Models of Complex Networks

Santa Fe Institute Summer School, 2009

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Modeling Complex Networks
Random networks
Basics
Configuration model
Scale-free networks
History
BA model
Redner & Krapivisky's model
Robustness
Small-world networks
References

Outline

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Some important models:

1. Generalized random networks
2. Scale-free networks (⊞)
3. Small-world networks (⊞)
4. Statistical generative models (p^*)
5. Generalized affiliation networks

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   - Arbitrary degree distribution P_k.
   - Wire nodes together randomly.
   - Create ensemble to test deviations from randomness.
   - Interesting, applicable, rich mathematically.
   - Much fun to be had with these guys...

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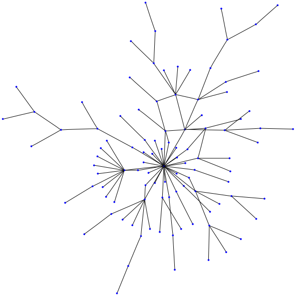

γ = 2.5
$\langle k \rangle$ = 1.8
$N = 150$

- Due to Barabasi and Albert [2]
- Generative model
- Preferential attachment model with growth
- P[attachment to node i] $\propto \mathrm{k}_\mathrm{i}^{\alpha}$.
- Produces $P_k \sim k^{-\gamma}$ when $\alpha = 1$.
- Trickiness: other models generate skewed degree distributions...

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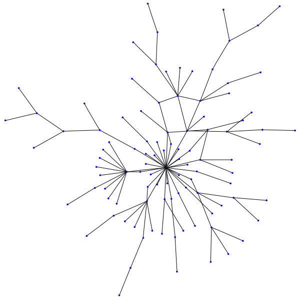

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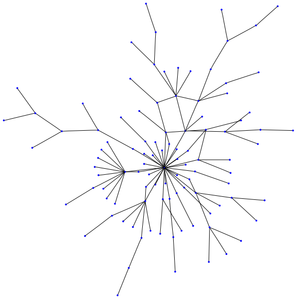

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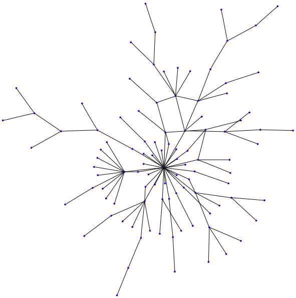

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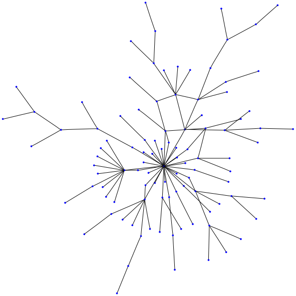

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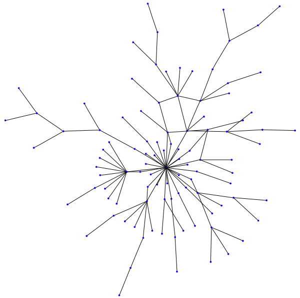

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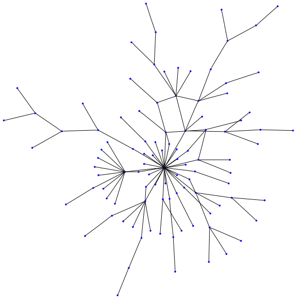

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3. Small-world networks

- Due to Watts and Strogatz [18]
- local regularity (high clustering—an individual's friends know each other)
- global randomness (shortcuts).

Strong effects:

- Shortcuts make world 'small'
- Shortcuts allow disease to jump
- Facilitates synchronization [7]

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Two scales:

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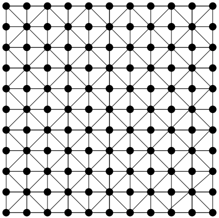

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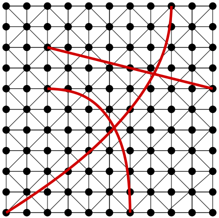

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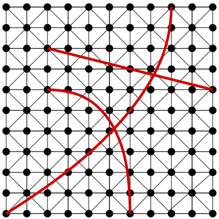

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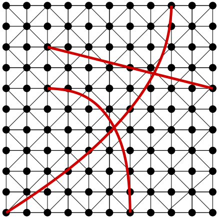

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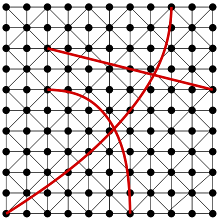

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4. Generative statistical models

- Idea is to realize networks based on certain tendencies:
  - Clustering (triadic closure)..
  - Types of nodes that like each other..
  - Anything really...
- Use statistical methods to estimate 'best' values of parameters.
- Drawback: parameters are not real, measurable quantities.
- Non-mechanistic and blackboxish.
- c.f., temperature in statistical mechanics.

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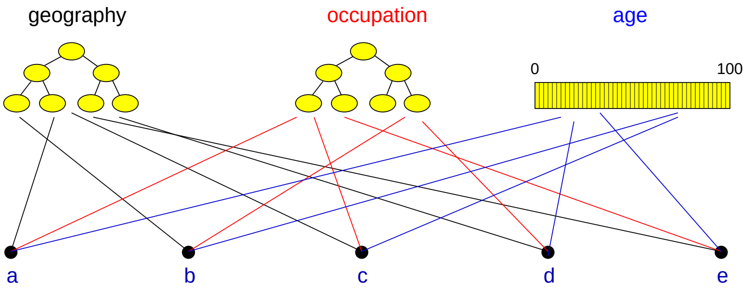

- Blau & Schwartz [3], Simmel [15], Breiger [4], Watts *et al.* [17]

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Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Horribly, there are $\binom{\binom{N}{2}}{m}$ of them.
- Standard random network = randomly chosen network from this set.
- To be clear: each network is equally probable.
- Known as Erdős-Rényi random networks
- Key structural feature of random networks is that they locally look like branching networks
- (No small cycles and zero clustering).

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Next slides:

Example realizations of random networks

- $N = 500$
- Vary m, the number of edges from 100 to 1000.
- Average degree $\langle k \rangle$ runs from 0.4 to 4.
- Look at full network plus the largest component.

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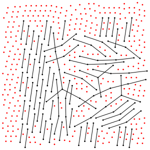
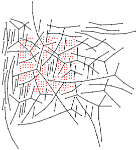
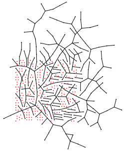


m = 100
$\langle k \rangle$ = 0.4



m = 200
$\langle k \rangle$ = 0.8

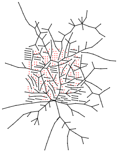
m = 230
$\langle k \rangle$ = 0.92



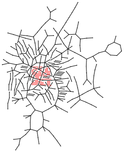
m = 240
$\langle k \rangle$ = 0.96

m = 250
$\langle k \rangle$ = 1

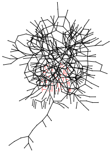
m = 260
$\langle k \rangle$ = 1.04



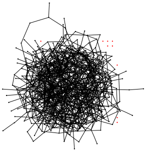
m = 280
$\langle k \rangle$ = 1.12



m = 300
$\langle k \rangle$ = 1.2



m = 500
$\langle k \rangle$ = 2



m = 1000
$\langle k \rangle$ = 4

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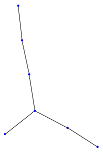
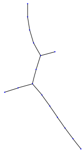
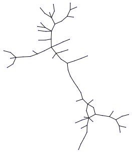


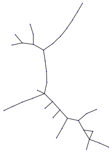
$m = 100$
$\langle k \rangle = 0.4$



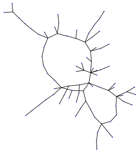
$m = 200$
$\langle k \rangle = 0.8$



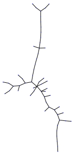
$m = 230$
$\langle k \rangle = 0.92$



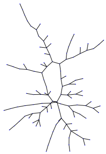
$m = 240$
$\langle k \rangle = 0.96$



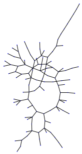
$m = 250$
$\langle k \rangle = 1$



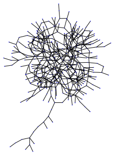
$m = 260$
$\langle k \rangle = 1.04$



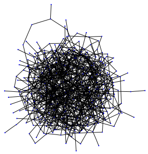
$m = 280$
$\langle k \rangle = 1.12$



$m = 300$
$\langle k \rangle = 1.2$

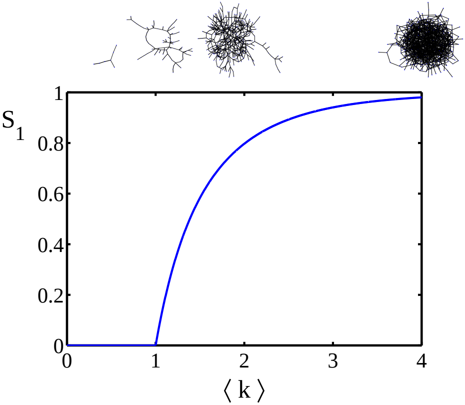


$m = 500$
$\langle k \rangle = 2$



$m = 1000$
$\langle k \rangle = 4$

Giant component:



- S_1 = fraction of nodes in largest component.
- Old school phase transition.
- Key idea in modeling contagion.

Properties

But:

- Erdős-Rényi random networks are a *mathematical construct*.
- Real networks are a microscopic subset of all networks...
- ex: 'Scale-free' networks are growing networks that form according to a plausible mechanism.

But but:

- Randomness is out there, just not to the degree of a completely random network.

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- Configuration model

Scale-free networks
- History
- BA model
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General random networks

- So... standard random networks have a Poisson degree distribution
- Can happily generalize to arbitrary degree distribution P_k.
- Also known as the configuration model.[11]
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution and form links with probability

$$P(\text{link between } i \text{ and } j) \propto w_i w_j.$$

- A more useful way:
  1. Randomly wire up (and rewire) already existing nodes with fixed degrees.
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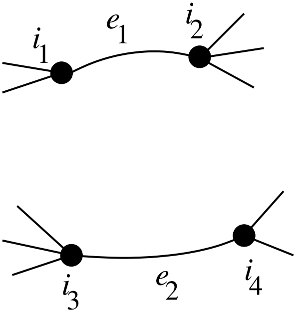
General random rewiring algorithm



- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.
- Rewire one end of each edge.
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- Works if e_1 is a self-loop or repeated edge.
- Same as finding on/off/on/off 4-cycles. and rotating them.

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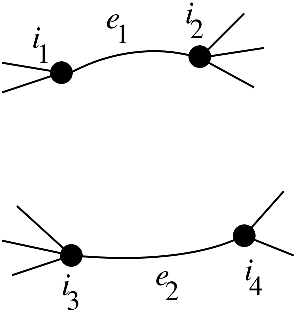

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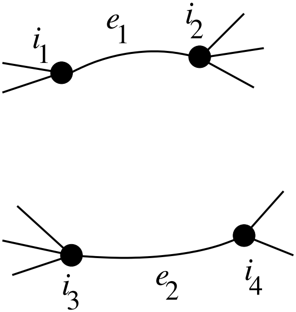

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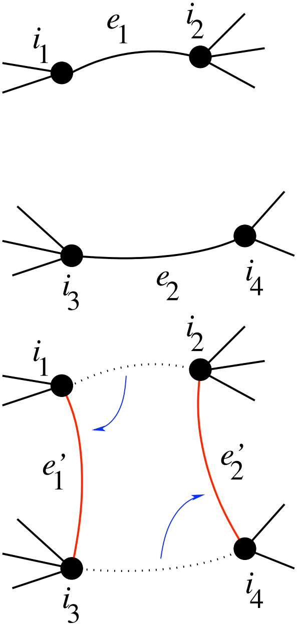

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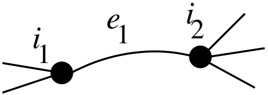

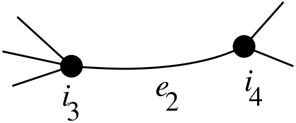

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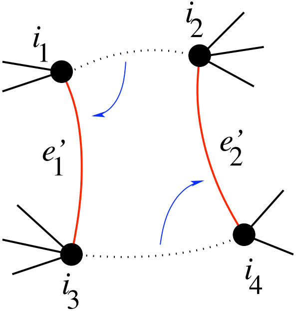

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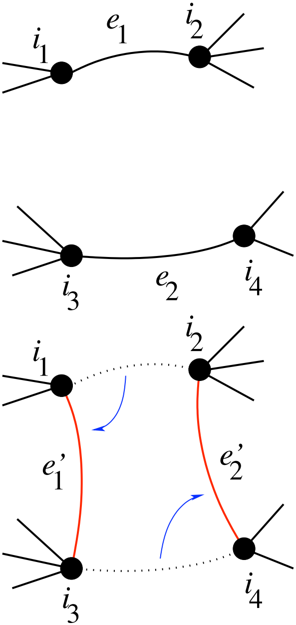

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Random networks: examples

Next slides:

Example realizations of random networks with power law degree distributions:

- $N = 1000$.
- $P_k \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_0 = 0$ (no isolated nodes).
- Vary exponent γ between 2.10 and 2.91.
- Apart from degree distribution, wiring is random.

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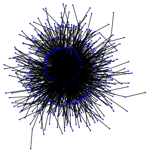
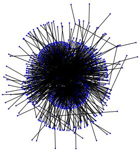
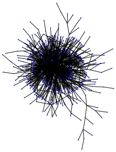
Random networks: largest components



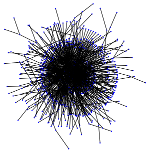
γ = 2.1
$\langle k \rangle$ = 3.448



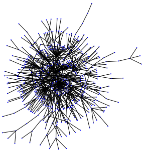
γ = 2.19
$\langle k \rangle$ = 2.986



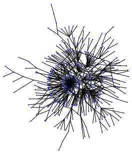
γ = 2.28
$\langle k \rangle$ = 2.306



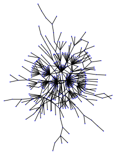
γ = 2.37
$\langle k \rangle$ = 2.504



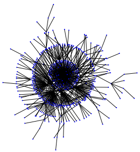
γ = 2.46
$\langle k \rangle$ = 1.856



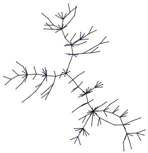
γ = 2.55
$\langle k \rangle$ = 1.712



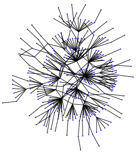
γ = 2.64
$\langle k \rangle$ = 1.6



γ = 2.73
$\langle k \rangle$ = 1.862



γ = 2.82
$\langle k \rangle$ = 1.386



γ = 2.91
$\langle k \rangle$ = 1.49

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The edge-degree distribution:

- The degree distribution P_k is fundamental for our description of many complex networks
- A related key distribution:
  R_k = probability that a friend of a random node has k other friends.
- $$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$
- Natural question: what's the expected number of other friends that one friend has?
- Find
  $$\langle k \rangle_R = \frac{1}{\langle k \rangle}\left(\langle k^2 \rangle - \langle k \rangle\right)$$
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$$\boxed{\langle k \rangle_R = \frac{1}{\langle k \rangle}\left(\langle k^2 \rangle - \langle k \rangle\right) > 1}$$

then our random network has a giant component.

- Exponential explosion in number of nodes as we move out from a random node.
- Number of nodes expected at n steps:

$$\langle k \rangle \cdot \langle k \rangle_R^{n-1} = \frac{1}{\langle k \rangle^{n-2}}\left(\langle k^2 \rangle - \langle k \rangle\right)^{n-1}$$

- We'll see this again for contagion models...

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$$\langle k \rangle \cdot \langle k \rangle_R^{n-1} = \frac{1}{\langle k \rangle^{n-2}} \left(\langle k^2 \rangle - \langle k \rangle \right)^{n-1}$$

- We'll see this again for contagion models...

Giant component condition

- If:

$$\boxed{\langle k \rangle_R = \frac{1}{\langle k \rangle}\left(\langle k^2 \rangle - \langle k \rangle\right) > 1}$$

then our random network has a giant component.

- Exponential explosion in number of nodes as we move out from a random node.

- Number of nodes expected at n steps:

$$\langle k \rangle \cdot \langle k \rangle_R^{n-1} = \frac{1}{\langle k \rangle^{n-2}}\left(\langle k^2 \rangle - \langle k \rangle\right)^{n-1}$$

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Mild weirdness...

- Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k^2 \rangle - \langle k \rangle.$$

- Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.
- Three peculiarities:
  1. We might guess $\langle k_2 \rangle = \langle k \rangle(\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.
  2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big.
  3. Your friends have more friends than you...

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Size distributions

The sizes of many systems' elements appear to obey an inverse power-law size distribution:

$$P(\text{size} = x) \sim c\, x^{-\gamma}$$

where $x_{\min} < x < x_{\max}$ and $\gamma > 1$.

- x can be continuous or discrete.
- Typically, $2 < \gamma < 3$.
- No dominant internal scale between $x_{\min}$ and $x_{\max}$.
- If $\gamma < 3$, variance and higher moments are 'infinite'
- If $\gamma < 2$, mean and higher moments are 'infinite'
- Negative linear relationship in log-log space:

$$\log P(x) = \log c - \gamma \log x$$

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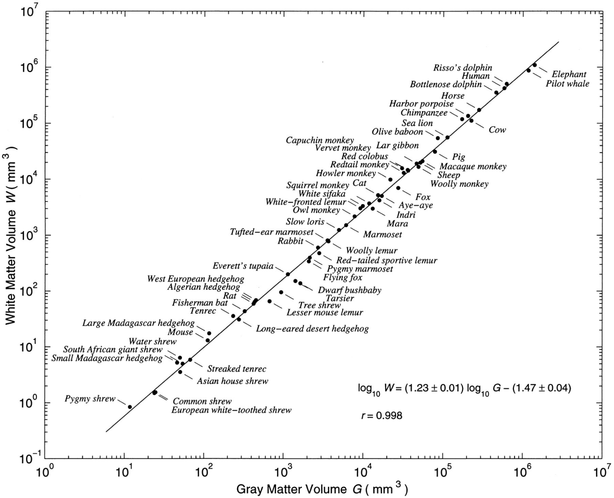
A beautiful, heart-warming example:



from Zhang & Sejnowski, PNAS (2000) [20]

$\alpha \simeq 1.23$

gray matter: 'computing elements'

white matter: 'wiring'

Size distributions

Power law size distributions are sometimes called Pareto distributions (⊞) after Italian scholar Vilfredo Pareto.

- Pareto noted wealth in Italy was distributed unevenly (80–20 rule).
- Term used especially by economists

Size distributions

Examples:

- Earthquake magnitude (Gutenberg Richter law): $P(M) \propto M^{-3}$
- Number of war deaths: $P(d) \propto d^{-1.8}$ [14]
- Sizes of forest fires
- Sizes of cities: $P(n) \propto n^{-2.1}$
- Number of links to and from websites

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Size distributions

Examples:

- Number of citations to papers: $P(k) \propto k^{-3}$.
- Individual wealth (maybe): $P(W) \propto W^{-2}$.
- Distributions of tree trunk diameters: $P(d) \propto d^{-2}$.
- The gravitational force at a random point in the universe: $P(F) \propto F^{-5/2}$.
- Diameter of moon craters: $P(d) \propto d^{-3}$.
- Word frequency: e.g., $P(k) \propto k^{-2.2}$ (variable)

Note: Exponents range in error;
see M.E.J. Newman arxiv.org/cond-mat/0412004v3 (⊞)

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- Random Additive/Copying Processes involving Competition.
- Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)
- Competing mechanisms (more trickiness)

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Essential Extract of a Growth Model

Random Competitive Replication (RCR):

1. Start with 1 element of a particular flavor at $t = 1$
2. At time $t = 2, 3, 4, \ldots$, add a new element in one of two ways:
   - With probability ρ, create a new element with a new flavor
   - With probability $1 - \rho$, randomly choose from all existing elements, and make a copy.
   - Elements of the same flavor form a group

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Random Competitive Replication

Example: Words in a text

- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
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- Please note: authors do not do this...

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- Yule's paper (1924) [19]:
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Merton was a catchphrase machine:

1. self-fulfilling prophecy
2. role model
3. unintended (or unanticipated) consequences
4. focused interview $\rightarrow$ focus group

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And just to rub it in...

Merton's son, Robert C. Merton, won the Nobel Prize for Economics in 1997.

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- Basic idea: a new node arrives every discrete time step and connects to an existing node i with probability $\propto k_i$.
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  Groups of a single flavor $\sim$ edges of a node
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- Networks with power-law degree distributions have become known as scale-free networks.
- Scale-free refers specifically to the degree distribution having a power-law decay in its tail:

$$P_k \sim k^{-\gamma} \text{ for 'large' } k$$

- Please note: not every network is a scale-free network...

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- Term ‘scale-free’ is somewhat confusing...
- Scale-free networks are not fractal in any sense.
- Usually talking about networks whose links are abstract, relational, informational, . . . (non-physical)
- Main reason is link cost.
- Primary example: hyperlink network of the Web
- Much arguing about whether or networks are ‘scale-free’ or not. . .

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The big deal:

- We move beyond describing networks to finding mechanisms for why certain networks arise.

A big deal for scale-free networks:

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From Barabási and Albert's original paper [2]:

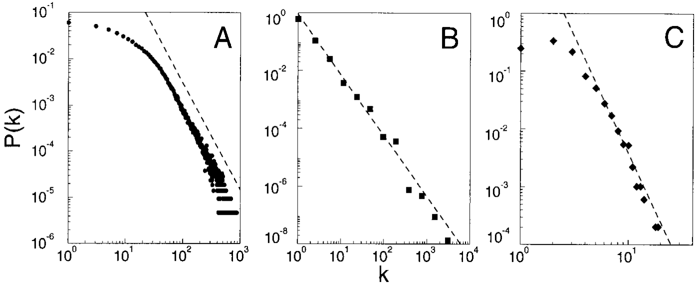

Fig. 1. The distribution function of connectivities for various large networks. (**A**) Actor collaboration graph with N = 212,250 vertices and average connectivity $\langle k \rangle$ = 28.78. (**B**) WWW, N = 325,729, $\langle k \rangle$ = 5.46 (6). (**C**) Power grid data, N = 4941, $\langle k \rangle$ = 2.67. The dashed lines have slopes (A) γ_{actor} = 2.3, (B) γ_{www} = 2.1 and (C) γ_{power} = 4.

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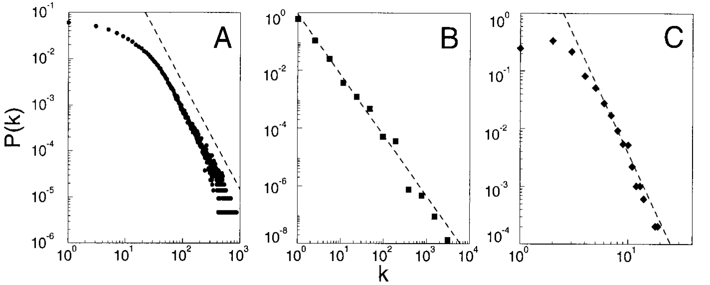

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Generalized model

Fooling with the mechanism:

- 2001: Redner & Krapivsky (RK) [8] explored the general attachment kernel:

$$\mathbf{Pr}(\text{attach to node } i) \propto A_k = k_i^{\nu}$$

where A_k is the attachment kernel and $\nu > 0$.

- RK also looked at changing very subtle details of the attachment kernel.
- e.g., keep $A_k \sim k$ for large k but tweak A_k for low k.
- RK's approach is to use rate equations (⊞).

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- Consider $A_1 = \alpha$ and $A_k = k$ for $k \geq 2$.
- Some unsettling calculations leads to $P_k \sim k^{-\gamma}$ where

$$\gamma = 1 + \frac{1 + \sqrt{1 + 8\alpha}}{2}.$$

- We then have

$$0 \leq \alpha < \infty \Rightarrow 2 \leq \gamma < \infty$$

- Craziness...

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$$0 \leq \alpha < \infty \Rightarrow 2 \leq \gamma < \infty$$
- Craziness...

Sublinear attachment kernels

- Rich-get-somewhat-richer:

$$A_k \sim k^{\nu} \text{ with } 0 < \nu < 1.$$

- General finding by Krapivsky and Redner:[8]

$$P_k \sim k^{-\nu} e^{-c_1 k^{1-\nu} + \text{correction terms}}.$$

- Weibull distribution*ish* (truncated power laws).
- Universality: now details of kernel do not matter.

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$$A_k \sim k^{\nu} \text{ with } \nu > 1.$$

- Now a winner-take-all mechanism.
- One single node ends up being connected to almost all other nodes.
- For $\nu > 2$, all but a finite # of nodes connect to one node.

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Outline

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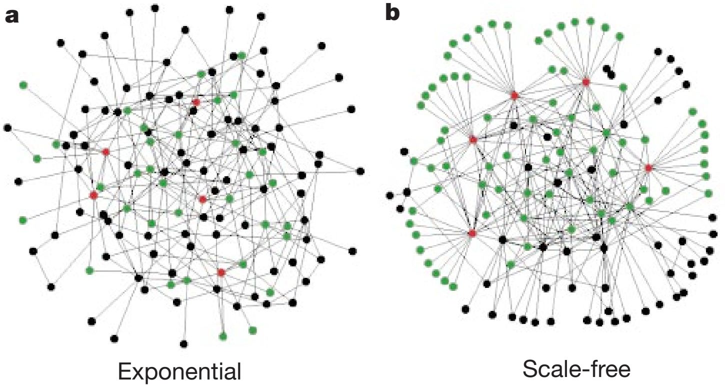
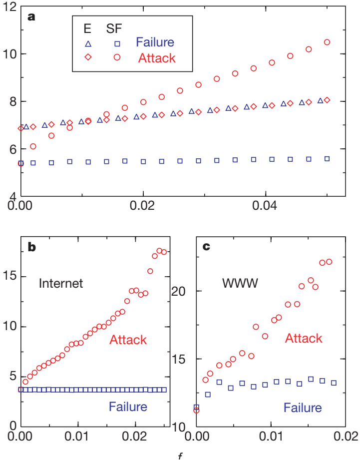
- Standard random networks (Erdős-Rényi) versus Scale-free networks



from Albert et al., 2000 "Error and attack tolerance of complex networks" [1]

Robustness



from Albert et al., 2000

- Plots of network diameter as a function of fraction of nodes removed
- Erdős-Rényi versus scale-free networks
- blue symbols = random removal
- red symbols = targeted removal (most connected first)

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- Scale-free networks are thus robust to random failures yet fragile to targeted ones.
- All very reasonable: Hubs are a big deal.
- But: next issue is whether hubs are vulnerable or not.
- Representing all webpages as the same size node is obviously a stretch (e.g., google vs. a random person's webpage)
- Most connected nodes are either:
  1. Physically larger nodes that may be harder to 'target'
  2. or subnetworks of smaller, normal-sized nodes.
- Need to explore cost of various targeting schemes.

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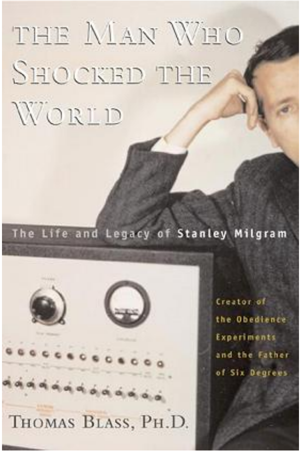
Milgram’s social search experiment (1960s)



http://www.stanleymilgram.com

- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- chain length $\simeq$ 6.5.

Popular terms:

- The Small World Phenomenon;
- “Six Degrees of Separation.”

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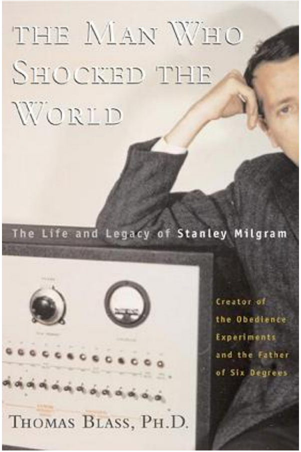

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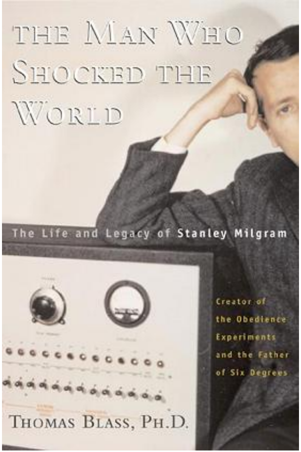

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Milgram's experiment with e-mail [5]

Participants:

- 60,000+ people in 166 countries
- 24,000+ chains
- Big media boost...

18 targets in 13 countries including

- a professor at an Ivy League university,
- an archival inspector in Estonia,
- a technology consultant in India,
- a policeman in Australia,
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Social search—the Columbia experiment

The world is smaller:

- $\langle L \rangle = 4.05$ for all completed chains
- L_* = Estimated 'true' median chain length (zero attrition)

- Intra-country chains: $L_* = 5$
- Inter-country chains: $L_* = 7$
- All chains: $L_* = 7$

- c.f. Milgram (zero attrition): $L_* \simeq 9$

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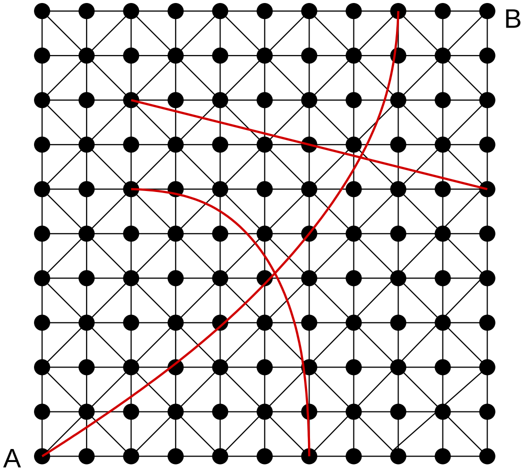
Randomness + regularity



$d_{AB} = 10$ without random paths
$d_{AB} = 3$ with random paths

$\langle d \rangle$ decreases overall

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Theory of Small-World networks

Introduced by
Watts and Strogatz (Nature, 1998) [18]
"Collective dynamics of 'small-world' networks."

Small-world networks are found everywhere:

- neural network of C. elegans,
- semantic networks of languages,
- actor collaboration graph,
- food webs,
- social networks of comic book characters,...

Very weak requirements:

- local regularity + random short cuts

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Nodes cannot find each other quickly
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- So what's special about social networks?

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The model

One approach: incorporate identity.

(See "Identity and Search in Social Networks." Science, 2002, Watts, Dodds, and Newman [17])

Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Religious beliefs
- Recreational activities.

Groups are formed by people with at least one similar attribute.

Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks.

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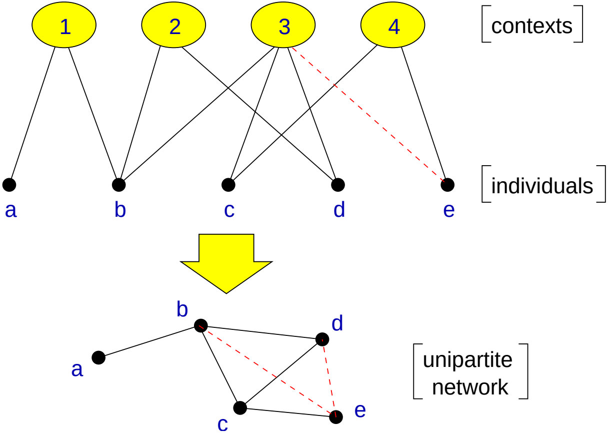
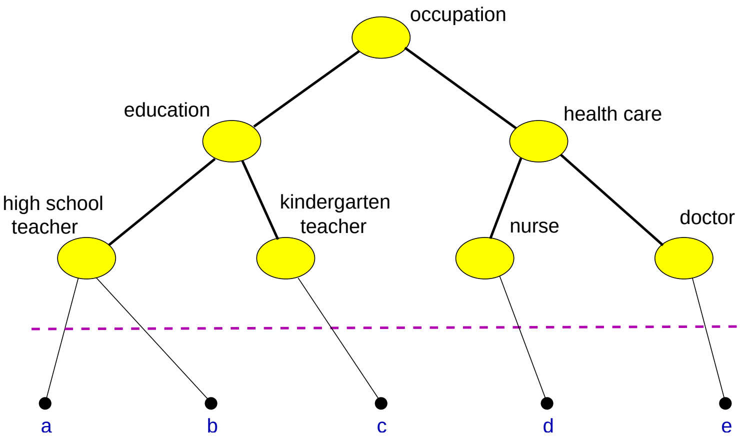
Social distance—Bipartite affiliation networks

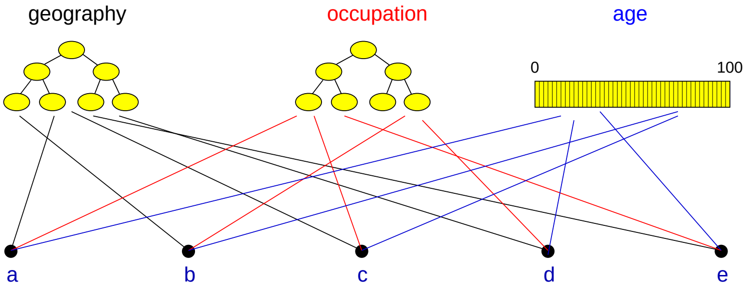


Bipartite affiliation networks: boards and directors, movies and actors.

Social distance as a function of identity



Homophily



(Blau & Schwartz, Simmel, Breiger)

- Networks built with 'birds of a feather...' are searchable.
- Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks.

Social Search—Real world uses

- Tagging: e.g., Flickr induces a network between photos
- Search in organizations for solutions to problems
- Peer-to-peer networks
- Synchronization in networked systems
- Motivation for search matters...

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