Complex Networks, CSYS/MATH 303—Assignment 4
University of Vermont, Spring 2010

Dispersed: Monday, March 1, 2010.
Due: By start of lecture, 10:00 am, Thursday, March 18, 2010.
Some useful reminders:
Instructor: Peter Dodds
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Office hours: 1:00 pm to 2:30 pm, Wednesday @ Farrell, and by appointment
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2010-01UVM-303/`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related variant).

Size-density laws:

1. For a uniformly distributed population how will facilities be distributed so as to minimize the average distance from individuals to their nearest facility?

2. In two dimensions, the size-density law for distributed source density $D(\vec{x})$ given a sink density $\rho(\vec{x})$ states that $D \propto \rho^{2/3}$. We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the 2/3 exponent ([1]; also see Supply Networks lecture notes).

   Repeat this argument for the d-dimensional case and find the general form of the exponent β in $D \propto \rho^{\beta}$.

3. Following Um et al.'s approach [2], obtain a more general scaling for mixed public-private facilities in two dimensions. Use the objective function:

   $$v_i = n_i \simeq n_i A_i^{\theta/2} \text{ with } 0 \leq \theta \leq 1,$$

   where, respectively, n_i and A_i are the population and the area of the ith Voronoi cell (which surrounds the ith facility).

   Note that $\theta = 0$ corresponds to purely commercial facilities, and $\theta = 1$ to strongly social ones. Also, for general dimension d, the $A_i^{\theta/2}$ is replaced by $V_i^{\theta/d}$.

References

[1] M. T. Gastner and M. E. J. Newman. Optimal design of spatial distribution networks. *Phys. Rev. E*, 74:016117, 2006.

[2] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim. Scaling laws between population and facility densities. *Proc. Natl. Acad. Sci.*, 106:14236–14240, 2009.