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Topological Modeling of Nonlinear Systems through Active Interrogation

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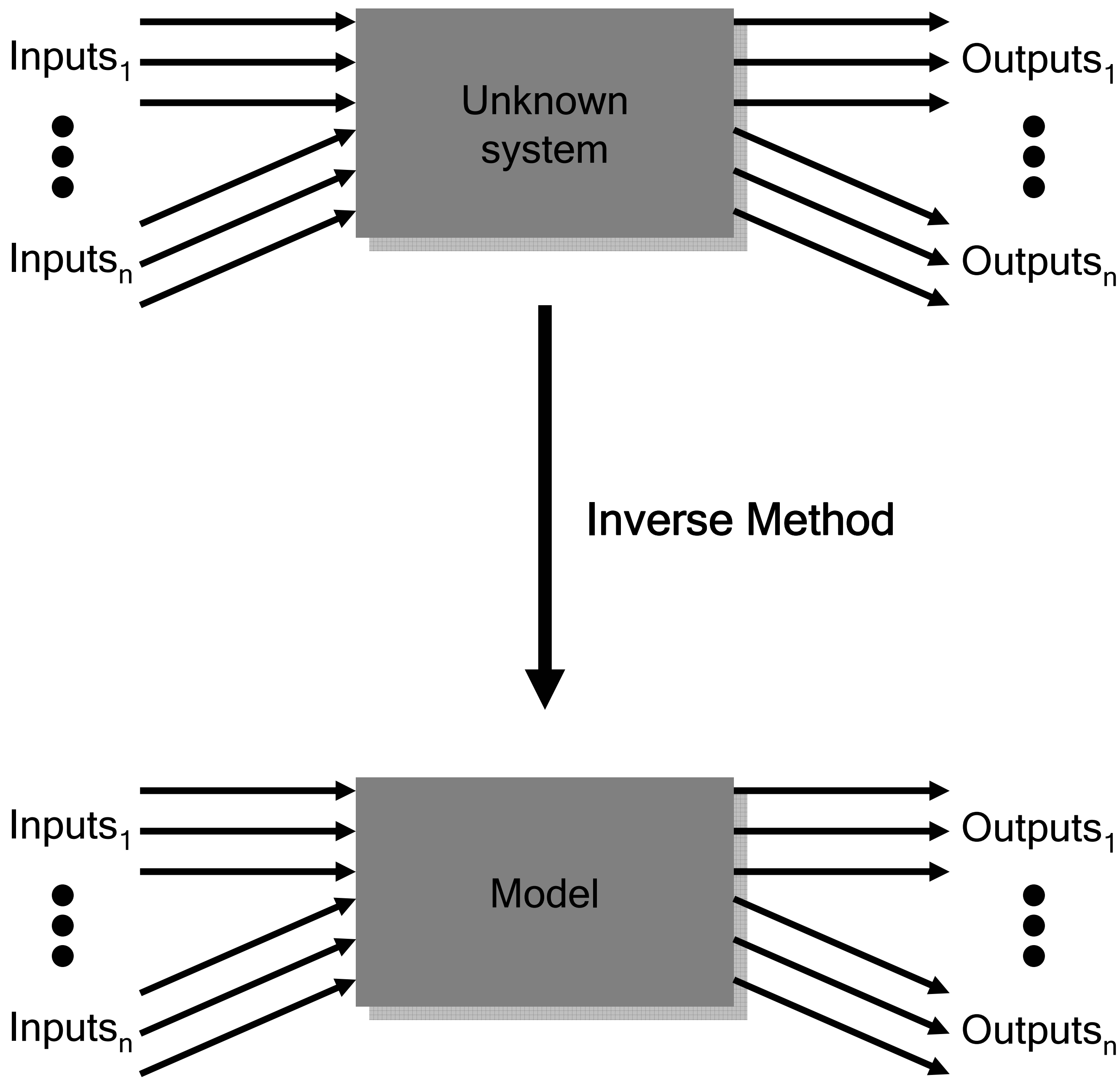
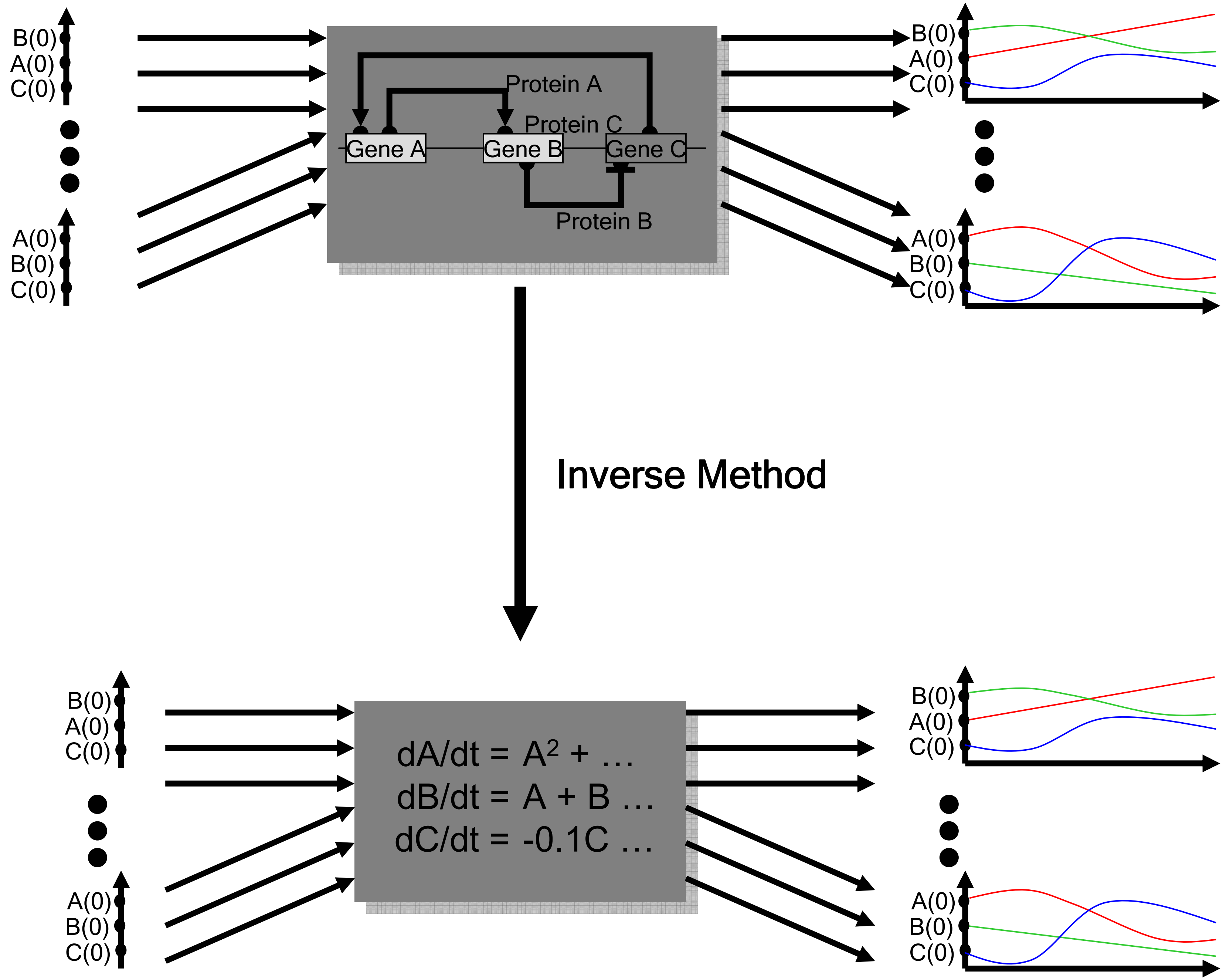
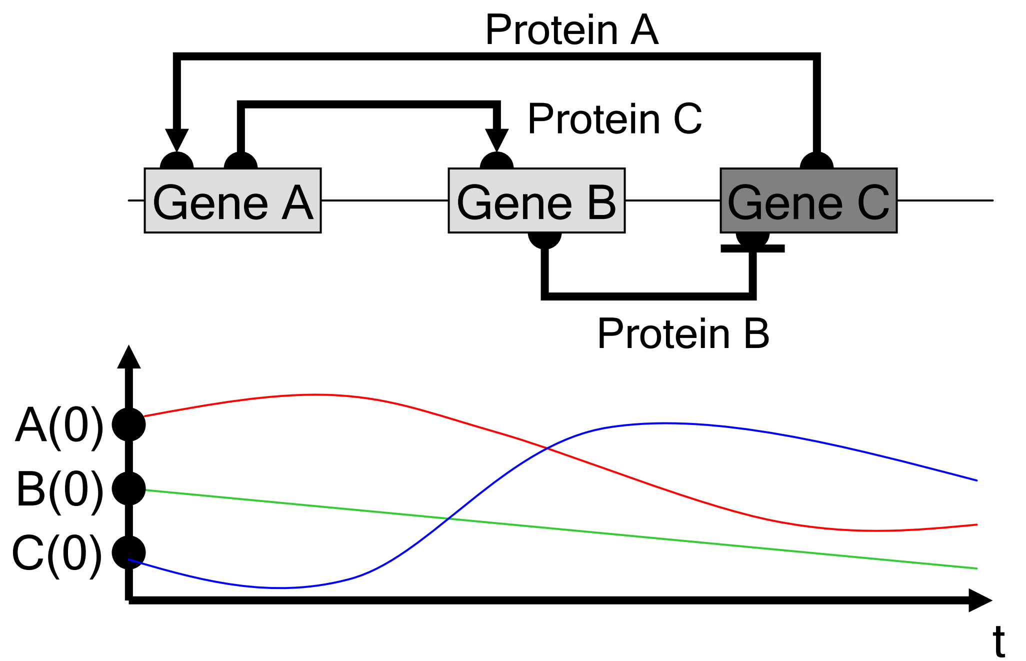

Inputs1
Inputsn
Unknown system
Outputs1
Outputsn
Inverse Method
Inputs1
Inputsn
Model
Outputs1
Outputsn

Inverse Methods



Target System



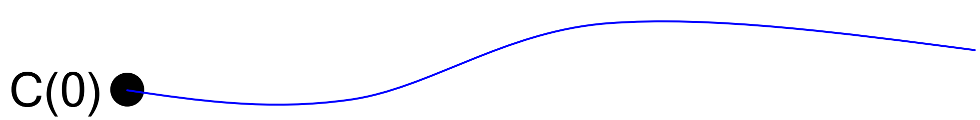
Sample Model

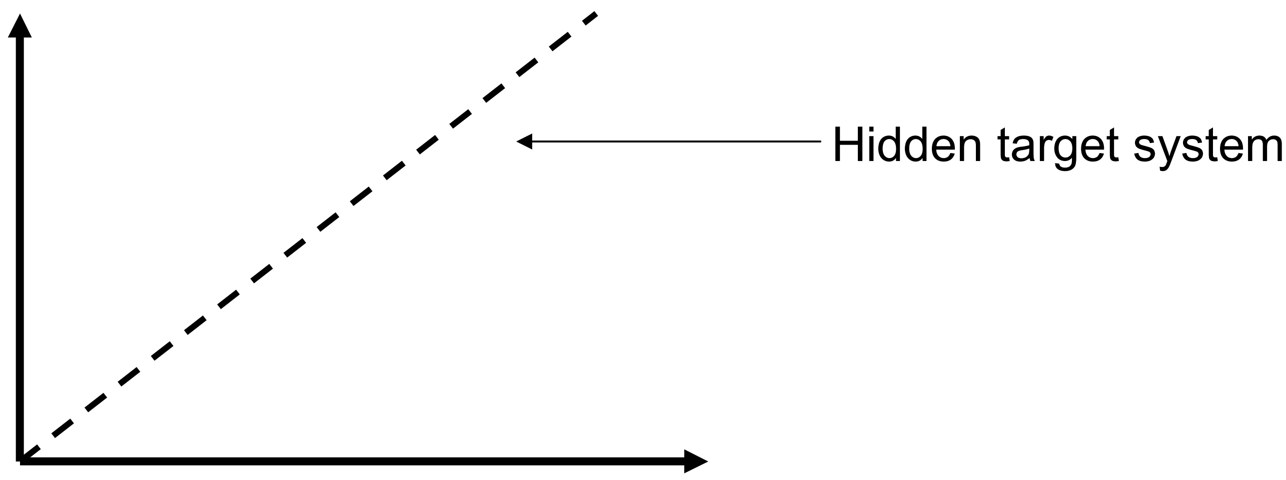
$$dA/dt = A^2 + B - C$$
$$dB/dt = -0.1B$$
$$dC/dt = C^3 + A/(A+1)$$

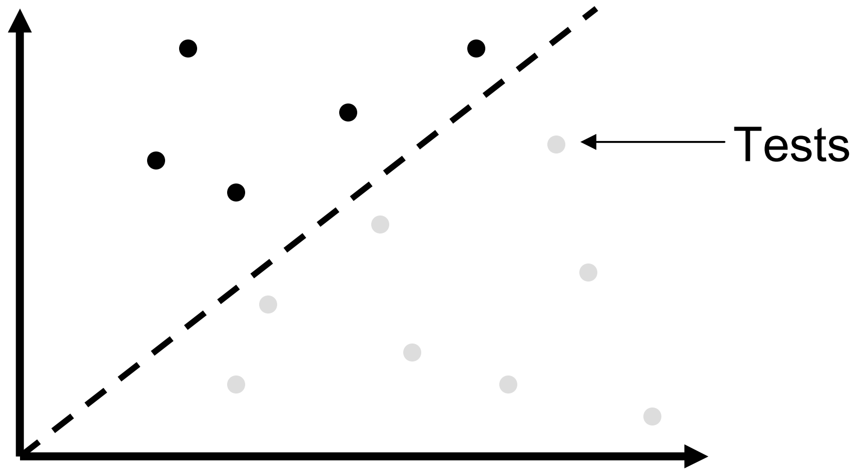
Sample Test

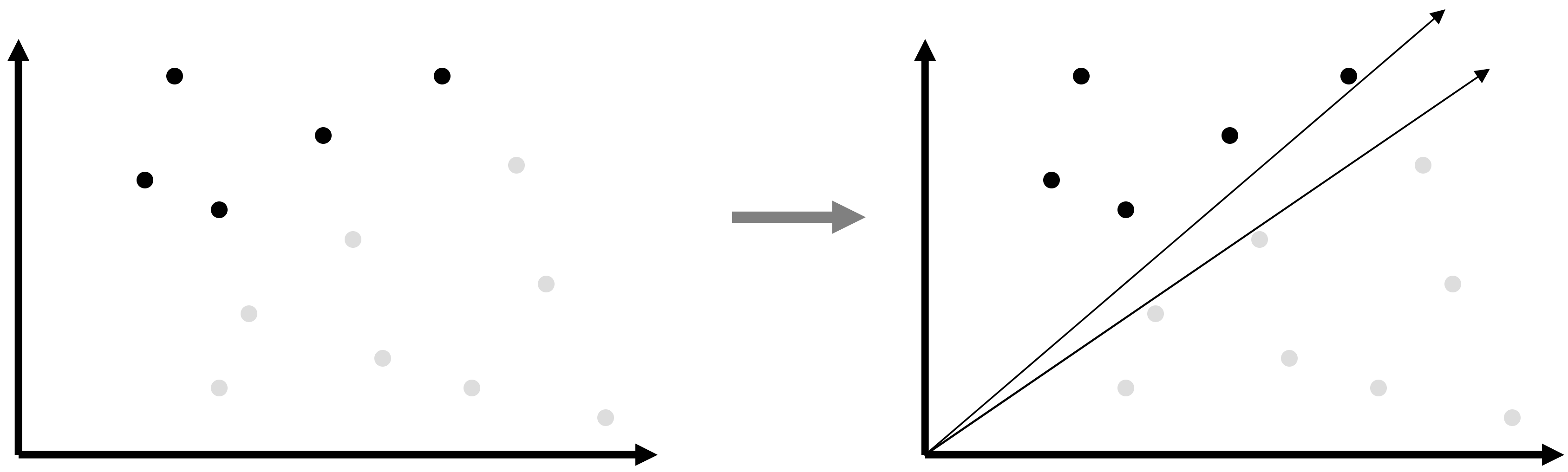
A(0) = 0.0
B(0) = 0.5
C(0) = 1.0

Sample Output

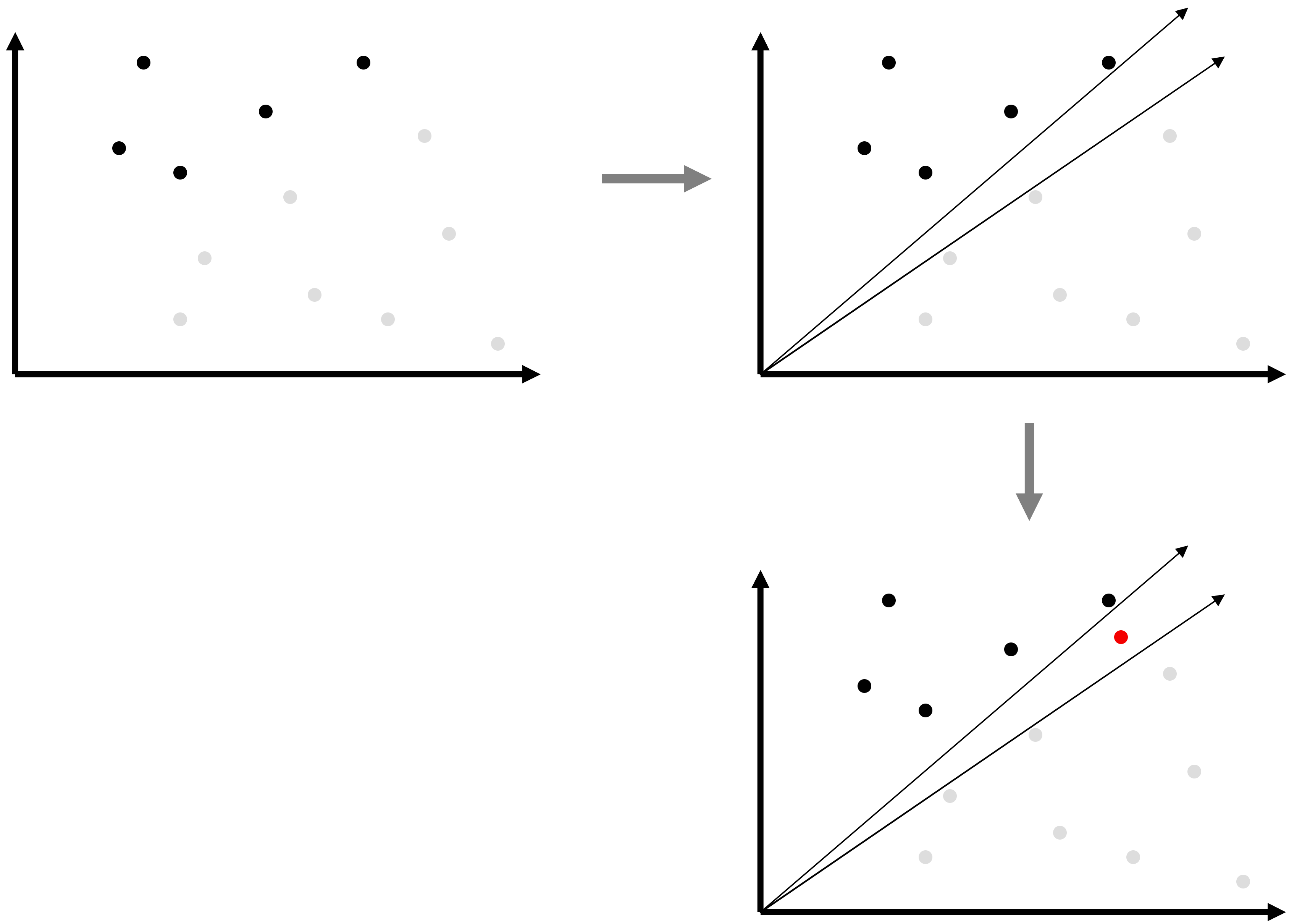



Hidden target system

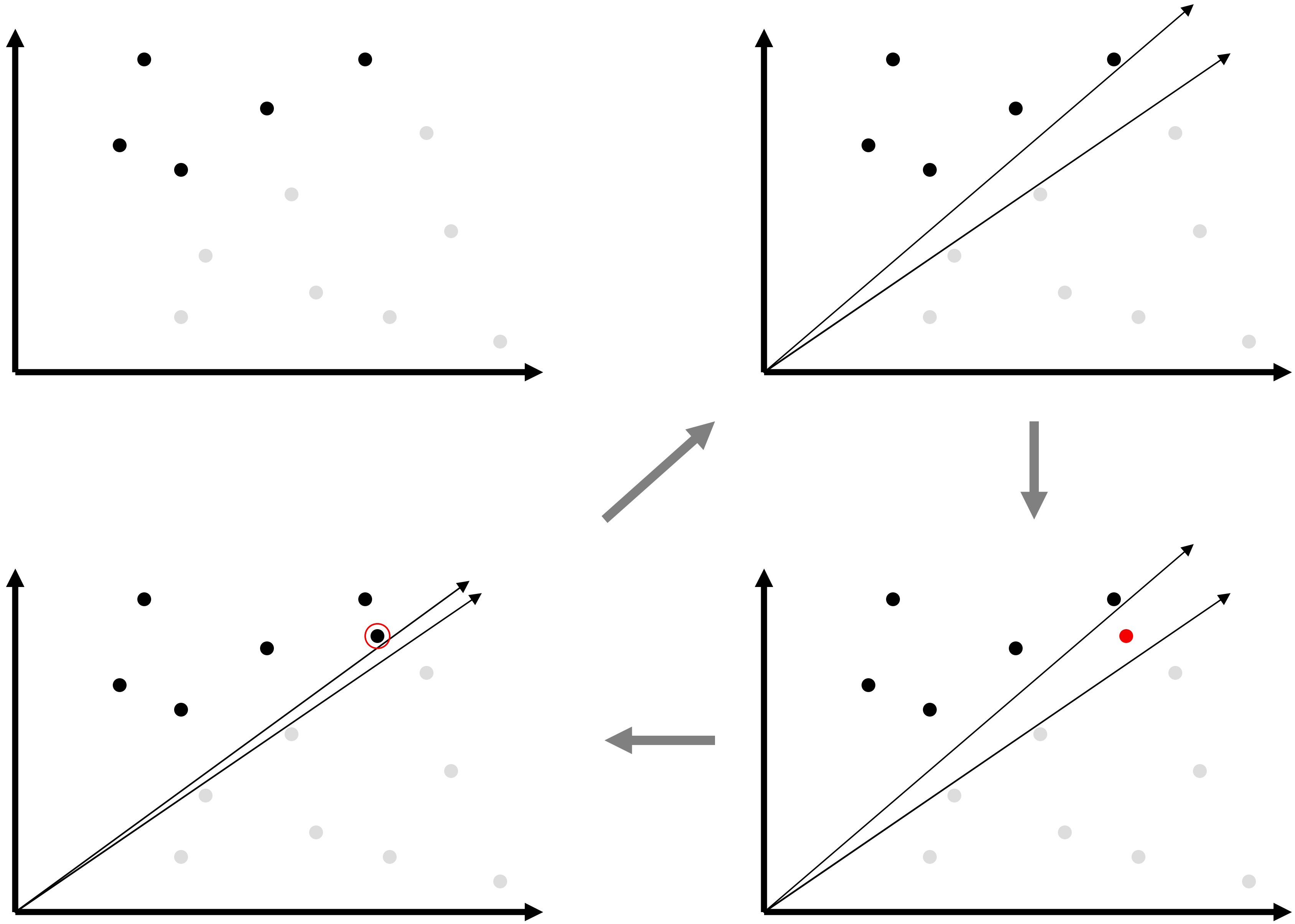

Tests



Query By Committee

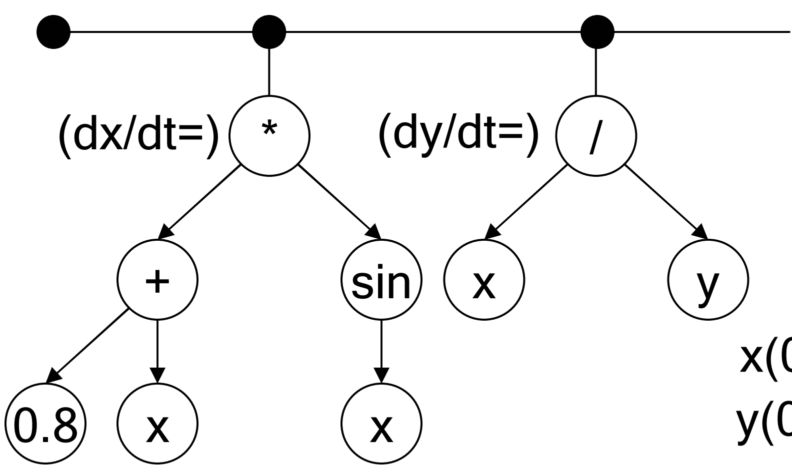


Query By Committee

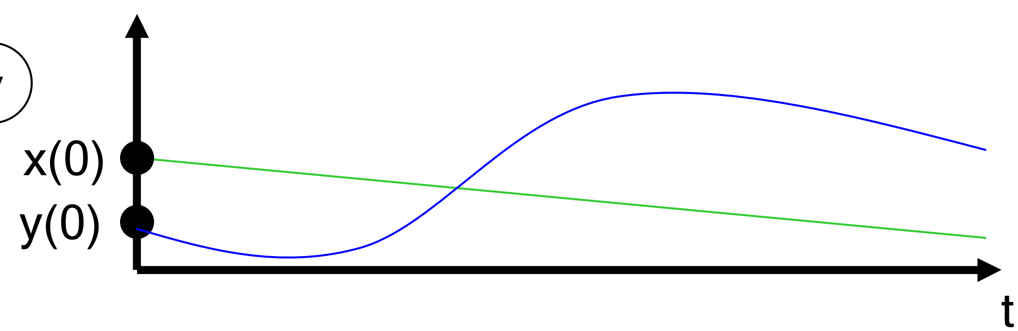


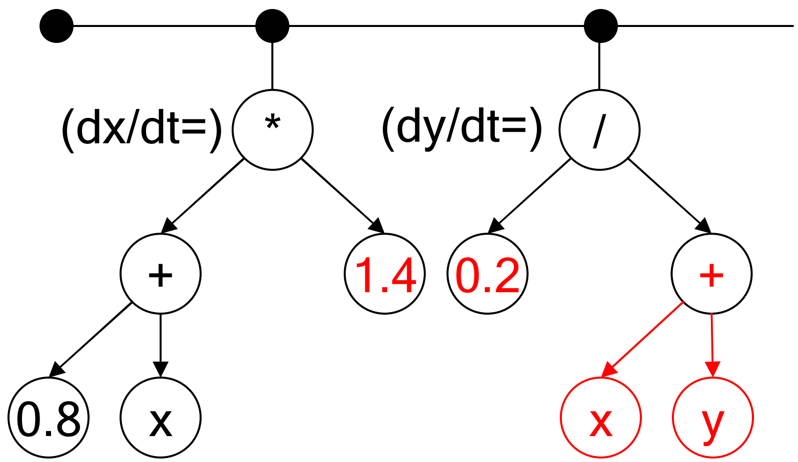
Encoding Models

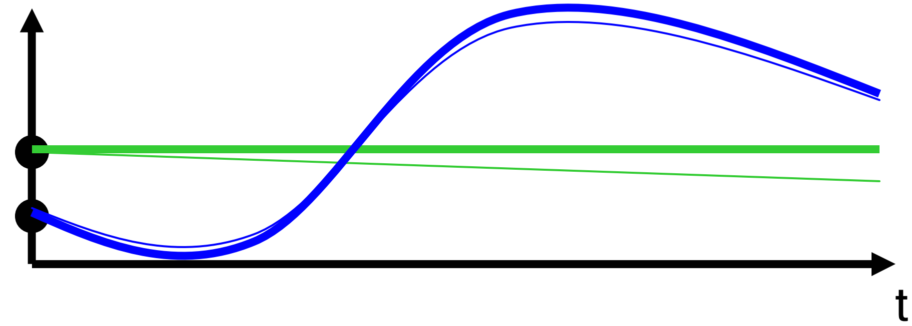
Possible branch nodes:	sin(a) cos(a) plus(a,b) minus(a,b) mult(a,b) div(a,b) pow(a,b) Hill(a,b,c)
Possible terminal nodes	x,y,… 0.1,-3.0,…



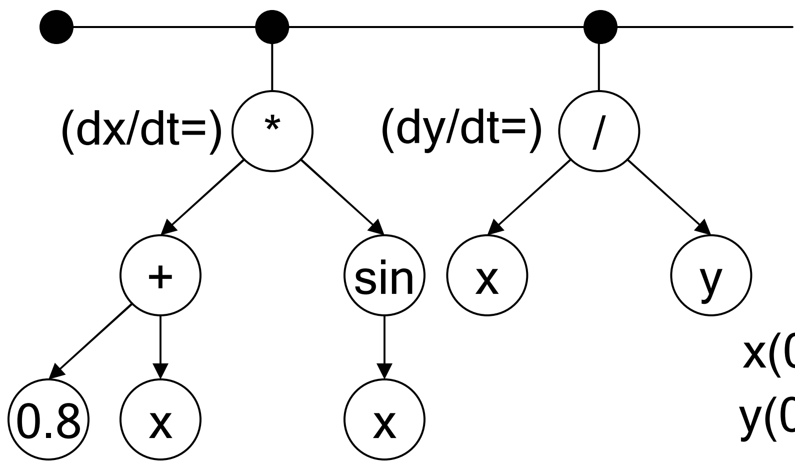
dx/dt = (0.8 + x) * sin(x)
dy/dt = x / y



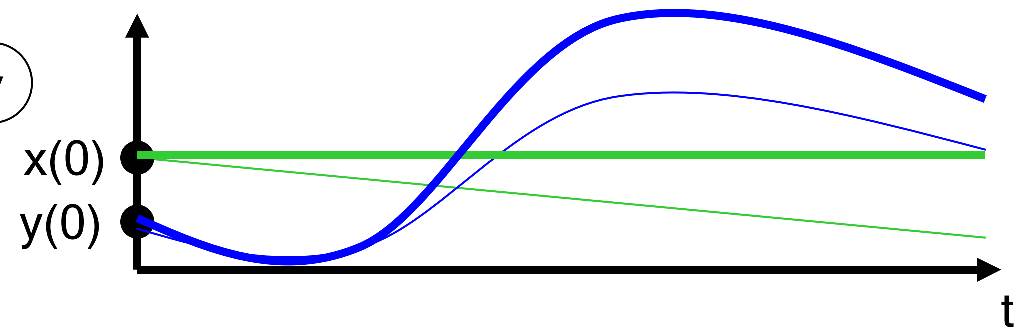




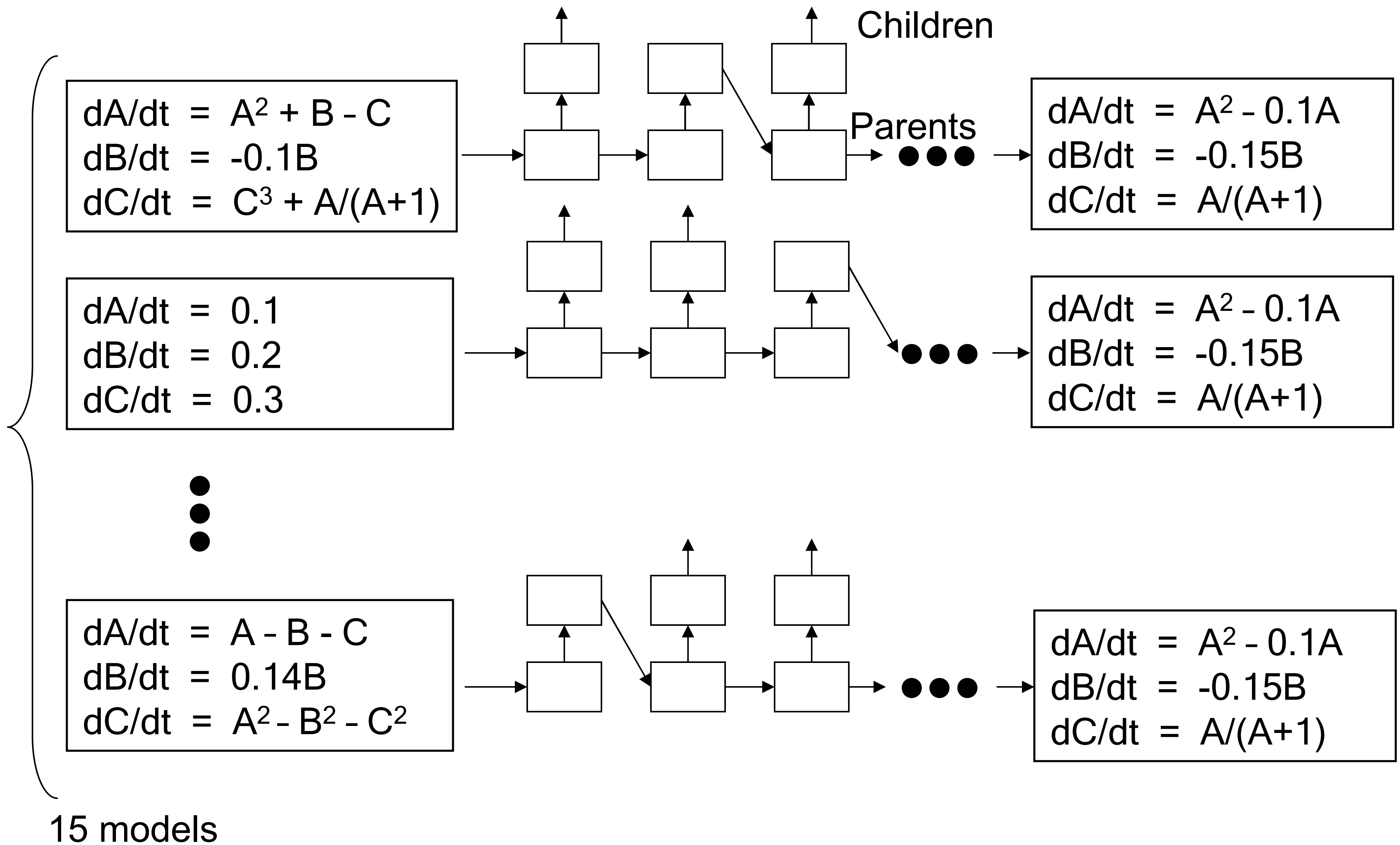
dx/dt = 1.4(0.8+x)
dy/dt = 0.2 / (x+y)

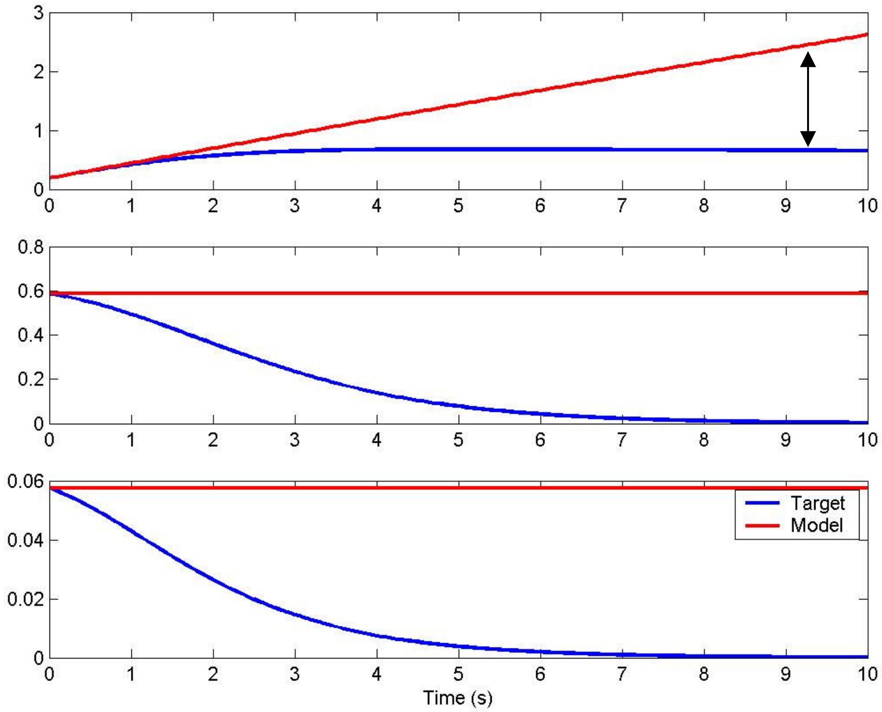


dx/dt = (0.8 + x) * sin(x)
dy/dt = x / y



Optimizing Models: Hill Climber





Target / Model1

dG/dt = $0.001 + A^2/(A^2+1^2) - 0.01G$

$0.001 + A^2/(A^2+1^2) - 0.01G$

dA/dt = $G(L/(L+1) - A(A+1))$

0

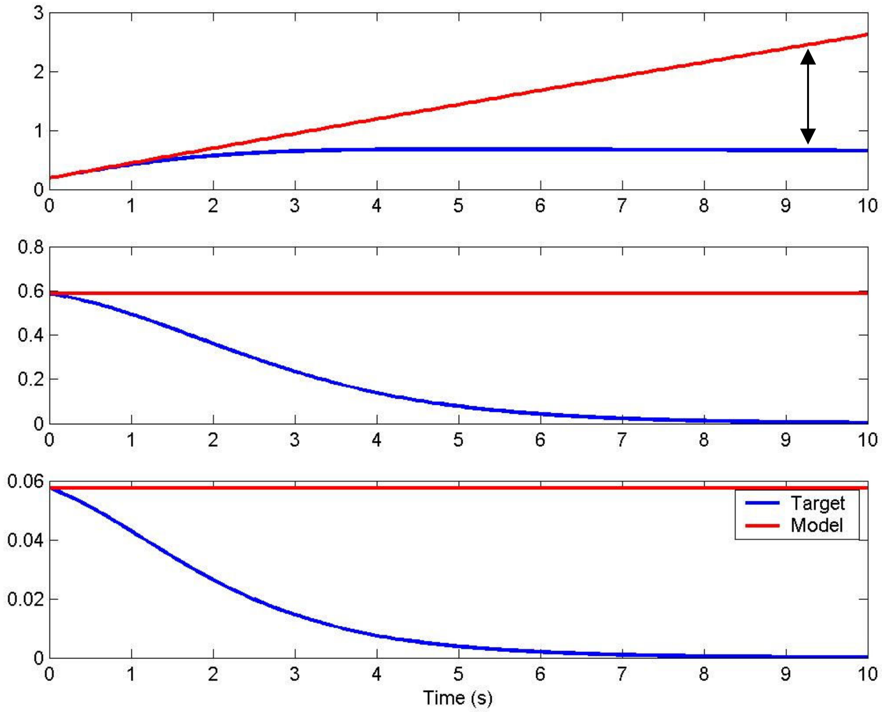
dL/dt = $-GL/(L+1)$

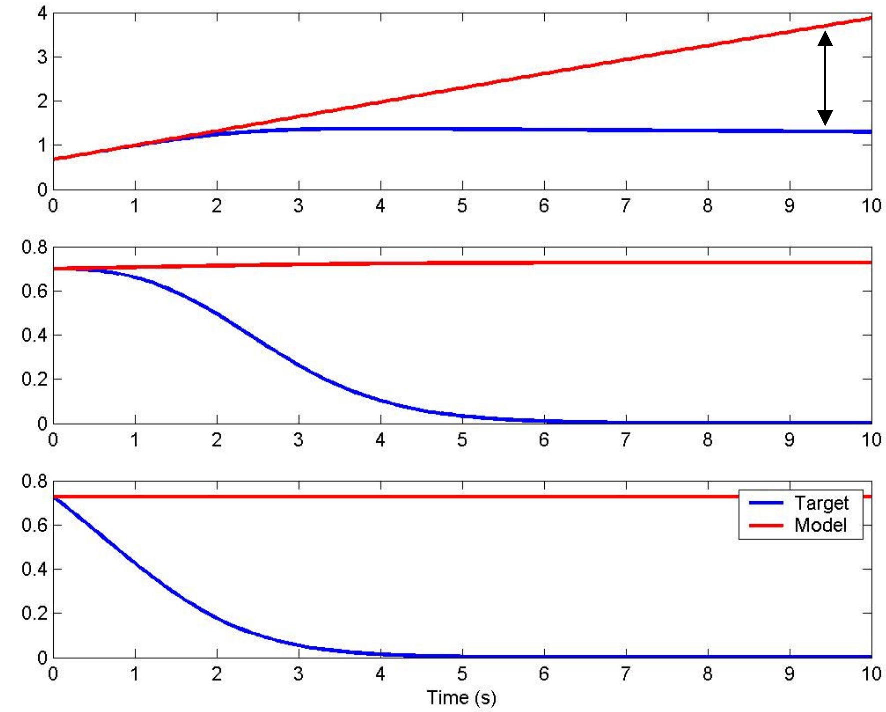
0

G = b-Galactosidase

A = Allolactose

L = Lactose





Target / Model1

dG/dt = $0.001 + A^2/(A^2+1^2) – 0.01G$

$0.001 + A^2/(A^2+1^2) – 0.01G$

dA/dt = $G(L/(L+1) – A(A+1))$

0

dL/dt = $-GL/(L+1)$

0

G = b-Galactosidase

A = Allolactose

L = Lactose

Target / Model2

dG/dt = $0.001 + A^2/(A^2+1^2) – 0.01G$

$0.001 + A^2/(A^2+1^2) – 0.01G$

dA/dt = $G(L/(L+1) – A(A+1))$

$G(L/(L+1) – A(A+1))$

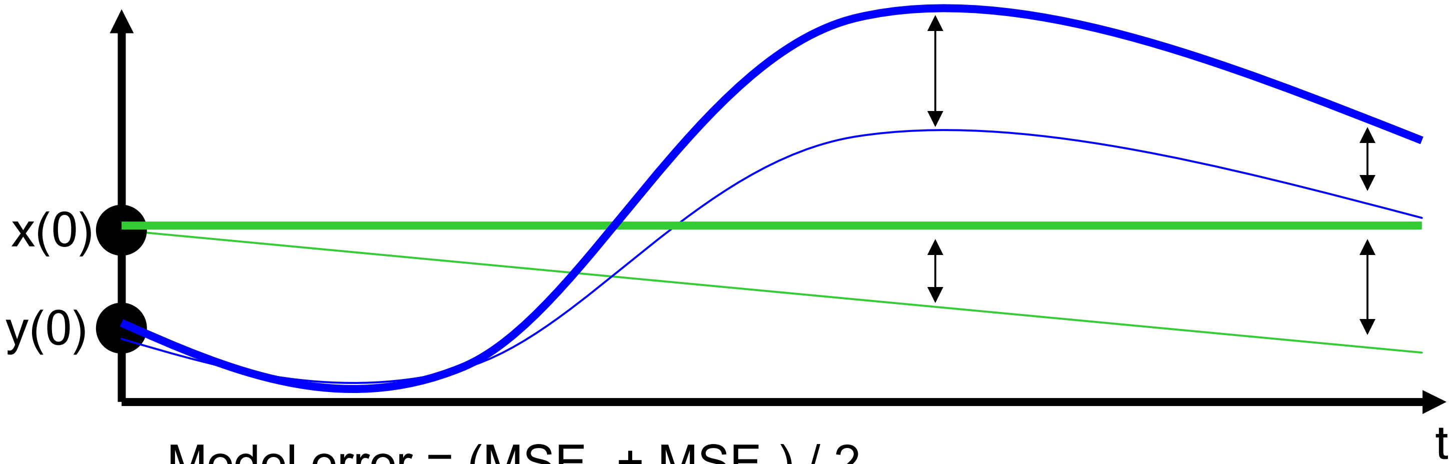
dL/dt = $-GL/(L+1)$

0

G = b-Galactosidase

A = Allolactose

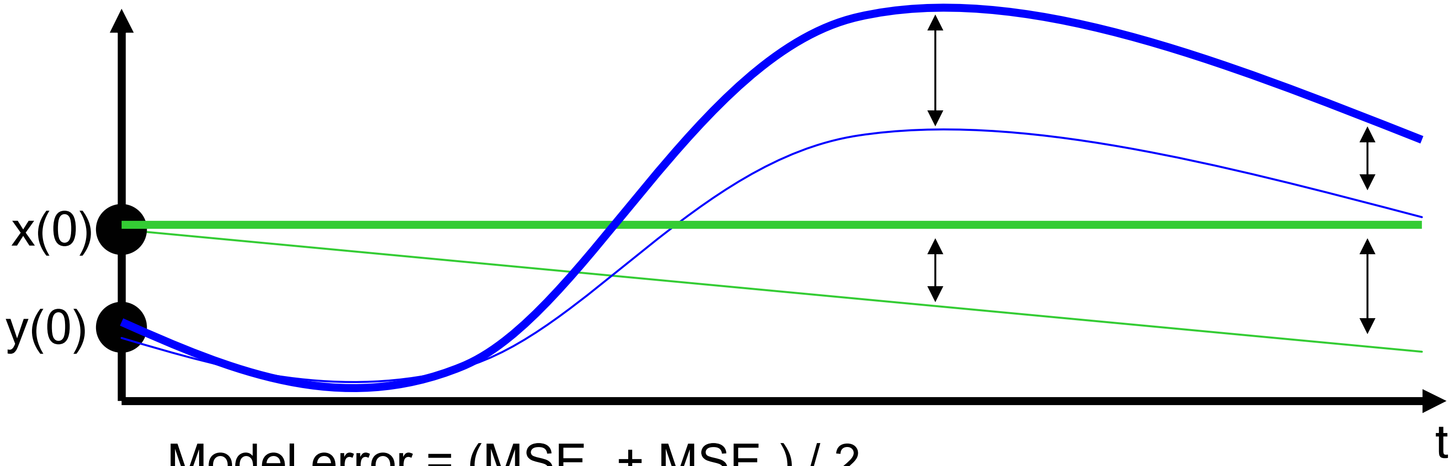
L = Lactose



Model error = $(MSE_x + MSE_y) / 2$

$dx/dt = 2x + 3y$
$dy/dt = -3x - y$

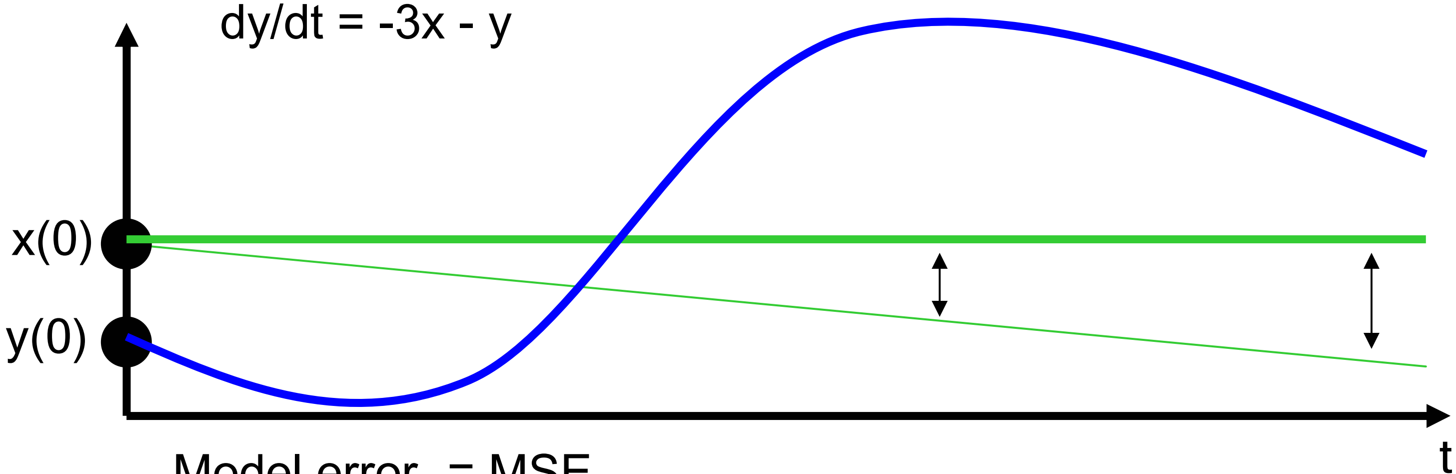
Partitioning



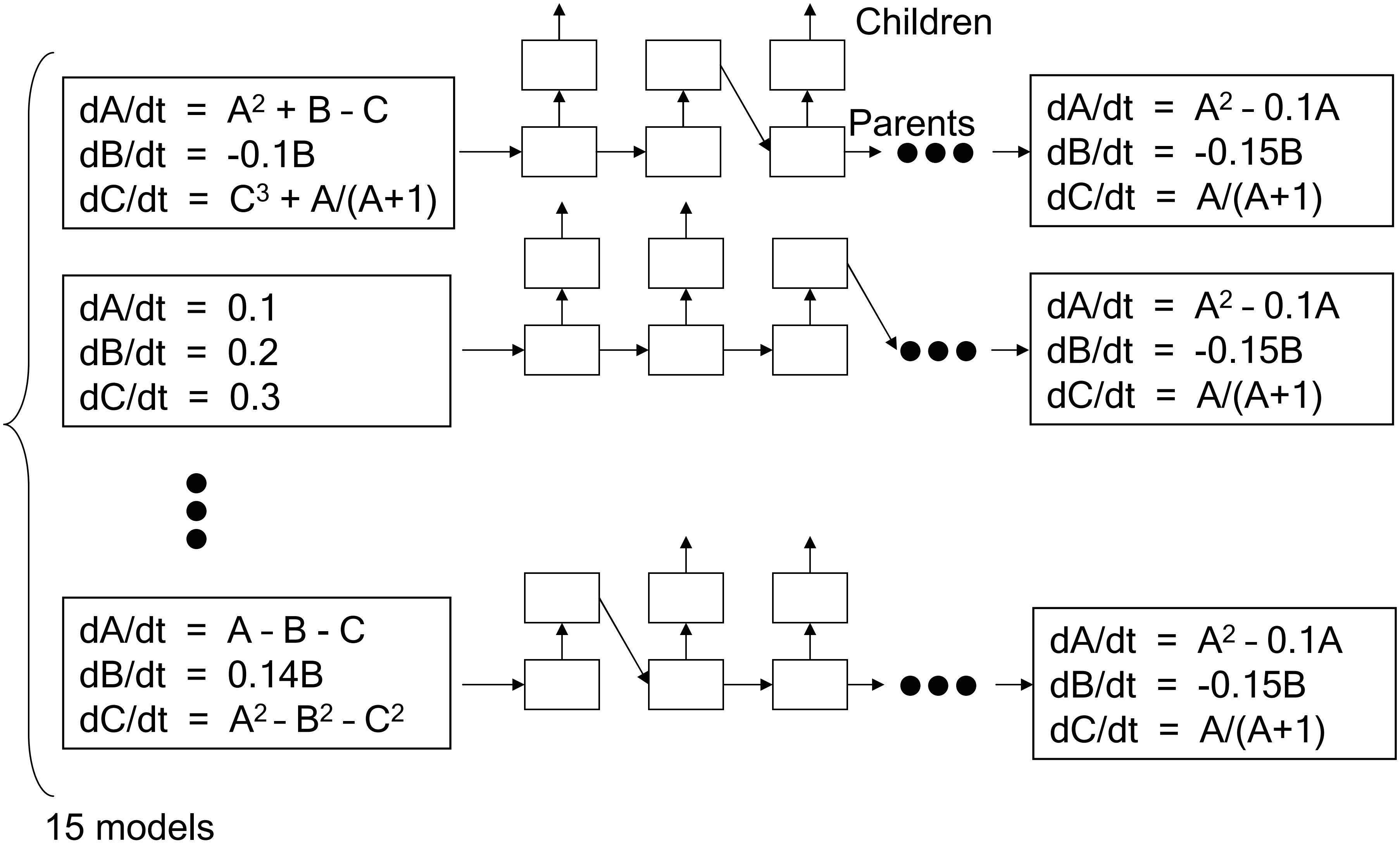
Model error = $(MSE_x + MSE_y) / 2$

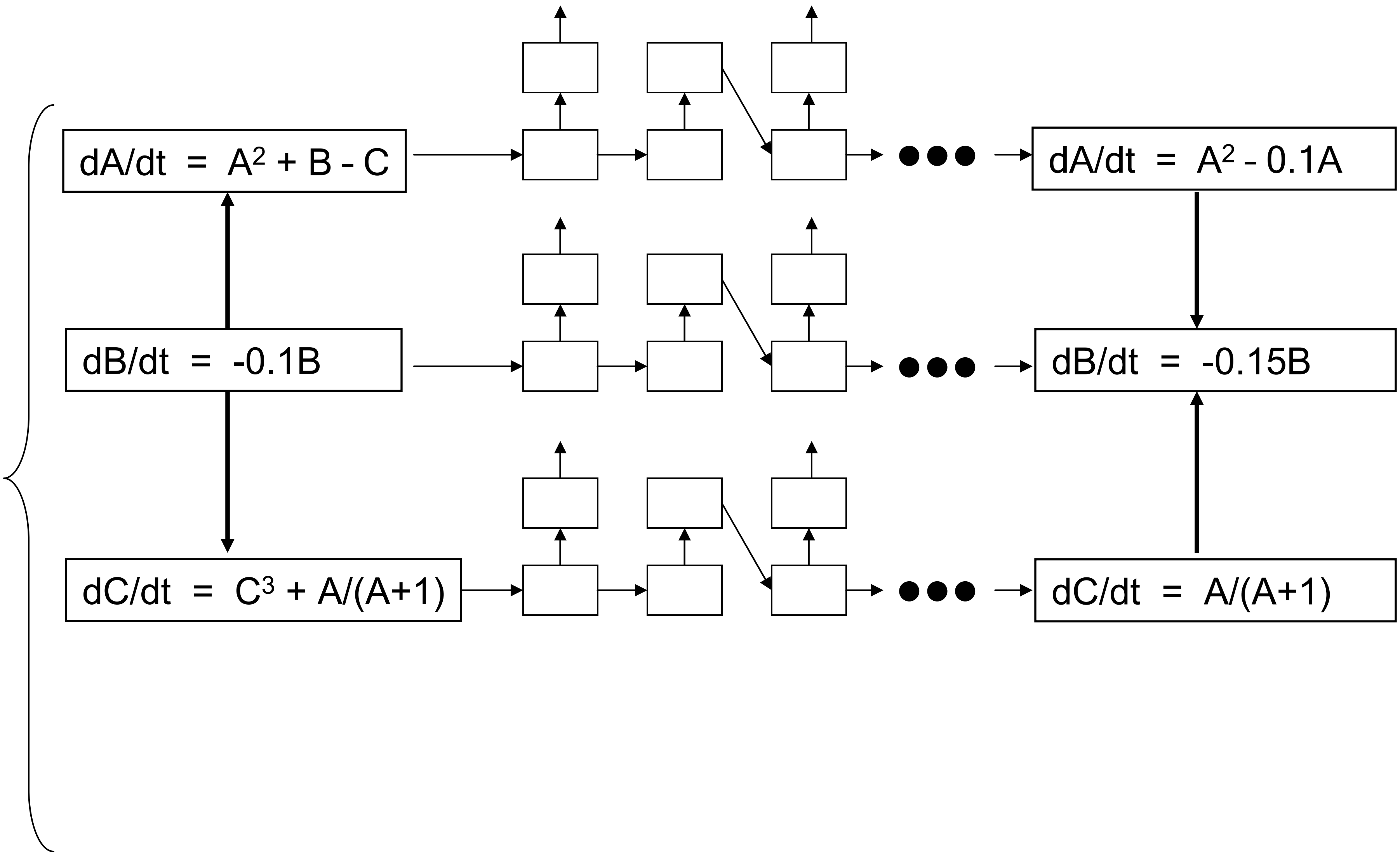
$dx/dt = 2x + 3y$

$dy/dt = -3x - y$

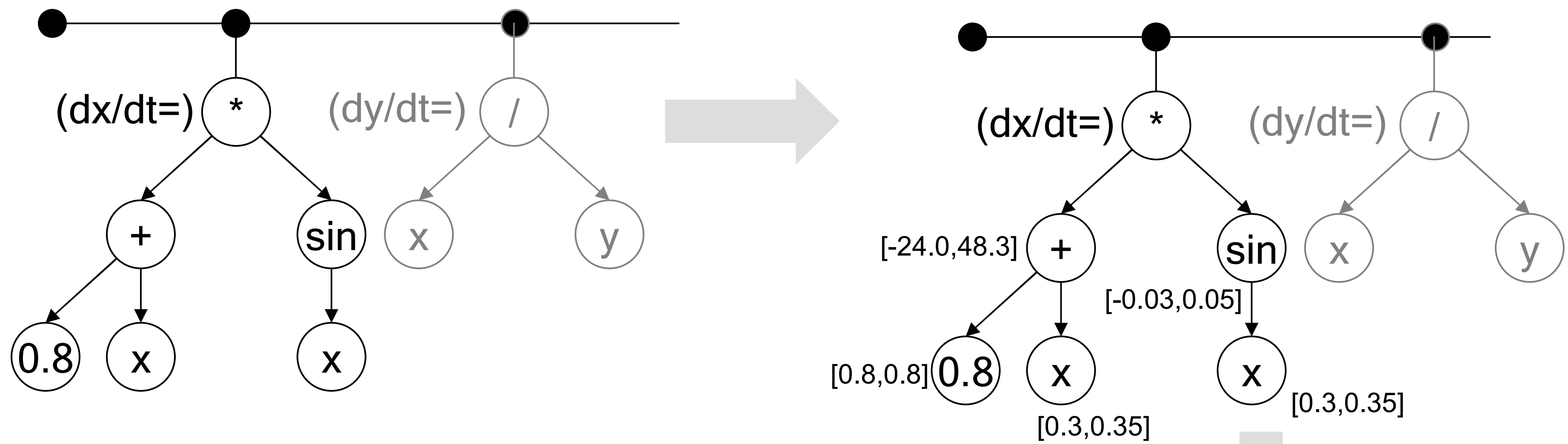


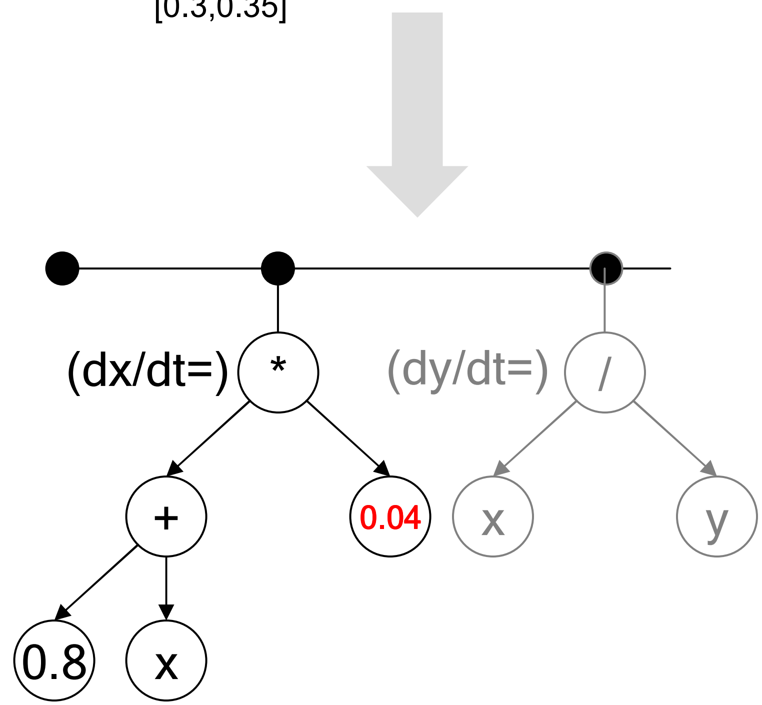
$\text{Model error}_x = MSE_x$


Children
Parents
dA/dt = A² + B - C
dB/dt = -0.1B
dC/dt = C³ + A/(A+1)
dA/dt = 0.1
dB/dt = 0.2
dC/dt = 0.3
dA/dt = A - B - C
dB/dt = 0.14B
dC/dt = A² - B² - C²
dA/dt = A² - 0.1A
dB/dt = -0.15B
dC/dt = A/(A+1)
dA/dt = A² - 0.1A
dB/dt = -0.15B
dC/dt = A/(A+1)
dA/dt = A² - 0.1A
dB/dt = -0.15B
dC/dt = A/(A+1)
15 models



15x3 = 45 models

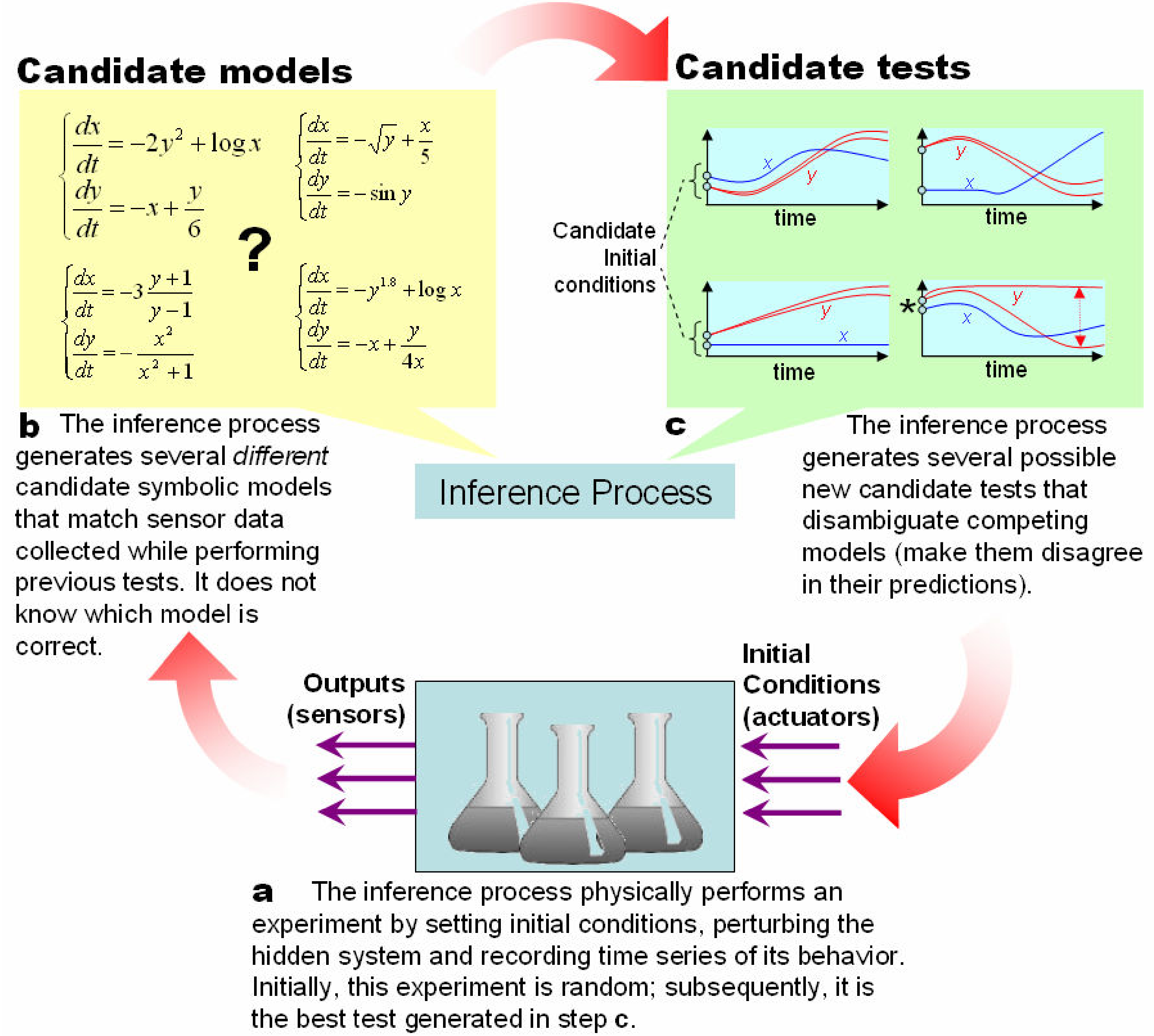




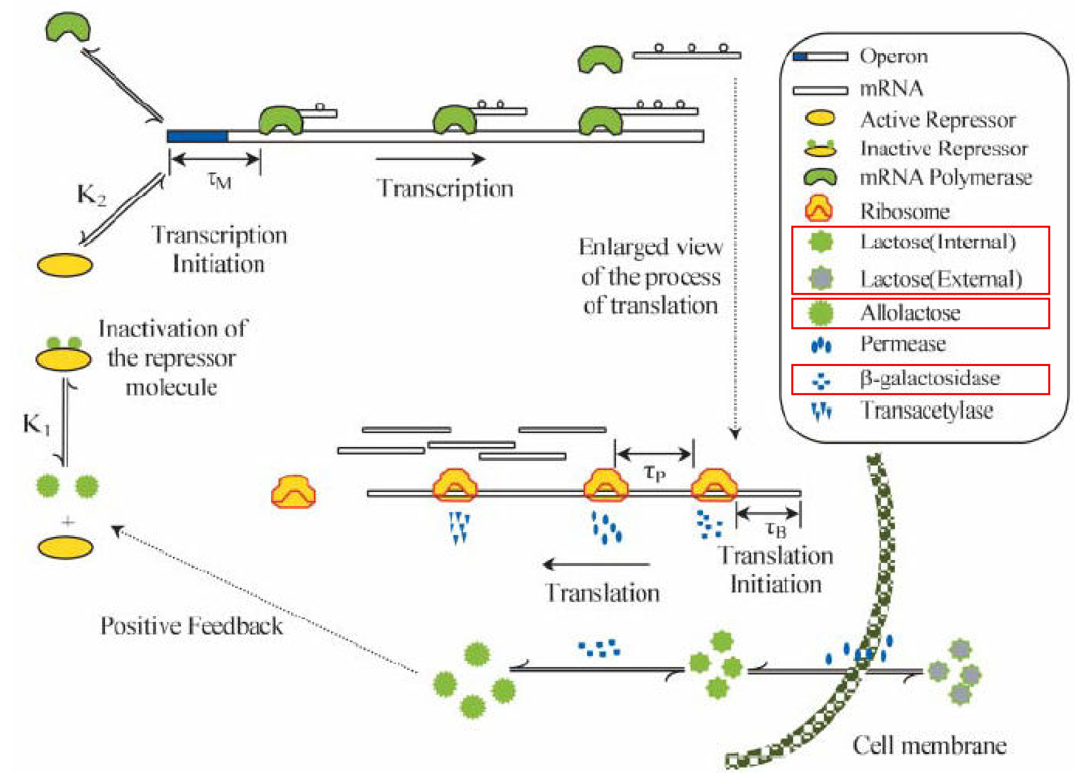
Ronald Fisher:
population biology,
modern statistics:

The Genetical Theory of Natural Selection (1930):

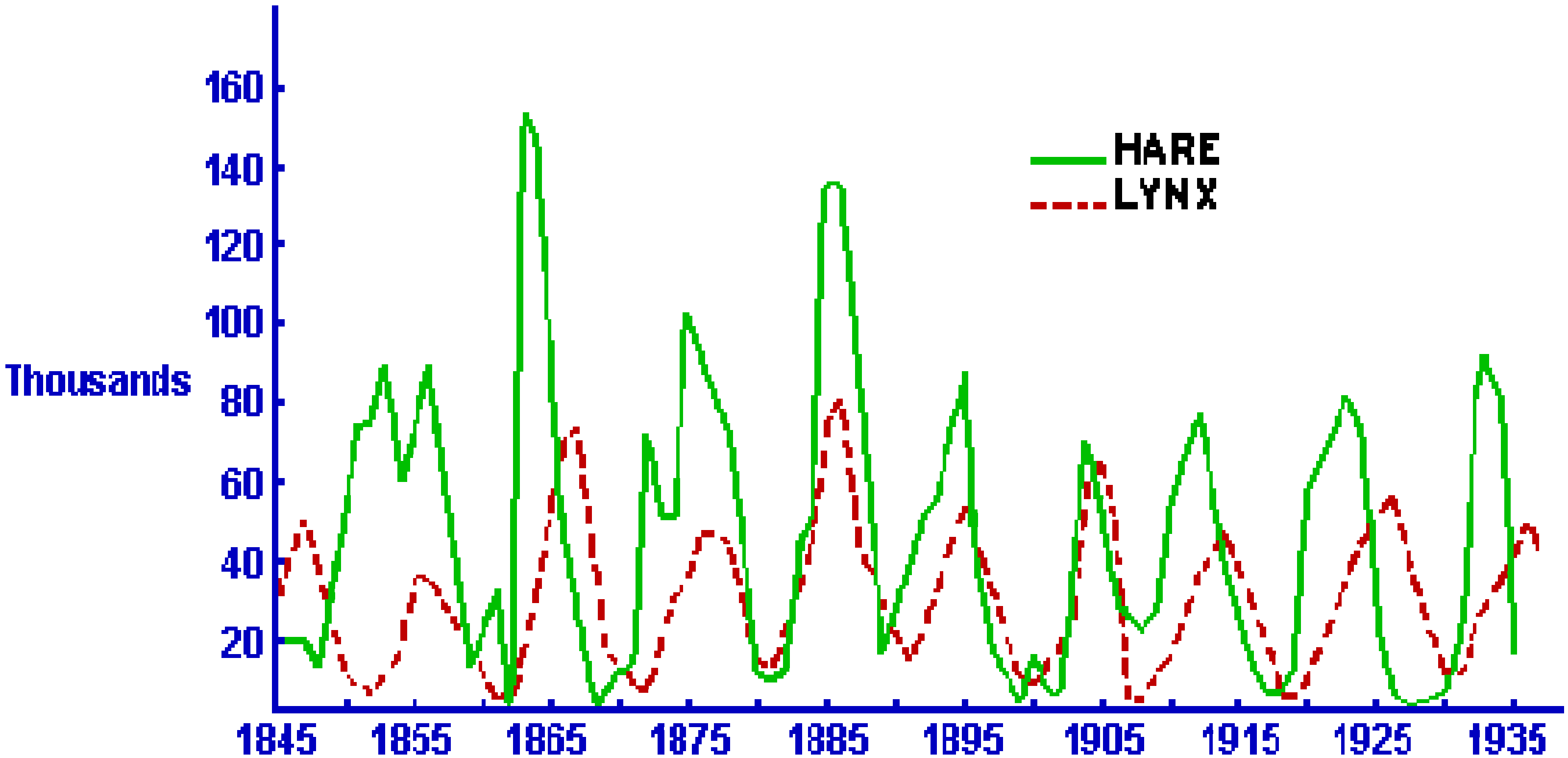
"Probability of a mutation being favorable is inversely proportional to its magnitude."



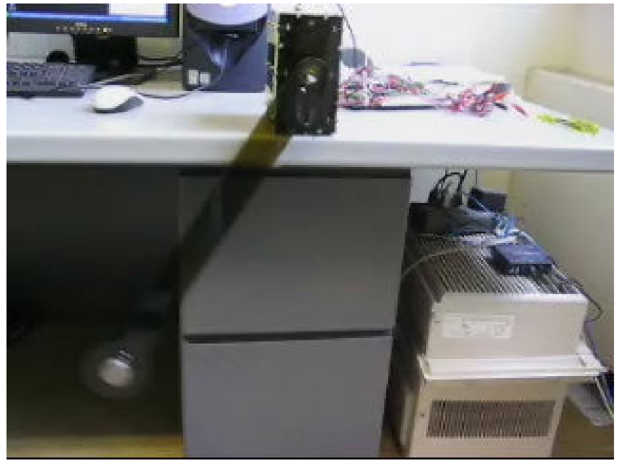
Bongard, J and Lipson, H (2007). Automated reverse engineering of nonlinear dynamical systems. ***Proceedings of the National Academy of Sciences***, to appear.

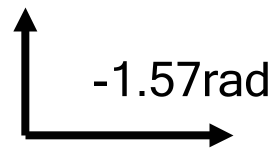


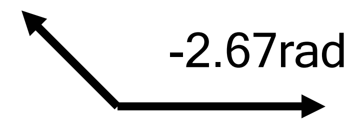
The *lac* operon from *E. coli* (G = concentration of beta-galactosidase; A = allolactose; L = lactose)	
Target system	dG/dt = $A^2/(A^2+1) - 0.01G + 0.001$ dA/dt = $G(L/(L+1) - A/(A+1))$ dL/dt = $-GL/(L+1)$
Best model	dG/dt = $0.96A^2/(0.96A^2+1)$ dA/dt = $G(L/(L+1) - A/(A+1))$ dL/dt = $-GL/(L+1)$

Historical data reporting approximated populations of snowshoe hare (*H*) and Canadian lynx (*L*)	
Target System	
Best model	dH/dt $= 3.42\times10^6 - 67.82H - 10.97L$ dL/dt $= 3.10\times10^5 + 32.66H - 63.16L$

Application: Mechanical pendulum







$d\theta/dt = 1.004\omega + 0.0001$
$d\omega/dt = -19.43\sin(1.104\theta\mathbf{+0})$

$d\theta/dt = 1.0039\omega - 0.0003$
$d\omega/dt = -22.61\sin(1.101\theta\mathbf{-2.673})$

$d\theta/dt = 1.008\omega + 0.0028$
$d\omega/dt = -19.43\sin(1.0009\theta\mathbf{-1.575})$

$d\theta/dt = \omega$
$d\omega/dt = -9.8L\sin(\theta)$

Idealized model for a single pendulum with no friction

Results

(**a**) $d\theta/dt = 0.45*-7.63$
$d\omega/dt = \sin(\theta-\omega) - \cos(\theta / (\omega / t))$

(**b**) $d\theta/dt = -3.23$
$d\omega/dt = \cos(\sin(\omega))^{-3.6\omega} / \cos(\theta) / -2.09 / \omega$

(**c**) $d\theta/dt = 1.14\omega$
$d\omega/dt = \cos(\sin(\omega))^{(\omega+\theta\omega) \wedge (-3.97t\,\theta - \omega)} / \omega$

(**d**) $d\theta/dt = \omega - ((((0.33+\theta) / \omega) / \omega)+6.78t)$
$d\omega/dt = \sin(\omega + (\theta-0.26) / (t+2.04) + 8.94)$

(**e**) $d\theta/dt = \omega - (0.33+ \theta/\omega) / (\omega+6.78t)$
$d\omega/dt = \sin(\omega + (\theta-0.27) / (t+2.04) + 8.94)$

(**f**) $d\theta/dt = \omega - (3.08/2t^{\cos(\theta)} +3.89t+t)$
$d\omega/dt = \sin(\omega + (\theta+t) / (t+2.04) + 8.94)$

(**g**) $d\theta/dt = \omega$
$d\omega/dt = -5.78 \sin(\sin(\theta)) - \theta$

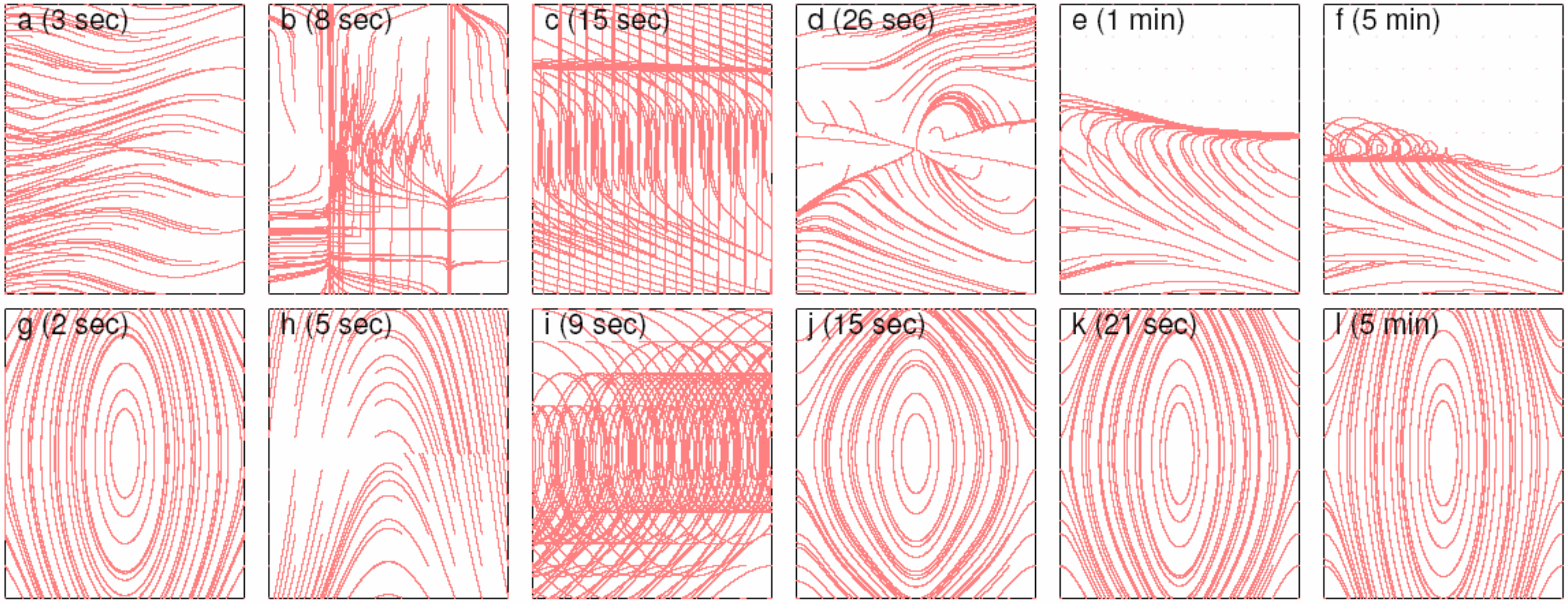
(**h**) $d\theta/dt = \omega$
$d\omega/dt = -1.54\, \theta \sin(0.58\omega)/0.38$

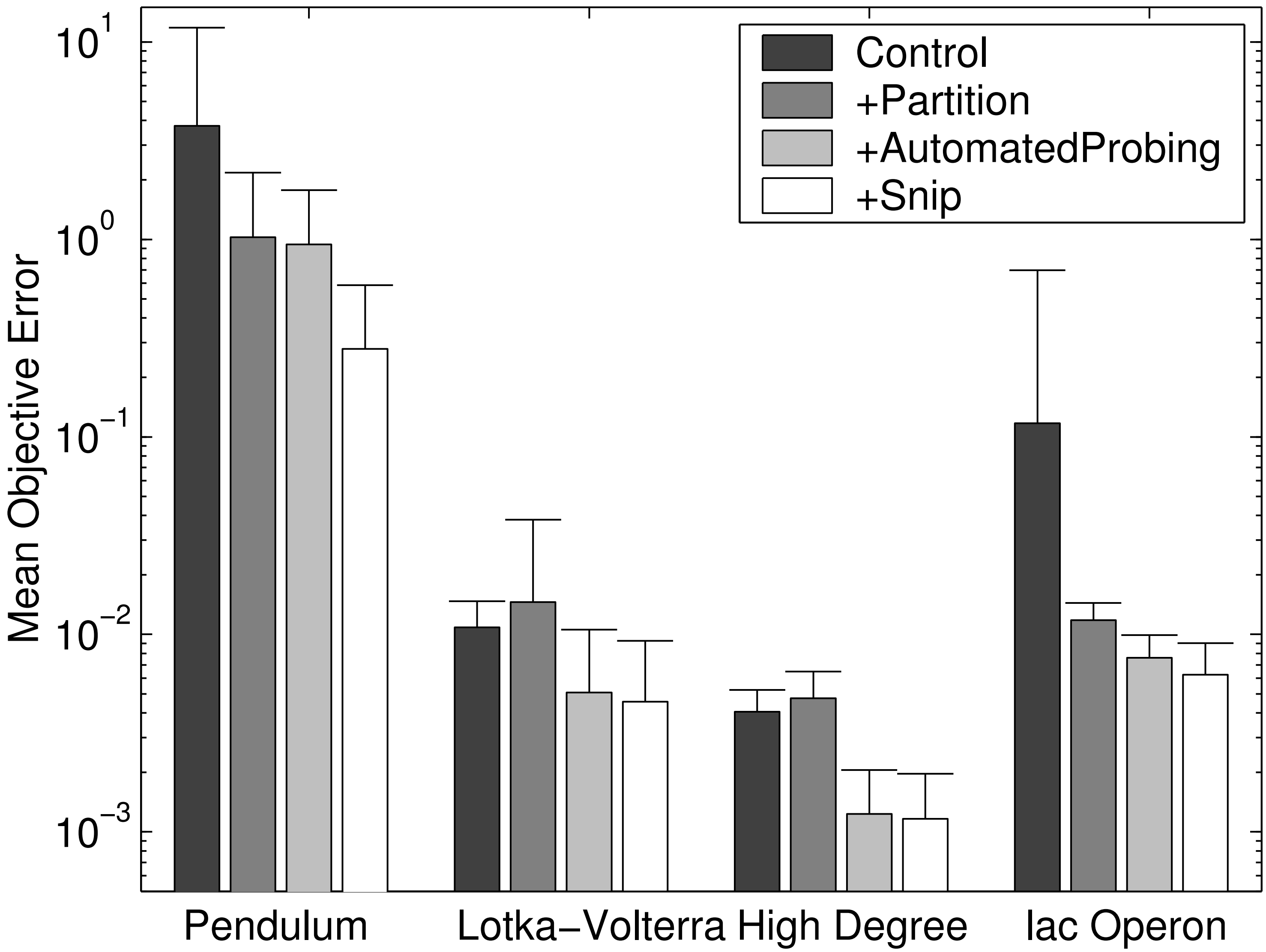
(**i**) $d\theta/dt = \omega$
$d\omega/dt = \sin(-13.37 / 7.85 / t) / 0.38$

(**j**) $d\theta/dt = \omega + t - t$
$d\omega/dt = \sin(-16.3 / 7.85 / \sin(\theta)) / 0.38$

(**k**) $d\theta/dt = \omega$
$d\omega/dt = \sin(-16.26 / 8.3 / \sin(\theta)) / 0.21$

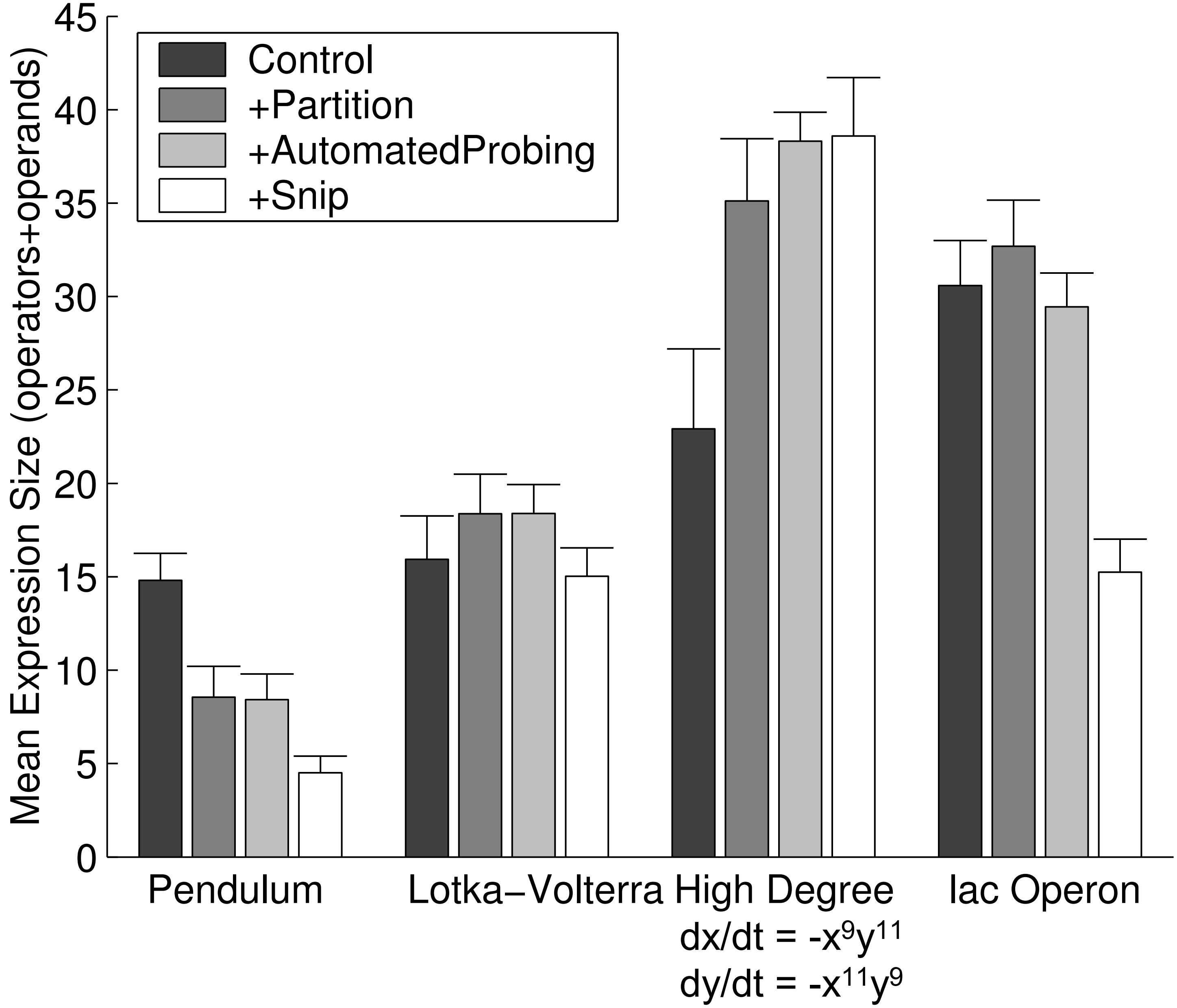
(**l**) $d\theta/dt = \omega$
$d\omega/dt = \sin(t-t) - 9.79987\sin(\theta)$





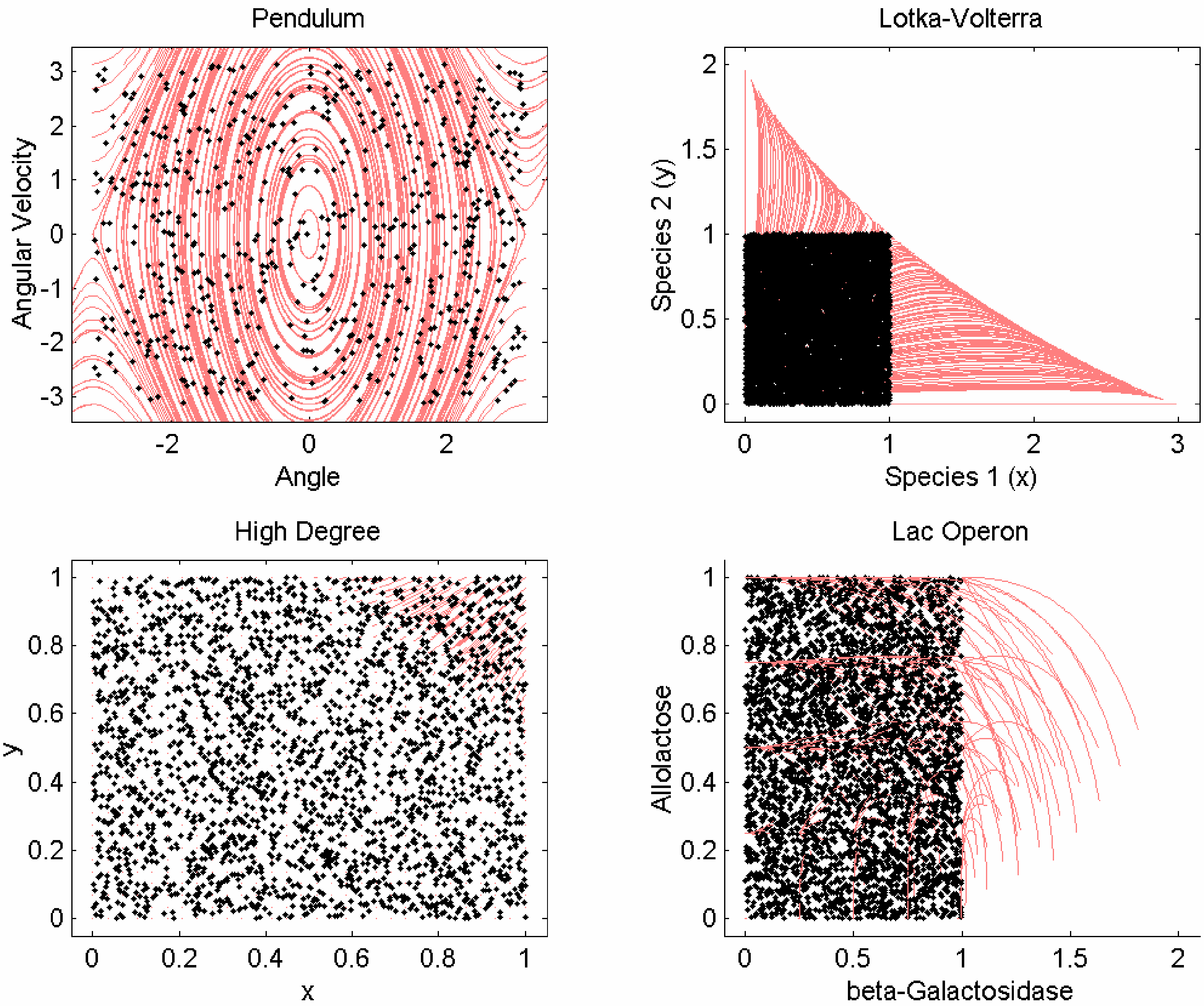
$dx/dt = -x^9y^{11}$

$dy/dt = -x^{11}y^9$


Mean Expression Size (operators+operands)
0
5
10
15
20
25
30
35
40
45
Control
+Partition
+AutomatedProbing
+Snip
Pendulum
Lotka−Volterra
High Degree
dx/dt = -x⁹y¹¹
dy/dt = -x¹¹y⁹
lac Operon

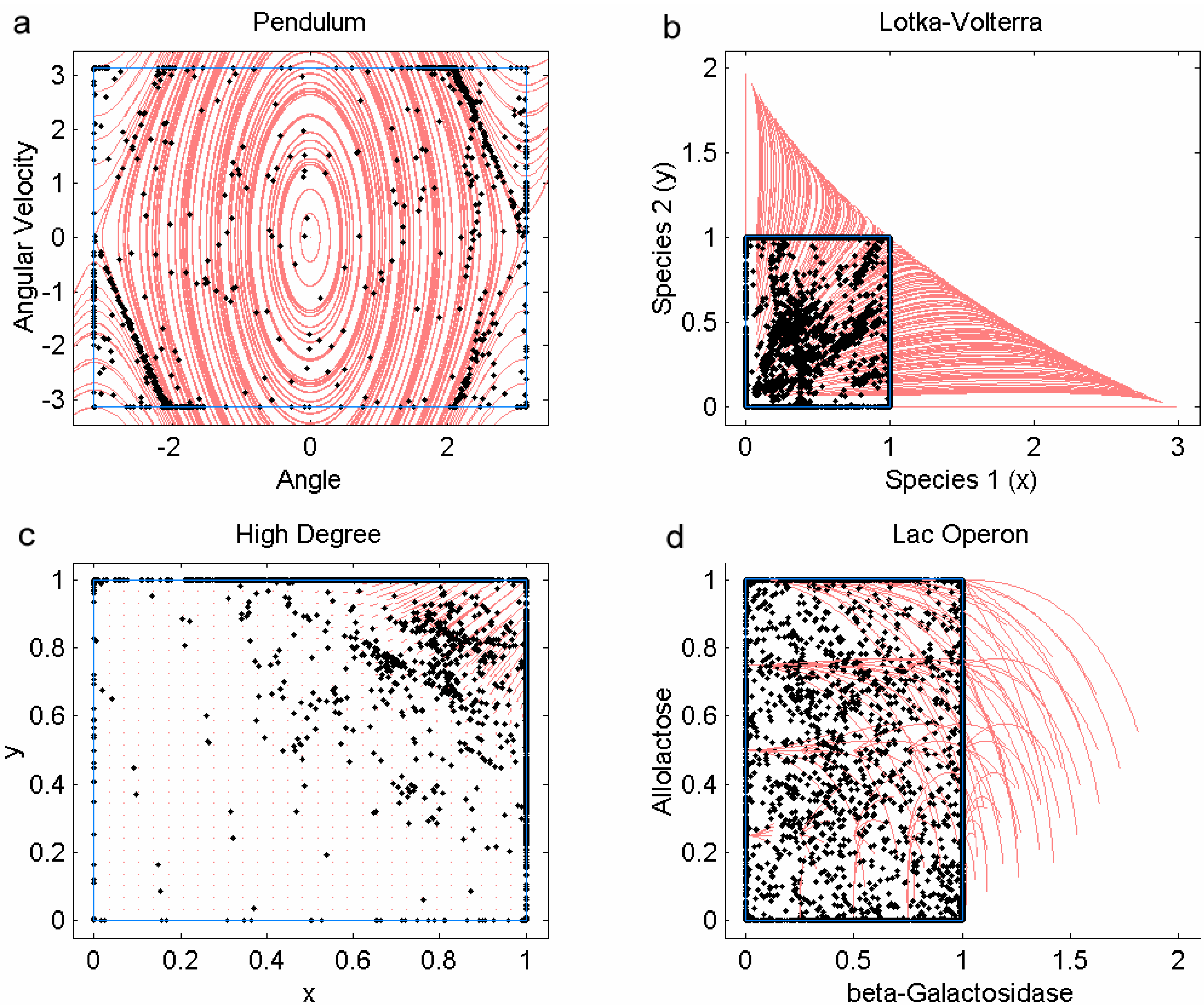
	System 1	System 2	System 3
2 variables	$dx_1/dt= -3x_1x_1 -3x_1x_2 +2x_2x_2$ $dx_2/dt= -x_1x_1 -3x_1x_2 -2x_2x_2$	$dx_1/dt=-3x_1x_1 +3x_1x_2 +3x_2x_2$ $dx_2/dt=-3x_1x_1-2x_1x_2 +2x_2x_2$	$dx_1/dt=3x_1x_1 -x_1x_2 -x_2x_2$ $dx_2/dt=x_1x_1 +3x_1y_2 -x_2x_2$
Success rate	20% (no partitioning) 100% (partitioning)	0% (no partitioning) 96.7% (partitioning)	0% (no partitioning) 100% (partitioning)
3 variables	$dx_1/dt=-3x_1x_3 -2x_2x_3 -3x_3x_3$ $dx_2/dt=-3x_1x_2 +x_1x_3 -3x_2x_3$ $dx_3/dt=3x_1x_2 +3x_1x_3 -x_2x_3$	$dx_1/dt=-x_1x_2 +x_1x_3 -x_2x_3$ $dx_2/dt=x_1x_1 +2x_1x_2 +2x_2x_3$ $dx_3/dt=-2x_1x_1 +x_1x_2 -3x_2x_3$	$dx_1/dt=-3x_1x_2 +x_1x_3 -x_3x_3$ $dx_2/dt=-2x_1x_3 +3x_2x_3 +3x_3x_3$ $dx_3/dt=2x_1x_2 -2x_1x_3 -2x_2x_3$
Success rate	0% (no partitioning) 63.3% (partitioning)	0% (no partitioning) 100% (partitioning)	0% (no partitioning) 96.7% (partitioning)
4 variables	$dx_1/dt=-x_1x_1+2x_2x_3+2x_3x_3$ $dx_2/dt=x_1x_2 -3x_1x_3 -3x_2x_3$ $dx_3/dt=-x_1x_1 -x_2x_4 +3x_4x_4$ $dx_4/dt=-3x_1x_2 -3x_1x_4 -3x_3x_4$	$dx_1/dt=x_1x_4+x_2x_4+x_4x_4$ $dx_2/dt=-3x_1x_2-2x_2x_3-3x_3x_4$ $dx_3/dt=2x_1x_2-x_1x_3+2x_2x_2$ $dx_4/dt=x_1x_3+3x_2x_3-x_3x_4$	$dx_1/dt=-3x_1x_1+3x_1x_2+3x_2x_4$ $dx_2/dt=-x_1x_1-2x_1x_3-3x_4x_4$ $dx_3/dt=-2x_1x_4+x_2x_2-3x_3x_4$ $dx_4/dt=-x_1x_2+2x_1x_4-3x_3x_4$
Success rate	0% (no partitioning) 90% (partitioning)	0% (no partitioning) 83.3% (partitioning)	0% (no partitioning) 90% (partitioning)
5 variables	$dx_1/dt=-3x_1x_5 +3x_2x_3 -3x_2x_5$ $dx_2/dt=-3x_1x_3 -2x_3x_4 -x_4x_5$ $dx_3/dt=x_1x_1 -3x_1x_4 +x_2x_4$ $dx_4/dt=3x_1x_3 -3x_1x_4 +2x_2x_2$ $dx_5/dt=3x_1x_4 +3x_3x_3 +3x_3x_4$	$dx_1/dt=-2x_2x_2+3x_3x_5+2x_4x_5$ $dx_2/dt=3x_1x_2 +x_1x_5 -2x_2x_5$ $dx_3/dt=x_1x_2 +2x_2x_5 +2x_4x_5$ $dx_4/dt=2x_1x_2 +3x_1x_5 -x_4x_5$ $dx_5/dt=2x_1x_5 -x_2x_5 -2x_5x_5$	$dx_1/dt=2x_1x_4 +2x_2x_3 -x_2x_4$ $dx_2/dt=x_1x_3 +3x_1x_4 +x_2x_4$ $dx_3/dt=-2x_1x_1 +2x_1x_2 -3x_1x_3$ $dx_4/dt=-3x_2x_5 +3x_3x_4 -x_3x_5$ $dx_5/dt=x_1x_1 +x_1x_5 +x_2x_3$
Success rate	0% (no partitioning) 76.7% (partitioning)	0% (no partitioning) 76.7% (partitioning)	0% (no partitioning) 76.7% (partitioning)
6 variables	$dx_1/dt=-2x_1x_6+x_2x_4-2x_2x_6$ $dx_2/dt=x_1x_4-x_1x_5-2x_4x_4$ $dx_3/dt=2x_2x_5-x_3x_4+x_5x_5$ $dx_4/dt=-3x_4x_5-2x_4x_6+2x_5x_5$ $dx_5/dt=x_3x_6-2x_4x_4-3x_4x_5$ $dx_6/dt=x_3x_4-x_3x_6+2x_4x_6$	$dx_1/dt=-2x_1x_3-3x_2x_4+2x_3x_6$ $dx_2/dt=-3x_2x_4+x_3x_4-x_3x_6$ $dx_3/dt=-x_1x_2-x_1x_3+x_4x_6$ $dx_4/dt=-x_1x_4+x_3x_5-2x_4x_6$ $dx_5/dt=3x_1x_2-3x_1x_6-x_5x_5$ $dx_6/dt=-3x_1x_3-2x_1x_6-3x_4x_6$	$dx_1/dt=x_1x_5+x_1x_6+x_4x_5$ $dx_2/dt=-2x_2x_5-2x_2x_6+2x_3x_6$ $dx_3/dt=-x_1x_5-2x_3x_4+x_4x_4$ $dx_4/dt=3x_1x_2+3x_2x_3-2x_4x_5$ $dx_5/dt=-3x_1x_5+x_2x_2+3x_2x_6$ $dx_6/dt=-x_2x_5-2x_3x_5-3x_5x_6$
Success rate	0% (no partitioning) 80% (partitioning)	0% (no partitioning) 70% (partitioning)	0% (no partitioning) 93% (partitioning)

Results



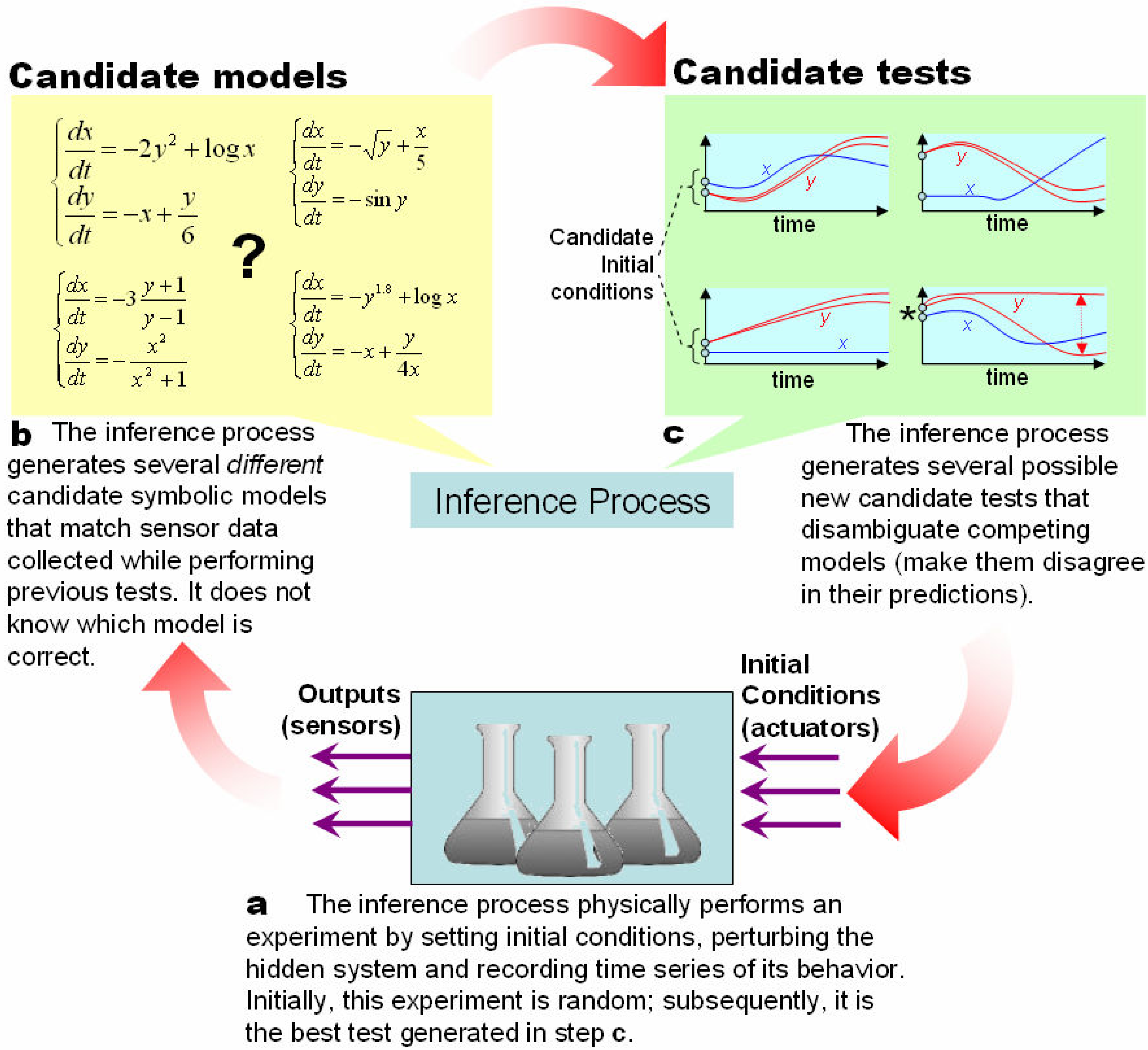
Pendulum
Angular Velocity
Angle
3
2
1
0
-1
-2
-3
-2
0
2
Lotka-Volterra
Species 2 (y)
Species 1 (x)
2
1.5
1
0.5
0
0
1
2
3
High Degree
y
x
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
Lac Operon
Allolactose
beta-Galactosidase
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2

Results



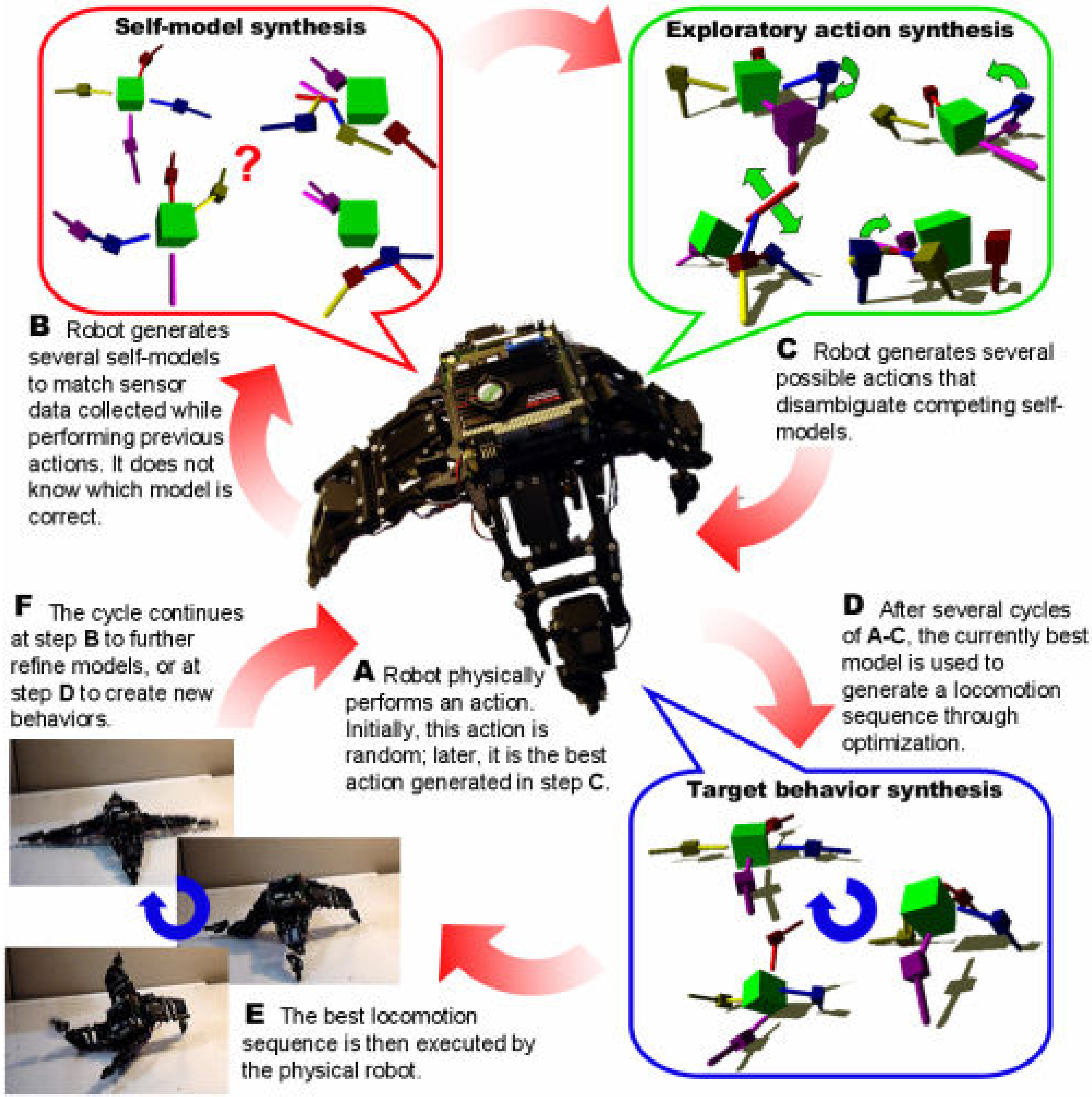
a
Pendulum
Angular Velocity
Angle
b
Lotka-Volterra
Species 2 (y)
Species 1 (x)
c
High Degree
y
x
d
Lac Operon
Allolactose
beta-Galactosidase

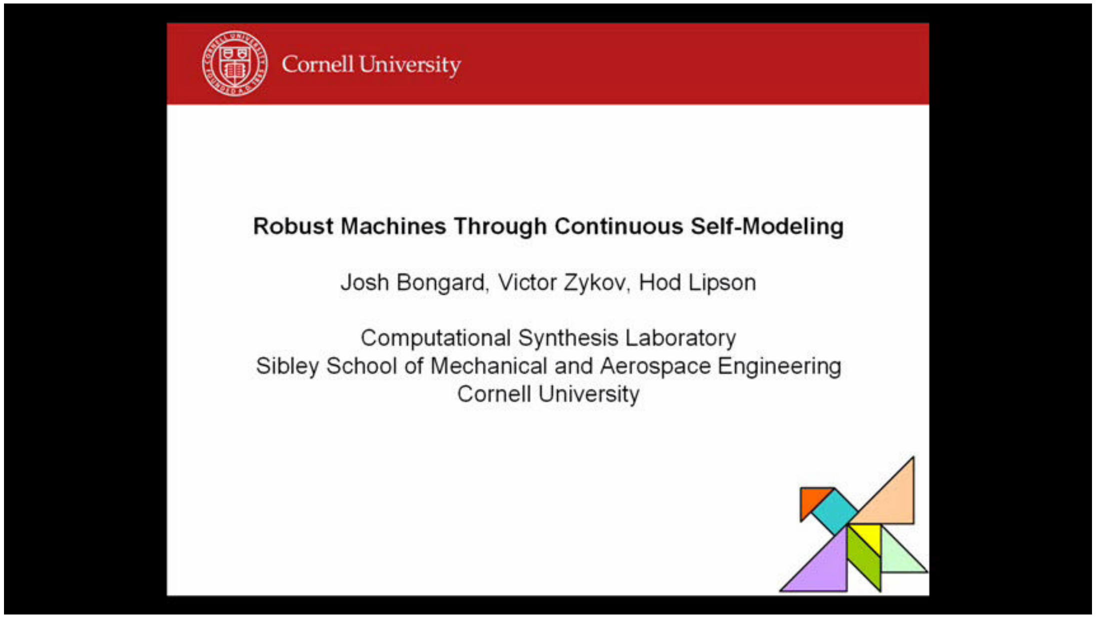
EEA for General Automated Modeling

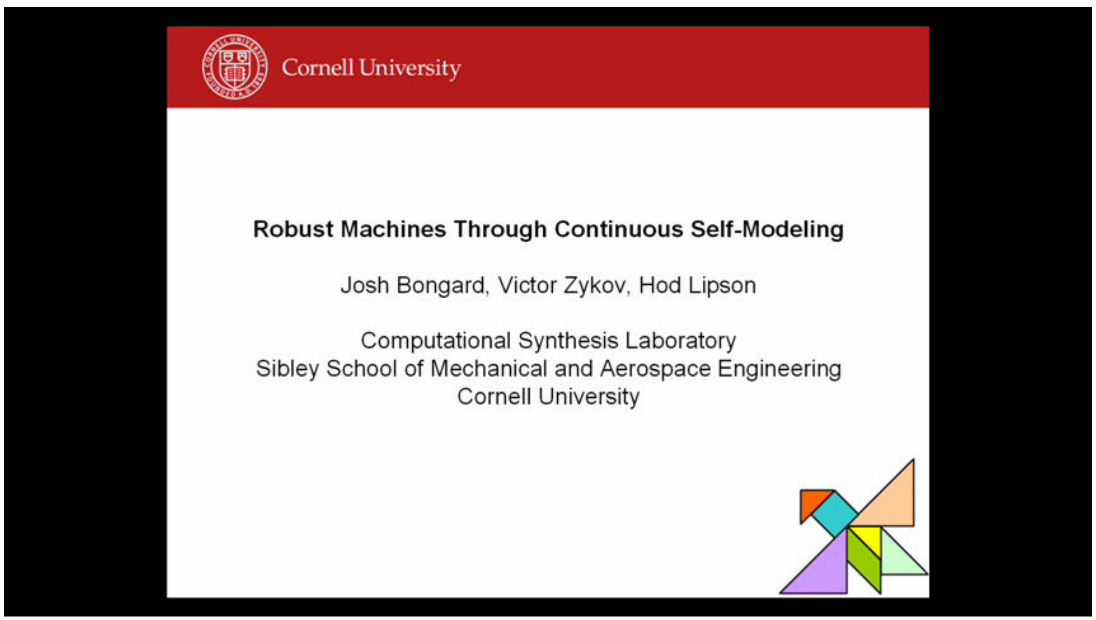


Bongard, J and Lipson, H (2007). Automated reverse engineering of nonlinear dynamical systems. ***Proceedings of the National Academy of Sciences***, to appear.

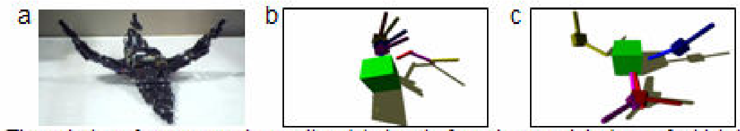
Estimation-Exploration Algorithm (EEA) for Robotics



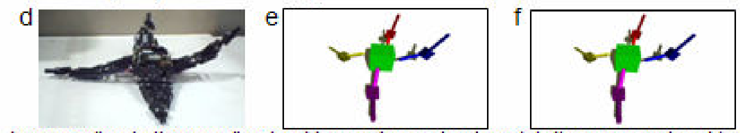




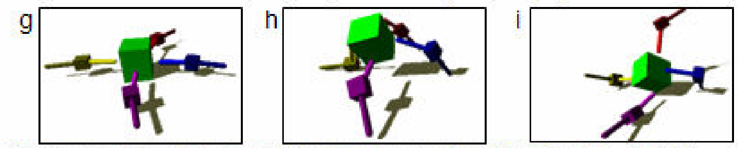
Bongard, J., V. Zykov and H. Lipson (2006) Resilient Machines Through Continuous Self-Modeling, *Science*, 314: 1118-1121.



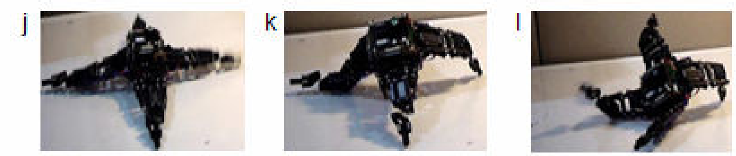
The robot performs a random action (**a**). A set of random models (one of which is shown in **b**) is synthesized into approximate models (one of which is shown in **c**).



A new action is then synthesized to create maximal model disagreement and is performed by the physical robot (**d**), after which further modeling ensues. This cycle continues until no further model improvement is possible (**e-f**).

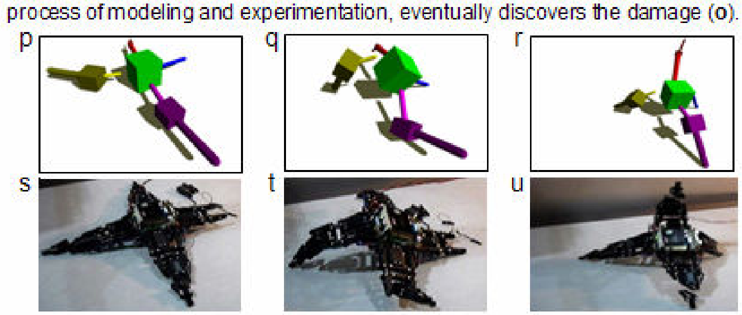


The best model is then used to synthesize a behavior (in this case, forward locomotion, the first few movements of which are shown in **g-i**).



This behavior is then executed by the physical robot (**j-l**).

The robot then suffers damage (the lower part of the right leg breaks off; **m**). Modeling then recommences with the best model so far (**n**), and using the same process of modeling and experimentation, eventually discovers the damage (**o**).



The new model is then used to synthesize a new behavior (**p-r**), which is executed by the physical robot (**s-u**), allowing it to recover functionality despite this unanticipated change.

Conclusions

The Estimation-Exploration Algorithm (EEA) generates sets of differential equations directly from time-series data.

Assumes all variables are observable.

Uses three new methods:

Active probing

generate new experiments to try based on current set of models

Partitioning

dissociate coupled variables, and approximate them separately

Snipping

Replace existing model with a similar, smaller model

Results

Can create models independent of domain (ecological, mechanical, etc.)

Active probing, partitioning and snipping improve modeling

Tests focus on bifurcations and extremal regions of the target system.