} \ldots V_{i_{j}} x^{j_{j}}}_{x^{k} \text { piece of }\left(\sum_{i^{\prime}=0}^{\infty} V_{i^{\prime}} x^{i^{\prime}}\right)^{j}}
$$

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## Proof of SR1:

With some concentration, observe:

$$
\begin{aligned}
& F_{W}(x)=\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty} \underbrace{\sum_{\substack{\left\{i_{i}, \ldots, \ldots, i_{2}\right\} \\
i_{1}+i_{2}+\ldots+i_{j}=k}} V_{i_{1}} x^{i_{1}} V_{i_{2}} x^{i_{2}} \ldots V_{i_{j} x^{j_{j}}}}_{x^{k} \text { piece of }\left(\sum_{i^{\prime}=0}^{\infty} V_{i^{\prime}} x^{i^{\prime}}\right)^{j}} \\
& \left(\sum_{i^{\prime}=0}^{\infty} v_{i^{\prime}} x^{i}\right)^{j}=\left(F_{V}(x)\right)^{j}
\end{aligned}
$$

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$$
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$$

## Proof of SR1:

With some concentration, observe:

$$
\begin{aligned}
& F_{W}(x)=\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty} \sum_{\left\{\left\{_{i}, 2, \ldots, \ldots, j\right\} \mid\right.} V_{i_{1}} x^{i_{1}} V_{i_{2}} x^{i_{2}} \ldots V_{i j} x^{i_{j}} \\
& i_{1}+i_{2}+\ldots+i_{i}=k \\
& x^{k} \text { piece of }\left(\sum_{i^{\prime}=0}^{\infty} V_{i^{\prime}} x^{\prime}\right)^{j} \\
& \left(\sum_{i^{\prime}=0}^{\infty} V_{i^{\prime}} x^{i^{\prime}}\right)^{j}=\left(F_{V}(x)\right)^{j} \\
& =\sum_{j=0}^{\infty} U_{j}\left(F_{V}(x)\right)^{j}
\end{aligned}
$$

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$$
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$$

## Proof of SR1：

With some concentration，observe：

$$
\begin{aligned}
& F_{W}(x)=\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty} \sum_{\left\{\left\{_{1}, 2_{2}, \ldots, \ldots, i_{1}\right\}\right.} V_{i_{1}} x^{i_{1}} V_{i_{2}} x^{i_{2}} \ldots V_{i_{j}} x^{i_{j}} \\
& i_{1}+i_{2}+\ldots+i_{j}=k \\
& x^{k} \text { piece of }\left(\sum_{i^{\prime}=0}^{\infty} v_{i^{\prime}} x^{i}\right)^{j} \\
& \left(\sum_{i^{\prime}=0}^{\infty} V_{i^{\prime}} x^{i^{\prime}}\right)^{j}=\left(F_{V}(x)\right)^{j} \\
& =\sum_{j=0}^{\infty} U_{j}\left(F_{V}(x)\right)^{j} \\
& =F_{U}\left(F_{V}(x)\right)
\end{aligned}
$$

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## Proof of SR1：

With some concentration，observe：

$$
\begin{aligned}
& F_{W}(x)=\sum_{j=0}^{\infty} U_{j} \sum_{k=0}^{\infty} \sum_{\substack{\left.\left\{1,,_{2}, \ldots, i_{i},\right\}\right\}}} V_{i_{1}} x^{i_{1}} V_{i_{2}} x^{i_{2}} \ldots V_{i_{j}} x^{j_{j}} \\
& i_{1}+i_{2}+\ldots+i_{j}=k \\
& x^{k} \text { piece of }\left(\sum_{i^{\prime}=0}^{\infty} v_{i^{\prime}} x^{i}\right)^{j} \\
& \left(\sum_{i^{\prime}=0}^{\infty} V_{i^{\prime}} x^{i^{\prime}}\right)^{j}=\left(F_{V}(x)\right)^{j} \\
& =\sum_{j=0}^{\infty} U_{j}\left(F_{V}(x)\right)^{j} \\
& =F_{U}\left(F_{V}(x)\right)
\end{aligned}
$$

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## Useful results we＇ll need for g．f．＇s

Random Networks

## Sneaky Result 2：

## －Start with a random variable $U$ with distribution $U_{k}$ （ $k=0,1,2, \ldots$ ） <br> －SR2：If a second random variable is defined as

Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$ （ $k=0,1,2, \ldots$ ）
－SR2：If a second random variable is defined as

$$
\text { Reason: } V_{k}=U_{k-1} \text { for } k \geq 1 \text { and } V_{0}=0 \text {. }
$$

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

$$
\therefore F_{V}(x)=\sum_{k=0}^{\infty} V_{k} x^{k}
$$

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

$$
\therefore F_{V}(x)=\sum_{k=0}^{\infty} V_{k} x^{k}=\sum_{k=1}^{\infty} U_{k-1} x^{k}
$$

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

$$
\begin{gathered}
\therefore F_{V}(x)=\sum_{k=0}^{\infty} V_{k} x^{k}=\sum_{k=1}^{\infty} U_{k-1} x^{k} \\
=x \sum_{j=0}^{\infty} U_{j} x^{j}=x F_{U}(x) .
\end{gathered}
$$

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

$$
\begin{gathered}
\therefore F_{V}(x)=\sum_{k=0}^{\infty} V_{k} x^{k}=\sum_{k=1}^{\infty} U_{k-1} x^{k} \\
=x \sum_{j=0}^{\infty} U_{j} x^{j}=x F_{U}(x) .
\end{gathered}
$$

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## Useful results we＇ll need for g．f．＇s

## Sneaky Result 2：

－Start with a random variable $U$ with distribution $U_{k}$

$$
(k=0,1,2, \ldots)
$$

－SR2：If a second random variable is defined as

$$
V=U+1 \text { then } F_{V}(x)=x F_{U}(x)
$$

－Reason：$V_{k}=U_{k-1}$ for $k \geq 1$ and $V_{0}=0$ ．

$$
\begin{gathered}
\therefore F_{V}(x)=\sum_{k=0}^{\infty} V_{k} x^{k}=\sum_{k=1}^{\infty} U_{k-1} x^{k} \\
=x \sum_{j=0}^{\infty} U_{j} x^{j}=x F_{U}(x) \cdot \checkmark
\end{gathered}
$$

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## Useful results we'll need for g.f.'s

Generalization of SR2:
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$\rightarrow(1)$ If $V=U+i$ then

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## Useful results we＇ll need for g．f．＇s

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Definitions
Generalization of SR2：
－（1）If $V=U+i$ then

$$
F_{V}(x)=x^{i} F_{U}(x) .
$$

－（2）If $V=U-i$ then
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## Useful results we＇ll need for g．f．＇s

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Generalization of SR2：
－（1）If $V=U+i$ then

$$
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－（2）If $V=U$－$i$ then

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$$

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$$
F_{V}(x)=x^{i} F_{U}(x) .
$$

－（2）If $V=U-i$ then

$$
\begin{aligned}
& F_{V}(x)=x^{-i} F_{U}(x) \\
& \quad=x^{-i} \sum_{k=0}^{\infty} U_{k} x^{k}
\end{aligned}
$$

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## Connecting generating functions

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－Goal：figure out forms of the component generating
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Therefore：$\quad F_{\pi}(x)=$
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－Extra factor of $x$ accounts for random node itself．

## Connecting generating functions

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$F_{\pi}(x)=$
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## Connecting generating functions

## Basics

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Therefore：


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## Connecting generating functions

－Goal：figure out forms of the component generating functions，$F_{\pi}$ and $F_{\rho}$ ．
－$\pi_{n}=$ probability that a random node belongs to a finite component of size $n$
$=\sum_{k=0}^{\infty} P_{k} \times \operatorname{Pr}\binom{$ sum of sizes of subcomponents }{ at end of $k$ random links $=n-1}$

Therefore：

$$
F_{\pi}(x)=\underbrace{x} \underbrace{F_{p}\left(F_{p}(x)\right)}
$$

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## Connecting generating functions

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$=\sum_{k=0}^{\infty} P_{k} \times \operatorname{Pr}\binom{$ sum of sizes of subcomponents }{ at end of $k$ random links $=n-1}$

Therefore：

$$
F_{\pi}(x)=\underbrace{x}_{\text {SRR2 }} \underbrace{F_{P}\left(F_{\rho}(x)\right)}_{\text {SRil }}
$$

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## Connecting generating functions

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－$\pi_{n}=$ probability that a random node belongs to a finite component of size $n$

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=\sum_{k=0}^{\infty} P_{k} \times \operatorname{Pr}\binom{\text { sum of sizes of subcomponents }}{\text { at end of } k \text { random links }=n-1}
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Therefore：

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F_{\pi}(x)=\underbrace{x}_{\text {shr2 }} \underbrace{F_{P}\left(F_{\rho}(x)\right)}_{\text {GRil }}
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－Extra factor of $x$ accounts for random node itself．

## Connecting generating functions

－$\rho_{n}=$ probability that a random link leads to a finite
subcomponent of size $n$ ．
Invoke one step of recursion：$\rho_{n}=$ probability that in
following a random edge，the outgoing edges of the
node reached lead to finite subcomponents of
combined size $n-1$ ，
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Therefore：$F_{\rho}(x)=$
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## Connecting generating functions

－$\rho_{n}=$ probability that a random link leads to a finite subcomponent of size $n$ ．
－Invoke one step of recursion：$\rho_{n}=$ probability that in following a random edge，the outgoing edges of the node reached lead to finite subcomponents of combined size $n-1$ ，

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## Therefore：


－Again，extra factor of $x$ accounts for random node itself．

## Connecting generating functions

－$\rho_{n}=$ probability that a random link leads to a finite subcomponent of size $n$ ．
－Invoke one step of recursion：$\rho_{n}=$ probability that in following a random edge，the outgoing edges of the node reached lead to finite subcomponents of combined size $n-1$ ，

$$
=\sum_{k=0}^{\infty} R_{k} \times \operatorname{Pr}\binom{\text { sum of sizes of subcomponents }}{\text { at end of } k \text { random links }=n-1}
$$

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Therefore：

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## Connecting generating functions

－$\rho_{n}=$ probability that a random link leads to a finite subcomponent of size $n$ ．
－Invoke one step of recursion：$\rho_{n}=$ probability that in following a random edge，the outgoing edges of the node reached lead to finite subcomponents of combined size $n-1$ ，

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Therefore：

$$
F_{\rho}(x)=\underbrace{x} \underbrace{F_{R}\left(F_{\rho}(x)\right)}
$$

## Again，extra factor of $x$ accounts for random node itself．

## Connecting generating functions

－$\rho_{n}=$ probability that a random link leads to a finite subcomponent of size $n$ ．
－Invoke one step of recursion：$\rho_{n}=$ probability that in following a random edge，the outgoing edges of the node reached lead to finite subcomponents of combined size $n-1$ ，

$$
=\sum_{k=0}^{\infty} R_{k} \times \operatorname{Pr}\binom{\text { sum of sizes of subcomponents }}{\text { at end of } k \text { random links }=n-1}
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Therefore：

$$
F_{\rho}(x)=\underbrace{x}_{\text {SR2 }} \underbrace{F_{R}\left(F_{\rho}(x)\right)}_{\text {SR1 }}
$$

Again，extra factor of $x$ accounts for random node itself．

## Connecting generating functions

－$\rho_{n}=$ probability that a random link leads to a finite subcomponent of size $n$ ．
－Invoke one step of recursion：$\rho_{n}=$ probability that in following a random edge，the outgoing edges of the node reached lead to finite subcomponents of combined size $n-1$ ，

$$
=\sum_{k=0}^{\infty} R_{k} \times \operatorname{Pr}\binom{\text { sum of sizes of subcomponents }}{\text { at end of } k \text { random links }=n-1}
$$

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Therefore：

$$
F_{\rho}(x)=\underbrace{x}_{\text {SR2 }} \underbrace{F_{R}\left(F_{\rho}(x)\right)}_{\text {SR1 }}
$$

－Again，extra factor of $x$ accounts for random node itself．

## Connecting generating functions

－We now have two functional equations connecting our generating functions：

$$
F_{\pi}(x)=x F_{P}\left(F_{\rho}(x)\right) \text { and } \quad F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)
$$

＞Taking stock：We know $F_{P}(x)$ and $F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)$ ．
－We first untangle the second equation to find $F_{\rho}$
－We can do this because it only involves $F_{\rho}$ and $F_{R}$ ．
－The first equation then immediately gives us $F_{\pi}$ in terms of $F_{\rho}$ and $F_{R}$ ．

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- The first equation then immediately gives us $F_{\pi}$ in terms of $F_{\rho}$ and $F_{R}$.


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－The first equation then immediately gives us $F_{\pi}$ in terms of $F_{\rho}$ and $F_{R}$ ．

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The first equation then immediately gives us $F_{\pi}$ in
terms of $F_{\rho}$ and $F_{R}$.

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－The first equation then immediately gives us $F_{\pi}$ in terms of $F_{\rho}$ and $F_{R}$ ．

## Component sizes

－Remembering vaguely what we are doing：
Finding $F_{\pi}$ to obtain the fractional size of the largest component $S_{1}=1-F_{\pi}(1)$ ．
－Set $x=1$ in our two equations：
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－Solve second equation numerically for $F_{\rho}(1)$ ．
－Plug $F_{\rho}(1)$ into first equation to obtain $F_{\pi}(1)$ ．

## Component sizes

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－Remembering vaguely what we are doing：
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Finding $F_{\pi}$ to obtain the fractional size of the largest component $S_{1}=1-F_{\pi}(1)$ ．
－Set $x=1$ in our two equations：

－Solve second equation numerically for $F_{\rho}(1)$ ．
－Plua $F_{\rho}(1)$ into first equation to obtain $F_{\pi}(1)$ ．

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## Component sizes

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－Remembering vaguely what we are doing：
Finding $F_{\pi}$ to obtain the fractional size of the largest component $S_{1}=1-F_{\pi}(1)$ ．
－Set $x=1$ in our two equations：

$$
F_{\pi}(1)=F_{P}\left(F_{\rho}(1)\right) \text { and } \quad F_{\rho}(1)=F_{R}\left(F_{\rho}(1)\right)
$$

－Solve second equation numerically for $F_{\rho}(1)$ ．
－Plug $F_{\rho}(1)$ into first equation to obtain $F_{\pi}(1)$ ．

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－Plug $F_{\rho}(1)$ into first equation to obtain $F_{\pi}(1)$ ．

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－Solve second equation numerically for $F_{\rho}(1)$ ．
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## Component sizes

Basics

Example: Standard random graphs.

- We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

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- RHS's of our two equations are the same.
- So $F_{\pi}(x)=F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)=x F_{R}\left(F_{\pi}(x)\right)$
- Why our dirty (but wrong) trick worked earlier...

Average Component Size

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## Component sizes

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Example：Standard random graphs．
－We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$
$\therefore F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)=e^{-\langle k)(1-x)} /\left.e^{-\langle k)\left(1-x^{\prime}\right)}\right|_{x^{\prime}=1}$
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－RHS＇s of our two equations are the same．
－So $F_{\pi}(x)=F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)=x F_{R}\left(F_{\pi}(x)\right)$
－Why our dirty（but wrong）trick worked earlier．．．

## Component sizes

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Definitions

## Example: Standard random graphs.

- We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

$$
\therefore F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)=e^{-\langle k\rangle(1-x)} /\left.e^{-\langle k\rangle\left(1-x^{\prime}\right)}\right|_{x^{\prime}=1}
$$



- RHS's of our two equations are the same.
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## Component sizes

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Example：Standard random graphs．
－We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

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\therefore F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)=e^{-\langle k\rangle(1-x)} /\left.e^{-\langle k\rangle\left(1-x^{\prime}\right)}\right|_{x^{\prime}=1}
$$

$$
=e^{-\langle k\rangle(1-x)}
$$

..aha!
－RHS＇s of our two equations are the same．
－So $F_{\pi}(x)=F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)=x F_{R}\left(F_{\pi}(x)\right)$
－Why our dirty（but wrong）trick worked earlier．．．

## Component sizes

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Definitions

## Example: Standard random graphs.

- We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

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\therefore F_{R}(x)=F_{P}^{\prime}(x) / F_{P}^{\prime}(1)=e^{-\langle k\rangle(1-x)} /\left.e^{-\langle k\rangle\left(1-x^{\prime}\right)}\right|_{x^{\prime}=1}
$$

$$
=e^{-\langle k\rangle(1-x)}=F_{P}(x) \quad \text {...aha! }
$$

- RHS's of our two equations are the same.
- So $F_{\pi}(x)=F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)=x F_{R}\left(F_{\pi}(x)\right)$
- Why our dirty (but wrong) trick worked earlier...


## Component sizes

Basics
Definitions
Example：Standard random graphs．
－We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

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\begin{aligned}
\therefore F_{R}(x)= & F_{P}^{\prime}(x) / F_{P}^{\prime}(1)=e^{-\langle k\rangle(1-x)} /\left.e^{-\langle k\rangle\left(1-x^{\prime}\right)}\right|_{x^{\prime}=1} \\
& =e^{-\langle k\rangle(1-x)}=F_{P}(x) \quad \ldots \text { aha! }
\end{aligned}
$$

－RHS＇s of our two equations are the same．
－So $F_{\pi}(x)=F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)=x F_{R}\left(F_{\pi}(x)\right)$
－Why our dirty（but wrong）trick worked earlier．．．

## Component sizes

Basics
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Example：Standard random graphs．
－We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

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\begin{aligned}
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－RHS＇s of our two equations are the same．
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Why our dirty（but wrong）trick worked earlier．．．

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## Component sizes

## Basics

Definitions
Example: Standard random graphs.

- We can show $F_{P}(x)=e^{-\langle k\rangle(1-x)}$

$$
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\therefore F_{R}(x)= & F_{P}^{\prime}(x) / F_{P}^{\prime}(1)=e^{-\langle k\rangle(1-x)} /\left.e^{-\langle k\rangle\left(1-x^{\prime}\right)}\right|_{x^{\prime}=1} \\
& =e^{-\langle k\rangle(1-x)}=F_{P}(x) \quad \ldots \text { aha! }
\end{aligned}
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- RHS's of our two equations are the same.
- So $F_{\pi}(x)=F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)=x F_{R}\left(F_{\pi}(x)\right)$
- Why our dirty (but wrong) trick worked earlier...


## Component sizes

－We are down to

$$
F_{\pi}(x)=x F_{R}\left(F_{\pi}(x)\right) \text { and } F_{R}(x)=e^{-\langle k\rangle(1-x)}
$$

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－We＇re first after $S_{1}=1-F_{\pi}(1)$ so set $x=1$ and replace $F_{\pi}(1)$ by $1-S_{1}$ ：

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－Just as we found with our dirty trick
－Again，we（usually）have to resort to numerics
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## Component sizes

－We are down to

$$
F_{\pi}(x)=x F_{R}\left(F_{\pi}(x)\right) \text { and } F_{R}(x)=e^{-\langle k\rangle(1-x)}
$$

$$
\therefore F_{\pi}(x)=x e^{-\langle k\rangle\left(1-F_{\pi}(x)\right)}
$$

－We＇re first after $S_{1}=1-F_{\pi}(1)$ so set $x=1$ and replace $F_{\pi}(1)$ by $1-S_{1}$ ：

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## Component sizes

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F_{\pi}(x)=x F_{R}\left(F_{\pi}(x)\right) \text { and } F_{R}(x)=e^{-\langle k\rangle(1-x)}
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$$
\therefore F_{\pi}(x)=x e^{-\langle k\rangle\left(1-F_{\pi}(x)\right)}
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## Component sizes

- We are down to

$$
F_{\pi}(x)=x F_{R}\left(F_{\pi}(x)\right) \text { and } F_{R}(x)=e^{-\langle k\rangle(1-x)}
$$

$$
\therefore F_{\pi}(x)=x e^{-\langle k\rangle\left(1-F_{\pi}(x)\right)}
$$

- We're first after $S_{1}=1-F_{\pi}(1)$ so set $x=1$ and replace $F_{\pi}(1)$ by $1-S_{1}$ :

$$
1-S_{1}=e^{-\langle k\rangle S_{1}}
$$

Or: $\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}}$


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## Component sizes

- We are down to

$$
F_{\pi}(x)=x F_{R}\left(F_{\pi}(x)\right) \text { and } F_{R}(x)=e^{-\langle k\rangle(1-x)}
$$

$$
\therefore F_{\pi}(x)=x e^{-\langle k\rangle\left(1-F_{\pi}(x)\right)}
$$

- We're first after $S_{1}=1-F_{\pi}(1)$ so set $x=1$ and replace $F_{\pi}(1)$ by $1-S_{1}$ :

$$
1-S_{1}=e^{-\langle k\rangle S_{1}}
$$

Or: $\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}}$


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## Component sizes

－We are down to

$$
F_{\pi}(x)=x F_{R}\left(F_{\pi}(x)\right) \text { and } F_{R}(x)=e^{-\langle k\rangle(1-x)}
$$

$$
\therefore F_{\pi}(x)=x e^{-\langle k\rangle\left(1-F_{\pi}(x)\right)}
$$

－We＇re first after $S_{1}=1-F_{\pi}(1)$ so set $x=1$ and replace $F_{\pi}(1)$ by $1-S_{1}$ ：

$$
1-S_{1}=e^{-\langle k\rangle S_{1}}
$$

Or：$\langle k\rangle=\frac{1}{S_{1}} \ln \frac{1}{1-S_{1}}$


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## Average Component Size

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## Average component size

－Next：find average size of finite components $\langle n\rangle$ ．
－Using standard G．F．result：$\langle n\rangle=F_{\pi}^{\prime}(1)$ ．
－Try to avoid finding $F_{\pi}(x) \ldots$
－Starting from $F_{\pi}(x)=x F_{P}\left(F_{\rho}(x)\right)$ ，we differentiate：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{P}^{\prime}\left(F_{\rho}(x)\right)
$$

－While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
F_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{p}^{\prime}(x) F_{R}^{\prime}\left(F_{p}(x)\right)
$$

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－Now set $x=1$ in both equations．
－We solve the second equation for $F_{\rho}^{\prime}(1)$（we must already have $\left.F_{\rho}(1)\right)$ ．
－Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

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## Average component size

- Next: find average size of finite components $\langle n\rangle$.
- Using standard G.F. result: $\langle n\rangle=F_{\pi}^{\prime}(1)$.
- Try to avoid finding $F_{\pi}(x)$...
- Starting from $F_{\pi}(x)=x F_{P}\left(F_{\rho}(x)\right)$, we differentiaate:

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{p}(x)\right)+x F_{p}^{\prime}(x) F_{p}^{\prime}\left(F_{p}(x)\right)
$$

- While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
F_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{R}^{\prime}\left(F_{\rho}(x)\right)
$$

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- Now set $x=1$ in both equations.
- We solve the second equation for $F_{\rho}^{\prime}(1)$ (we must already have $F_{\rho}(1)$ ).
- Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$.


## Average component size

－Next：find average size of finite components $\langle n\rangle$ ．
－Using standard G．F．result：$\langle n\rangle=F_{\pi}^{\prime}(1)$ ．
－Try to avoid finding $F_{\pi}(x) \ldots$
－Starting from $F_{\pi}(x)=x F_{P}\left(F_{\rho}(x)\right)$ ，we differentiate：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{P}^{\prime}\left(F_{\rho}(x)\right)
$$

－While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
\Gamma_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{R}^{\prime}\left(F_{\rho}(x)\right)
$$

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－We solve the second equation for $F_{p}^{\prime}(1)$（we must already have $\left.F_{\rho}(1)\right)$ ．
－Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

References

## Average component size

－Next：find average size of finite components $\langle n\rangle$ ．
－Using standard G．F．result：$\langle n\rangle=F_{\pi}^{\prime}(1)$ ．
－Try to avoid finding $F_{\pi}(x) \ldots$
－Starting from $F_{\pi}(x)=x F_{P}\left(F_{\rho}(x)\right)$ ，we differentiate：

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F_{\pi}^{\prime}(x)=F_{P}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{P}^{\prime}\left(F_{\rho}(x)\right)
$$

－While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
F_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{R}^{\prime}\left(F_{\rho}(x)\right)
$$

－Now set $x=1$ in both equations．

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－We solve the second equation for $F_{p}^{\prime}(1)$（we must already have $\left.F_{\rho}(1)\right)$ ．
－Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

## Average component size

－Next：find average size of finite components $\langle n\rangle$ ．
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F_{\pi}^{\prime}(x)=F_{P}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{P}^{\prime}\left(F_{\rho}(x)\right)
$$

－While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
F_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{R}^{\prime}\left(F_{\rho}(x)\right)
$$

$\Rightarrow$ Now set $x=1$ in both equations．

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－We solve the second equation for $F_{\rho}^{\prime}(1)$（we must already have $F_{\rho}(1)$ ）．
$\rightarrow$ Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

## Average component size

－Next：find average size of finite components $\langle n\rangle$ ．
－Using standard G．F．result：$\langle n\rangle=F_{\pi}^{\prime}(1)$ ．
－Try to avoid finding $F_{\pi}(x)$ ．．．
－Starting from $F_{\pi}(x)=x F_{P}\left(F_{\rho}(x)\right)$ ，we differentiate：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{P}^{\prime}\left(F_{\rho}(x)\right)
$$

－While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
F_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{R}^{\prime}\left(F_{\rho}(x)\right)
$$

－Now set $x=1$ in both equations．
We solve the second equation for $F_{\rho}^{\prime}(1)$（we must already have $F_{\rho}(1)$ ）．
－Plug $F^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

## Average component size

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－Using standard G．F．result：$\langle n\rangle=F_{\pi}^{\prime}(1)$ ．
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$$

－While $F_{\rho}(x)=x F_{R}\left(F_{\rho}(x)\right)$ gives

$$
F_{\rho}^{\prime}(x)=F_{R}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{R}^{\prime}\left(F_{\rho}(x)\right)
$$

－Now set $x=1$ in both equations．
－We solve the second equation for $F_{\rho}^{\prime}(1)$（we must already have $\left.F_{\rho}(1)\right)$ ．
－Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

## Average component size

－Next：find average size of finite components $\langle n\rangle$ ．
－Using standard G．F．result：$\langle n\rangle=F_{\pi}^{\prime}(1)$ ．
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F_{\pi}^{\prime}(x)=F_{P}\left(F_{\rho}(x)\right)+x F_{\rho}^{\prime}(x) F_{P}^{\prime}\left(F_{\rho}(x)\right)
$$

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$$
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$$

－Now set $x=1$ in both equations．
－We solve the second equation for $F_{\rho}^{\prime}(1)$（we must already have $F_{\rho}(1)$ ）．
－Plug $F_{\rho}^{\prime}(1)$ and $F_{\rho}(1)$ into first equation to find $F_{\pi}^{\prime}(1)$ ．

## Average component size

Example：Standard random graphs．
－Use fact that $F_{P}=F_{R}$ and $F_{\pi}=F_{\rho}$ ．
－Two differentiated equations reduce to only one：

$$
F_{\pi}^{\prime}(x)=F_{p}\left(F_{\pi}(x)\right)+x F_{\pi}^{\prime}(x) F_{p}^{\prime}\left(F_{\pi}(x)\right)
$$

－Simplify denominator using $F_{P}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．
－Set $x=1$ and replace $F_{\pi}(1)$ with $1-S_{1}$ ．

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## Average component size

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－Set $x=1$ and replace $F_{\pi}(1)$ with $1-S_{1}$ ．

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$$


－Simplify denominator using $F_{P}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．

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## Average component size

Example：Standard random graphs．
－Use fact that $F_{P}=F_{R}$ and $F_{\pi}=F_{\rho}$ ．
－Two differentiated equations reduce to only one：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\pi}(x)\right)+x F_{\pi}^{\prime}(x) F_{P}^{\prime}\left(F_{\pi}(x)\right)
$$

Rearrange：$\quad F_{\pi}^{\prime}(x)=\frac{F_{P}\left(F_{\pi}(x)\right)}{1-x F_{P}^{\prime}\left(F_{\pi}(x)\right)}$
－Simplify denominator using $F_{p}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．

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## Average component size

Example：Standard random graphs．
－Use fact that $F_{P}=F_{R}$ and $F_{\pi}=F_{\rho}$ ．
－Two differentiated equations reduce to only one：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\pi}(x)\right)+x F_{\pi}^{\prime}(x) F_{P}^{\prime}\left(F_{\pi}(x)\right)
$$

$$
\text { Rearrange: } \quad F_{\pi}^{\prime}(x)=\frac{F_{P}\left(F_{\pi}(x)\right)}{1-x F_{P}^{\prime}\left(F_{\pi}(x)\right)}
$$

－Simplify denominator using $F_{P}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．

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－Set $x=1$ and replace $F_{\pi}(1)$ with $1-S_{1}$ ．

## Average component size

Example：Standard random graphs．
－Use fact that $F_{P}=F_{R}$ and $F_{\pi}=F_{\rho}$ ．
－Two differentiated equations reduce to only one：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\pi}(x)\right)+x F_{\pi}^{\prime}(x) F_{P}^{\prime}\left(F_{\pi}(x)\right)
$$

$$
\text { Rearrange: } \quad F_{\pi}^{\prime}(x)=\frac{F_{P}\left(F_{\pi}(x)\right)}{1-x F_{P}^{\prime}\left(F_{\pi}(x)\right)}
$$

－Simplify denominator using $F_{P}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．

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## Average component size

Example：Standard random graphs．
－Use fact that $F_{P}=F_{R}$ and $F_{\pi}=F_{\rho}$ ．
－Two differentiated equations reduce to only one：

$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\pi}(x)\right)+x F_{\pi}^{\prime}(x) F_{P}^{\prime}\left(F_{\pi}(x)\right)
$$

$$
\text { Rearrange: } \quad F_{\pi}^{\prime}(x)=\frac{F_{P}\left(F_{\pi}(x)\right)}{1-x F_{P}^{\prime}\left(F_{\pi}(x)\right)}
$$

－Simplify denominator using $F_{P}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．

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－Set $x=1$ and replace $F_{\pi}(1)$ with $1-S_{1}$ ．


## Average component size

Example：Standard random graphs．
－Use fact that $F_{P}=F_{R}$ and $F_{\pi}=F_{\rho}$ ．
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$$
F_{\pi}^{\prime}(x)=F_{P}\left(F_{\pi}(x)\right)+x F_{\pi}^{\prime}(x) F_{P}^{\prime}\left(F_{\pi}(x)\right)
$$

$$
\text { Rearrange: } \quad F_{\pi}^{\prime}(x)=\frac{F_{P}\left(F_{\pi}(x)\right)}{1-x F_{P}^{\prime}\left(F_{\pi}(x)\right)}
$$

－Simplify denominator using $F_{P}^{\prime}(x)=\langle k\rangle F_{P}(x)$
－Replace $F_{P}\left(F_{\pi}(x)\right)$ using $F_{\pi}(x)=x F_{P}\left(F_{\pi}(x)\right)$ ．

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－Set $x=1$ and replace $F_{\pi}(1)$ with $1-S_{1}$ ．
End result：$\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}$

## Average component size

－Our result for standard random networks：

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

－Recall that $\langle k\rangle=1$ is the critical value of average degree for standard random networks．
－Look at what happens when we increase $\langle k\rangle$ to 1 from below．
－We have $S_{1}=0$ for all $\langle k\rangle<1$
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This blows up as $\langle k\rangle \rightarrow 1$ ．
－Reason：we have a power law distribution of component sizes at $\langle k\rangle=1$ ．
－Typical critical point behavior．．．．
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## Average component size

－Our result for standard random networks：

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
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Typical critical point behavior．．．．
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## Average component size

- Our result for standard random networks:

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

- Recall that $\langle k\rangle=1$ is the critical value of average degree for standard random networks.
- Look at what happens when we increase $\langle k\rangle$ to 1 from below.
- We have $S_{1}=0$ for all $\langle k\rangle<1$

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- This blows up as $\langle k\rangle \rightarrow 1$
- Reason: we have a power law distribution of component sizes at $\langle k\rangle=1$.
- Typical critical point behavior....


## Average component size

－Our result for standard random networks：

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

－Recall that $\langle k\rangle=1$ is the critical value of average degree for standard random networks．
－Look at what happens when we increase $\langle k\rangle$ to 1 from below．
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－This blows up as $\langle k\rangle \rightarrow 1$
－Reason：we have a power law distribution of component sizes at $\langle k\rangle=1$ ．
－Typical critical point behavior．．．．
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## Average component size

- Our result for standard random networks:

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

- Recall that $\langle k\rangle=1$ is the critical value of average degree for standard random networks.
- Look at what happens when we increase $\langle k\rangle$ to 1 from below.
- We have $S_{1}=0$ for all $\langle k\rangle<1$ so

$$
\langle n\rangle=\frac{1}{1-\langle k\rangle}
$$

- This blows up as $\langle k\rangle \rightarrow 1$.
- Reason: we have a power law distribution of component sizes at $\langle k\rangle=1$.
- Tynical critical noint behavior....


## Average component size

- Our result for standard random networks:

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

- Recall that $\langle k\rangle=1$ is the critical value of average degree for standard random networks.
- Look at what happens when we increase $\langle k\rangle$ to 1 from below.
- We have $S_{1}=0$ for all $\langle k\rangle<1$ so

$$
\langle n\rangle=\frac{1}{1-\langle k\rangle}
$$

- This blows up as $\langle k\rangle \rightarrow 1$.
- Reason: we have a power law distribution of component sizes at $\langle k\rangle=1$.


## Average component size

－Our result for standard random networks：

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

－Recall that $\langle k\rangle=1$ is the critical value of average degree for standard random networks．
－Look at what happens when we increase $\langle k\rangle$ to 1 from below．
－We have $S_{1}=0$ for all $\langle k\rangle<1$ so

$$
\langle n\rangle=\frac{1}{1-\langle k\rangle}
$$

－This blows up as $\langle k\rangle \rightarrow 1$ ．
－Reason：we have a power law distribution of component sizes at $\langle k\rangle=1$ ．
－Typical critical point behavior．．．．

Frame 87／89
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## Average component size

－Limits of $\langle k\rangle=0$ and $\infty$ make sense for

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

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## Average component size

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$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
$$

－As $\langle k\rangle \rightarrow 0, S_{1}=0$ ，and $\langle n\rangle \rightarrow 1$ ．
－All nodes are isolated．
－As $\langle k\rangle \rightarrow \infty, S_{1} \rightarrow 1$ and $\langle n\rangle \rightarrow 0$ ．
－No nodes are outside of the giant component．

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－For $\langle k\rangle=1, S_{1} \sim N^{2 / 3}$ ．
$\Rightarrow$ For $\langle k\rangle<1, S_{1} \sim \log N$ ．

## Average component size

－Limits of $\langle k\rangle=0$ and $\infty$ make sense for

$$
\langle n\rangle=F_{\pi}^{\prime}(1)=\frac{\left(1-S_{1}\right)}{1-\langle k\rangle\left(1-S_{1}\right)}
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－For $\langle k\rangle=1, S_{1} \sim N^{2 / 3}$ ．
－For $\langle k\rangle<1, S_{1} \sim \log N$ ．

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