## Contagion

## Santa Fe Institute Summer School, 2009

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### Contagion

ntroduction

Simple Disease Spreading Models

Social Contagion Models

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## **Outline**

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## Contagion

## Definition:

- ▶ (1) The spreading of a quality or quantity between individuals in a population.
- ▶ (2) A disease itself: the plague, a blight, the dreaded lurgi, ...

## Two main classes of contagion:

- Infectious diseases: tuberculosis, HIV, ebola, SARS, influenza, ...
- 2. Social contagion: fashion, word usage, rumors, riots, religion, ...

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## Contagion models

## Some large questions concerning network contagion:

- 1. For a given spreading mechanism on a given network, what's the probability that there will be global spreading?
- 2. If spreading does take off, how far will it go?
- 3. How do the details of the network affect the outcome?
- 4. How do the details of the spreading mechanism affect the outcome?
- 5. What if the seed is one or many nodes?

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### Introduction

Simple Disease Spreading Models

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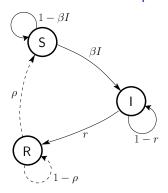


## Mathematical Epidemiology

## The standard SIR model:

- Three states:
  - ► S = Susceptible
  - ► I = Infected
  - ► R = Recovered
- S(t) + I(t) + R(t) = 1
- Presumes random interactions

## Discrete time example:



Transition Probabilities:

 $\beta$  for being infected given contact with infected r for recovery  $\rho$  for loss of immunity

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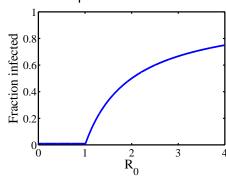
Spreading Models

Social Contagion

Independent Interaction models

## Reproduction Number $R_0$ :

- ► R<sub>0</sub> = expected number of infected individuals resulting from a single initial infective.
- ▶ Epidemic threshold: If  $R_0 > 1$ , 'epidemic' occurs.
- ► Example:



- Continuous phase transition.
- Fine idea from a simple model.

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## Disease spreading models

## For 'novel' diseases:

- 1. Can we predict the size of an epidemic?
- 2. How important/useful is the reproduction number  $R_0$ ?
- 3. What is the population size *N*?

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## $R_0$ and variation in epidemic sizes

## $R_0$ approximately the same for all of the following:

- ▶ 1918-19 "Spanish Flu"  $\sim$  500,000 deaths in US
- ▶ 1957-58 "Asian Flu"  $\sim$  70,000 deaths in US
- $\blacktriangleright$  1968-69 "Hong Kong Flu"  $\sim$  34,000 deaths in US
- $\blacktriangleright$  2003 "SARS Epidemic"  $\sim$  800 deaths world-wide

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## Size distributions

## Elsewhere, event size distributions are important:

- earthquakes (Gutenberg-Richter law)
- city sizes, forest fires, war fatalities
- wealth distributions
- 'popularity' (books, music, websites, ideas)
- What about Epidemics?

Power laws distributions are common but not obligatory...

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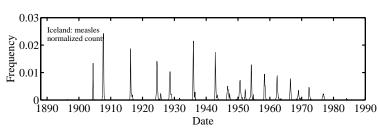
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## Feeling icky in Iceland

Caseload recorded monthly for range of diseases in Iceland, 1888-1990



Treat outbreaks separated in time as 'novel' diseases.

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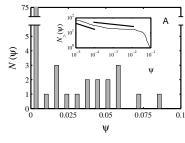
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## Measles



## Insert plots:

Complementary cumulative frequency distributions:

$$N_{>}(\Psi) \propto \Psi^{-\gamma+1}$$

 $\Psi$  = fractional epidemic size

## Measured values of $\gamma$ :

- ▶ measles: 1.40 (low Ψ) and 1.13 (high Ψ)
- $\blacktriangleright$  Expect 2  $\leq \gamma <$  3 (finite mean, infinite variance)
- ► Distribution is rather flat...

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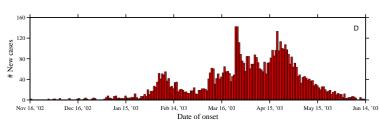
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## Resurgence—example of SARS



- ► Epidemic discovers new 'pools' of susceptibles: Resurgence.
- ► Importance of rare, stochastic events.

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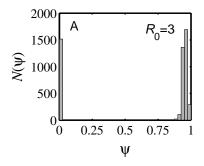
## A challenge

## So... can a simple model produce

- 1. broad epidemic distributions and
- 2. resurgence?

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## Size distributions



Simple models typically produce bimodal or unimodal size distributions.

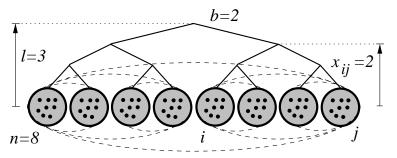
- This includes network models: random, small-world, scale-free, ...
- ► Some exceptions:
  - 1. Forest fire models
  - 2. Sophisticated metapopulation models

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## A toy agent-based model

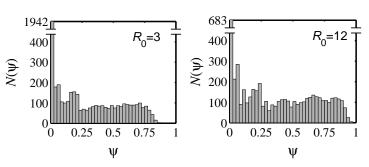
## Geography: allow people to move between contexts:



- ► *P* = probability of travel
- ▶ Movement distance:  $Pr(d) \propto exp(-d/\xi)$
- $\triangleright$   $\xi$  = typical travel distance

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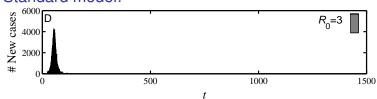
## Example model output: size distributions



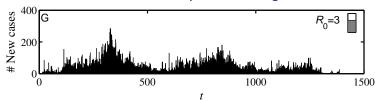
- ▶ Flat distributions are possible for certain  $\xi$  and P.
- ▶ Different R<sub>0</sub>'s may produce similar distributions
- $\triangleright$  Same epidemic sizes may arise from different  $R_0$ 's



## Standard model:



## Standard model with transport: Resurgence



- Disease spread highly sensitive to population structure
- ► Rare events may matter enormously

## Simple disease spreading models

## Attempts to use beyond disease:

- Adoption of ideas/beliefs (Goffman & Newell, 1964)
- Spread of rumors (Daley & Kendall, 1965)
- ▶ Diffusion of innovations (Bass, 1969)
- Spread of fanatical behavior (Castillo-Chávez & Song, 2003)

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## **Social Contagion**



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Simple Disease Spreading Models

Social Contagion



## **Social Contagion**

## Examples abound:

- being polite/rude
- strikes
- innovation
- residential segregation
- ipods
- obesity

- Harry Potter
- voting
- gossip
- Rubik's cube §
- religious beliefs
- leaving lectures

## SIR and SIRS contagion possible

Classes of behavior versus specific behavior: dieting

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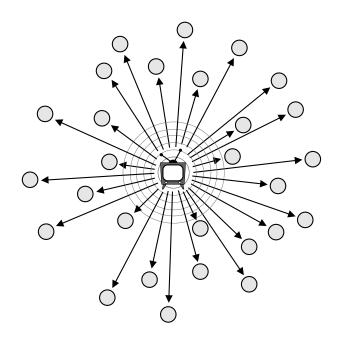
## **Social Contagion**

## Two focuses for us:

- ▶ Widespread media influence
- ► Word-of-mouth influence

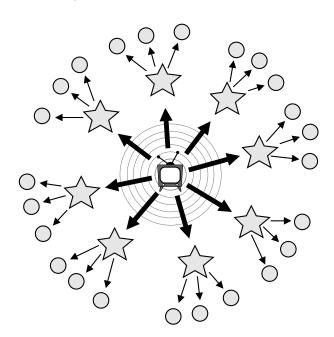


## The hypodermic model of influence:



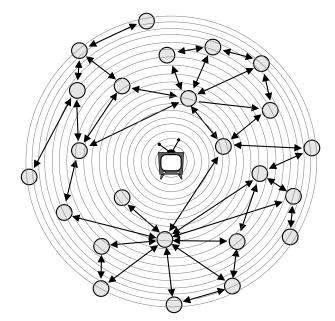


## The two step model of influence:





## The general model of influence:





## **Social Contagion**

## Why do things spread?

- ▶ Because of system level properties?
- ► Or properties of special individuals?
- ► Is the match that lights the forest fire the key? (Katz and Lazarsfeld; Gladwell)
- ➤ Yes. But only because we are narrative-making machines...
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

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## The Mona Lisa:



- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- ▶ Not the world's greatest painting from the start...
- ► Escalation through theft, vandalism, parody, ...

## The completely unpredicted fall of Eastern Europe:



Timur Kuran: "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"



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## **Social Contagion**

## Some important models:

- ► Tipping models—Schelling (1971)
  - Simulation on checker boards
  - Idea of thresholds
- ► Threshold models—Granovetter (1978)
- ► Herding models—Bikhchandani, Hirschleifer, Welch (1992)
  - Social learning theory, Informational cascades,...



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## Social contagion models

## Thresholds:

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- ▶ Response can be probabilistic or deterministic.
- Individual thresholds vary.

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## **Social Contagion**

## Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
  - ▶ Telephones, Facebook, operating systems, ...





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## **Imitation**



"When people are free to do as they please, they usually imitate each other."

-Eric Hoffer "The Passionate State of Mind" [11]

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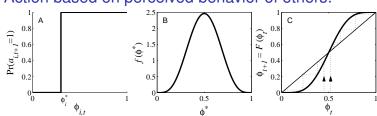
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## Granovetter's threshold model:

## Action based on perceived behavior of others:



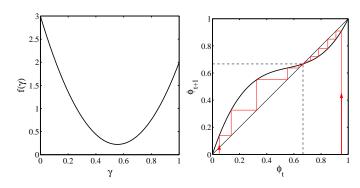
- Two states: S and I.
- $\phi$  = fraction of contacts 'on' (e.g., rioting)

$$\phi_{t+1} = \int_0^{\phi_t} f(\gamma) d\gamma = F(\gamma)|_0^{\phi_t} = F(\phi_t)$$

▶ This is a Critical Mass model

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## Social Sciences: Threshold models



► Example of single stable state model

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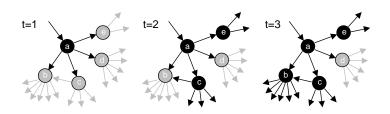
## Social Sciences—Threshold models

## Implications for collective action theory:

- 1. Collective uniformity ⇒ individual uniformity
- 2. Small individual changes ⇒ large global changes



## Threshold model on a network



- ▶ All nodes have threshold  $\phi = 0.2$ .
- "A simple model of global cascades on random networks"

D. J. Watts. Proc. Natl. Acad. Sci., 2002



## Snowballing

## The Cascade Condition:

- ▶ If one individual is initially activated, what is the probability that an activation will spread over a network?
- ► What features of a network determine whether a cascade will occur or not?



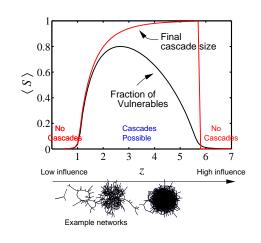
## The most gullible

## Vulnerables:

- ▶ = Individuals who can be activated by just one 'infected' contact
- ▶ For global cascades on random networks, must have a global cluster of vulnerables
- Cluster of vulnerables = critical mass
- ▶ Network story: 1 node → critical mass → everyone.

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## Cascades on random networks



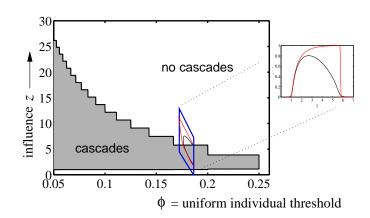
- Cascades occur only if size of max vulnerable cluster > 0.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

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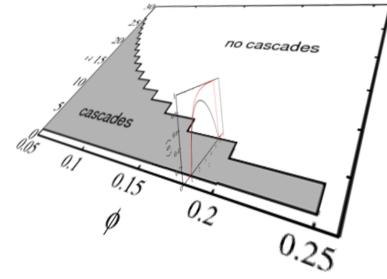
## Cascade window for random networks



- 'Cascade window' widens as threshold  $\phi$  decreases.
- Lower thresholds enable spreading.

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## Cascade window for random networks





## Analytic work

- ▶ Threshold model completely solved (by 2008):
- ► Cascade condition: [22]

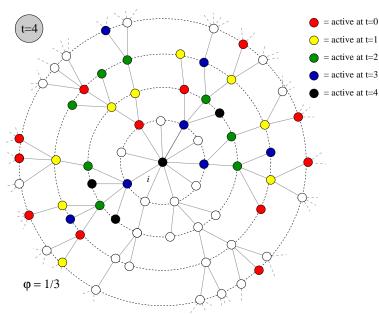
$$\sum_{k=1}^{\infty} k(k-1)\beta_k P_k/z \geq 1.$$

where  $\beta_k$  = probability a degree k node is vulnerable.

- ► Final size of spread figured out by Gleeson and Calahane [9, 8].
- Solution involves finding fixed points of an iterative map of the interval.
- ► Spreading takes off: expansion
- ► Spreading reaches a particular node: contraction

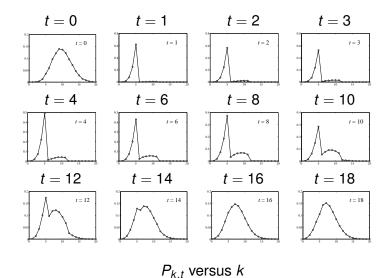


## Expected size of spread



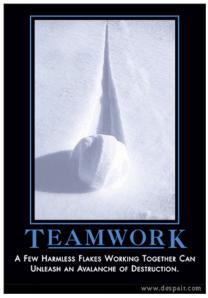


## Early adopters—degree distributions





## The power of groups...

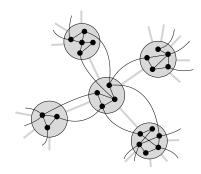


despair.com

"A few harmless flakes working together can unleash an avalanche of destruction."



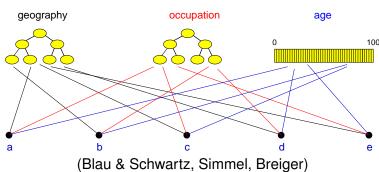
## Group structure—Ramified random networks



p = intergroup connection probability q = intragroup connection probability.

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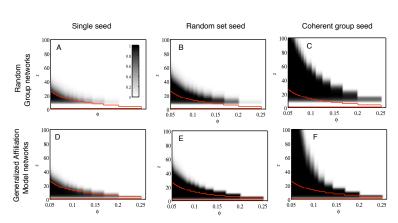
## Generalized affiliation model





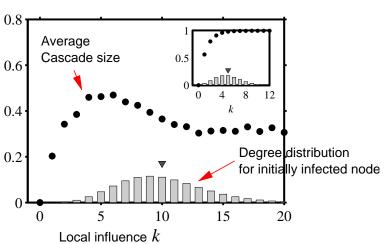
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## Cascade windows for group-based networks





## Assortativity in group-based networks



- ► The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.



## Social contagion

## Summary:

- 'Influential vulnerables' are key to spread.
- ► Early adopters are mostly vulnerables.
- ▶ Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Extreme/unexpected cascades may occur in highly connected networks
- Many potential 'influentials' exist.
- Average individuals may be more influential system-wise than locally influential individuals.
- ▶ 'Influentials' are posterior constructs.



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## Social contagion

## Implications:

- Focus on the influential vulnerables.
- Create entities that many individuals 'out in the wild' will adopt and display rather than broadcast from a few 'influentials.'
- ► Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Accept that movement of entities will be out of originator's control.
- ▶ Possibly only simple ideas can spread by word-of-mouth.

(Idea of opinion leaders has spread well...)







## **Social Contagion**

## Messing with social connections:

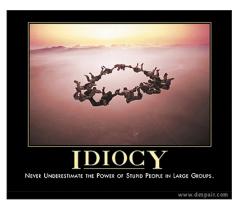
- ► Ads based on message content (e.g., Google and email)
- ▶ Buzz media
- Facebook's advertising (Beacon)

Arguably not always a good idea...



## The collective...

despair.com



"Never Underestimate the Power of Stupid People in Large Groups."





## Where do superstars come from?

Rosen (1981): "The Economics of Superstars"

## Examples:

- ▶ Full-time Comedians (≈ 200)
- Soloists in Classical Music
- ► Economic Textbooks (the usual myopic example)
- ▶ Highly skewed distributions again...



## **Superstars**

## Rosen's theory:

- ▶ Individual quality *q* maps to reward *R*(*q*)
- ightharpoonup R(q) is 'convex'  $(d^2R/dq^2 > 0)$
- Two reasons:
  - Imperfect substitution:
     A very good surgeon is worth many mediocre ones
  - Technology:
     Media spreads & technology reduces cost of reproduction of books, songs, etc.
- No social element—success follows 'inherent quality'



## Superstars

Adler (1985): "Stardom and Talent"

- Assumes extreme case of equal 'inherent quality'
- Argues desire for coordination in knowledge and culture leads to differential success
- Success is then purely a social construction



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## Dominance hierarchies

Chase et al. (2002): "Individual differences versus social dynamics in the formation of animal dominance hierarchies"

The aggressive female Metriaclima zebra (⊞):

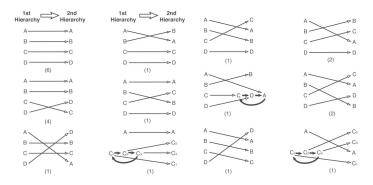


Pecking orders for fish...



## Dominance hierarchies

► Fish forget—changing of dominance hierarchies:



▶ 22 observations: about 3/4 of the time, hierarchy changed



## Music Lab Experiment



48 songs 30,000 participants BAND NAME

[Help] [Log off] for both codes

GROWTH PEOPLE: 66

Finance: 9

Tother people: 9

Tother people: 9

Town own out 9

SONG TIFLE NUMBER OF DOWNLOADS

multiple 'worlds'
Inter-world variability

► How probable is the world?

- ▶ Can we estimate variability?
- Superstars dominate but are unpredictable. Why?



## Music Lab Experiment



Salganik et al. (2006) "An experimental study of inequality and unpredictability in an artificial cultural market"



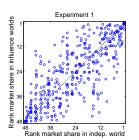
## Music Lab Experiment

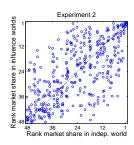






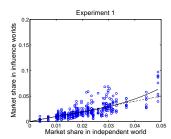
## Music Lab Experiment

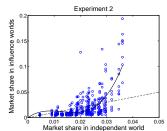




▶ Variability in final rank.

## Music Lab Experiment





Variability in final number of downloads.

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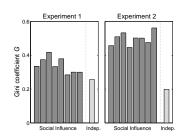


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▶ Inequality as measured by Gini coefficient:

$$G = \frac{1}{(2N_s - 1)} \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} |m_i - m_j|$$

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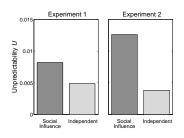
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## Music Lab Experiment



Unpredictability

$$U = \frac{1}{N_{\rm s} \binom{N_{\rm w}}{2}} \sum_{i=1}^{N_{\rm s}} \sum_{j=1}^{N_{\rm w}} \sum_{k=j+1}^{N_{\rm w}} |m_{i,j} - m_{i,k}|$$

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## Music Lab Experiment

## Sensible result:

Stronger social signal leads to greater following and greater inequality.

## Peculiar result:

 Stronger social signal leads to greater unpredictability.

## Very peculiar observation:

- The most unequal distributions would suggest the greatest variation in underlying 'quality.'
- But success may be due to social construction through following...

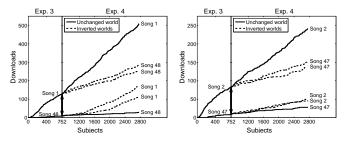
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## Music Lab Experiment—Sneakiness



- Inversion of download count
- The 'pretend rich' get richer ...
- ... but at a slower rate

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