The Small-World Phenomenon

Complex Networks, Course 303A, Spring, 2009

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Outline

Some problems for sociologists

How are social networks structured?

- How do we define connections?
- How do we measure connections?
- (remote sensing, self-reporting)

What about the dynamics of social networks?

- How do social networks evolve?
- How do social movements begin?
- How does collective problem solving work?
- How is information transmitted through social networks?

Social Search

A small slice of the pie:

- Q. Can people pass messages between distant individuals using only their existing social connections?
- A. Apparently yes...

Handles:

- The Small World Phenomenon
- or “Six Degrees of Separation.”

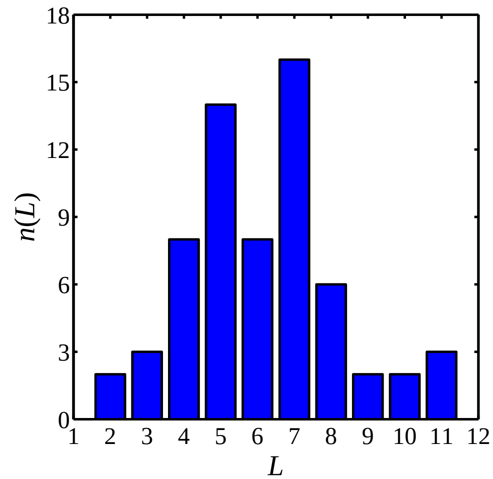
The problem

Stanley Milgram et al., late 1960's:

- Target person worked in Boston as a stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- average chain length $\simeq 6.5$.

The problem

Lengths of successful chains:



From Travers and Milgram (1969) in Sociometry:[4] "An Experimental Study of the Small World Problem."

The problem

Two features characterize a social 'Small World':

1. Short paths exist

   and
2. People are good at finding them.

Social Search

Milgram's small world experiment with e-mail [2]

Social search—the Columbia experiment

- 60,000+ participants in 166 countries
- 18 targets in 13 countries including
  - a professor at an Ivy League university,
  - an archival inspector in Estonia,
  - a technology consultant in India,
  - a policeman in Australia,
    and
  - a veterinarian in the Norwegian army.
- 24,000+ chains

Social search—the Columbia experiment

- Milgram's participation rate was roughly 75%
- Email version: Approximately 37% participation rate.
- Probability of a chain of length 10 getting through:

$$.37^{10} \simeq 5 \times 10^{-5}$$

- $\Rightarrow$ 384 completed chains (1.6% of all chains).

Social search—the Columbia experiment

- Motivation/Incentives/Perception matter.
- If target *seems* reachable
  $\Rightarrow$ participation more likely.
- Small changes in attrition rates
  $\Rightarrow$ large changes in completion rates
- e.g., $\searrow$ 15% in attrition rate
  $\Rightarrow \nearrow$ 800% in completion rate

Social search—the Columbia experiment

Successful chains disproportionately used

- weak ties (Granovetter)
- professional ties (34% vs. 13%)
- ties originating at work/college
- target's work (65% vs. 40%)

. . . and disproportionately avoided

- hubs (8% vs. 1%) (+ no evidence of funnels)
- family/friendship ties (60% vs. 83%)

Geography $\rightarrow$ Work

Social search—the Columbia experiment

Senders of successful messages showed little absolute dependency on

- age, gender
- country of residence
- income
- religion
- relationship to recipient

Range of completion rates for subpopulations:
30% to 40%

Social search—the Columbia experiment

Nevertheless, some weak discrepencies do exist...

An above average connector:

Norwegian, secular male, aged 30-39, earning over $100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

A below average connector:

Italian, Islamic or Christian female earning less than $2K, with elementary school education and retired, who uses strong ties to family members.

The Small-World Phenomenon

History
An online experiment
Previous theoretical work
An improved model
References

Social search—the Columbia experiment

Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain."

Why:

- Specificity important
- Successful links used relevant information. (e.g. connecting to someone who shares same profession as target.)

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Basic results:

- $\langle L \rangle = 4.05$ for all completed chains
- L_* = Estimated 'true' median chain length (zero attrition)
- Intra-country chains: $L_* = 5$
- Inter-country chains: $L_* = 7$
- All chains: $L_* = 7$
- Milgram: $L_* \simeq 9$

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Previous work—short paths

- Connected random networks have short average path lengths:

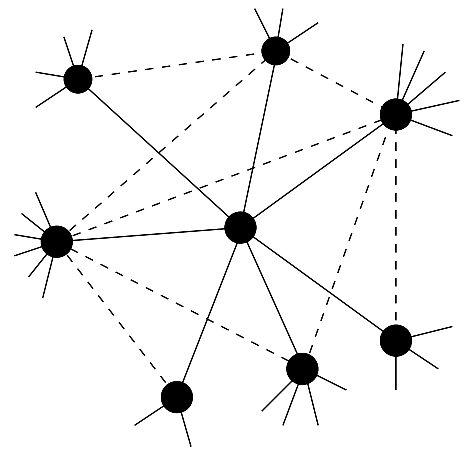
$$\langle d_{AB} \rangle \sim \log(N)$$

N = population size,

d_{AB} = distance between nodes A and B.

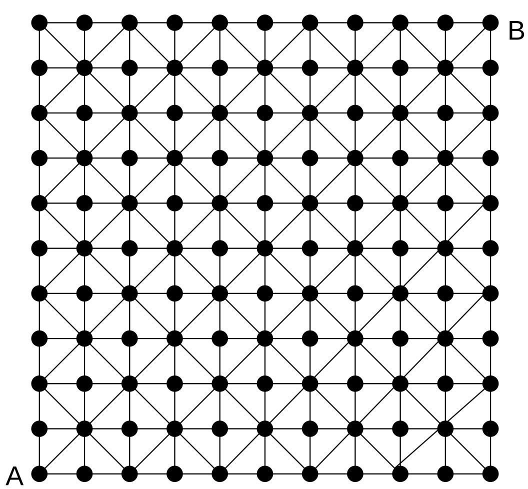
- But: social networks aren't random...

Previous work—short paths



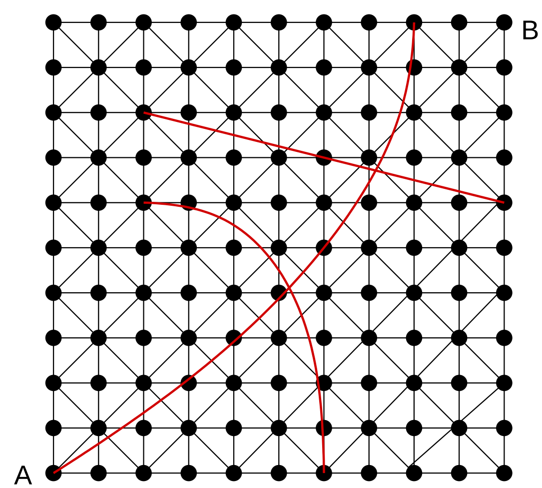
Need "clustering" (your friends are likely to know each other):

Non-randomness gives clustering



$d_{AB} = 10 \rightarrow$ too many long paths.

Randomness + regularity



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall

Small-world networks

Introduced by
Watts and Strogatz (Nature, 1998) [6]
"Collective dynamics of 'small-world' networks."

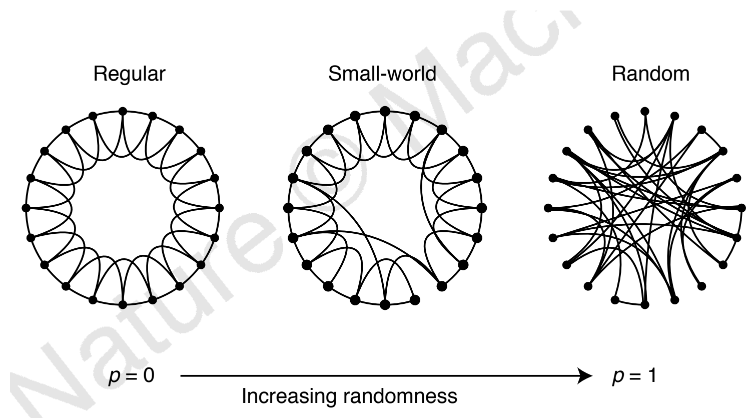
Small-world networks were found everywhere:

- neural network of C. elegans,
- semantic networks of languages,
- actor collaboration graph,
- food webs,
- social networks of comic book characters,...

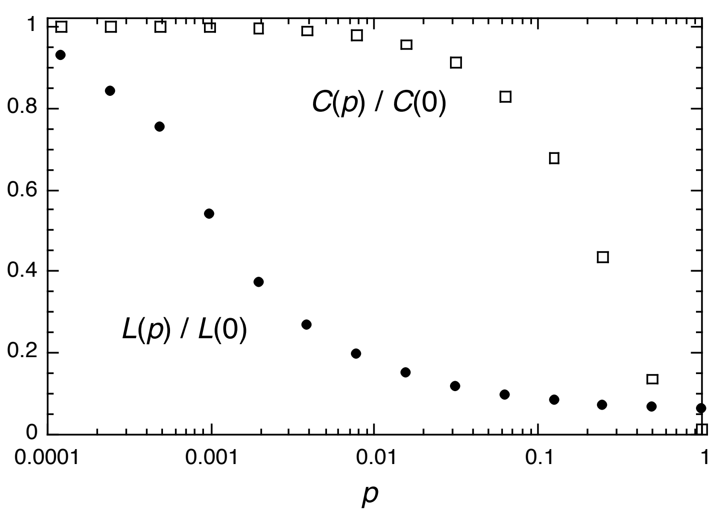
Very weak requirements:

- local regularity + random short cuts

Toy model



The structural small-world property



Previous work—finding short paths

But are these short cuts findable?

No.

Nodes cannot find each other quickly
with any local search method.

Previous work—finding short paths

- What can a local search method reasonably use?
- How to find things without a map?
- Need some measure of distance between friends and the target.

Some possible knowledge:

- Target's identity
- Friends' popularity
- Friends' identities
- Where message has been

Previous work—finding short paths

Jon Kleinberg (Nature, 2000) [3]
"Navigation in a small world."

Allowed to vary:

1. local search algorithm
   and
2. network structure.

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Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d-dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q.
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto d_{ij}^{-\alpha}.$$

- $\alpha = 0$: random connections.
- α large: reinforce local connections.
- $\alpha = d$: same number of connections at all scales.

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Previous work—finding short paths

Theoretical optimal search:

- "Greedy" algorithm.
- Same number of connections at all scales: $\alpha = d$.

Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.

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Previous work—finding short paths

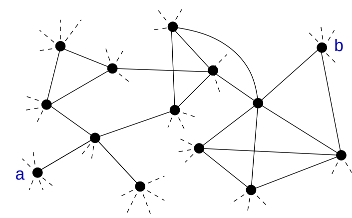
- If networks have hubs can also search well: Adamic et al. (2001) [1]

$$P(k_i) \propto k_i^{-\gamma}$$

  where k = degree of node i (number of friends).
- Basic idea: get to hubs first (airline networks).
- But: hubs in social networks are limited.

The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of a is closest to the target b?

What does ‘closest’ mean?

What is ‘social distance’?

The model

One approach: incorporate identity.
(See “Identity and Search in Social Networks.” Science, 2002, Watts, Dodds, and Newman [5])

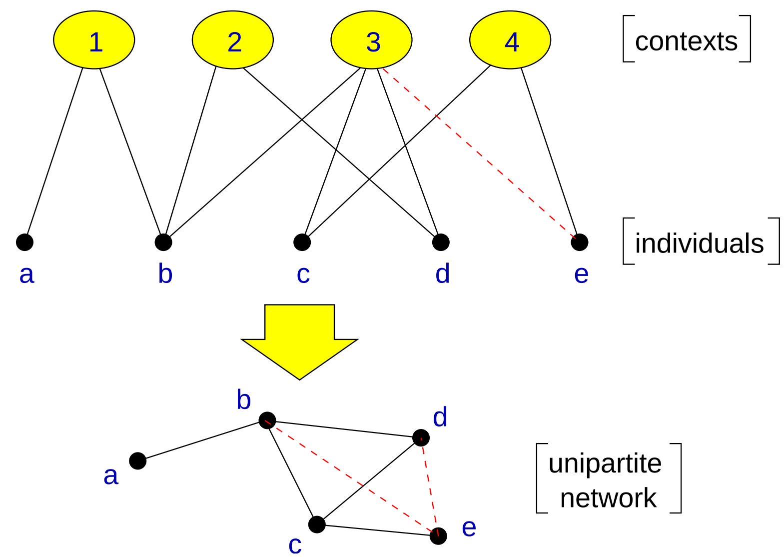
Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Religious beliefs
- Recreational activities.

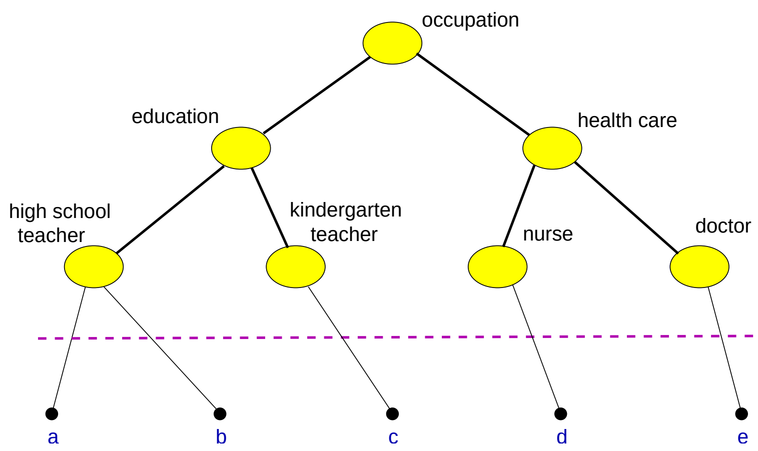
Groups are formed by people with at least one similar attribute.

Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks.

Social distance—Bipartite affiliation networks

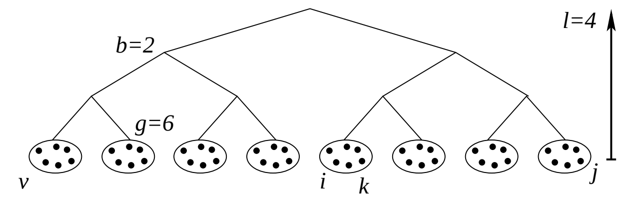


Social distance—Context distance



The model

Distance between two individuals x_{ij} is the height of lowest common ancestor.



$x_{ij} = 3$, $x_{ik} = 1$, $x_{iv} = 4$.

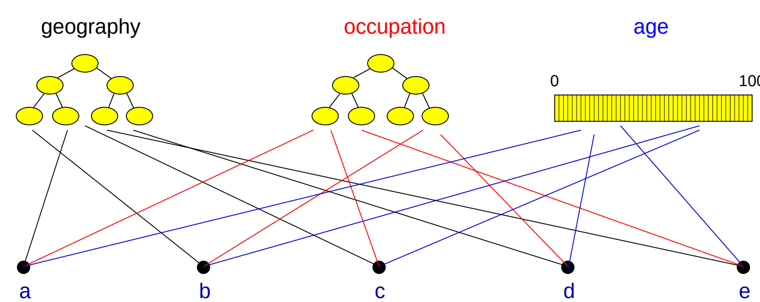
The model

- Individuals are more likely to know each other the closer they are within a hierarchy.
- Construct z connections for each node using

$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$

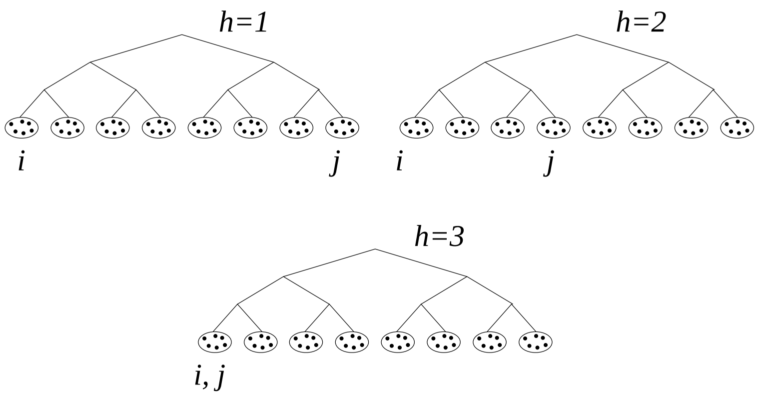
- $\alpha = 0$: random connections.
- α large: local connections.

Social distance—Generalized context space



(Blau & Schwartz, Simmel, Breiger)

The model



$\vec{v}_i = [1\ 1\ 1]^T$, $\vec{v}_j = [8\ 4\ 1]^T$

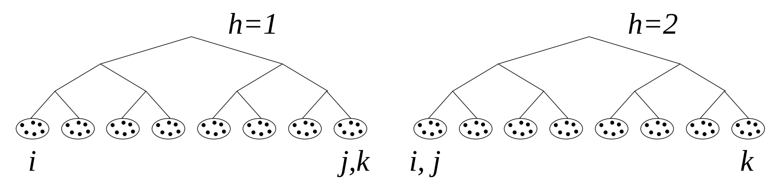
$x_{ij}^1 = 4$, $x_{ij}^2 = 3$, $x_{ij}^3 = 1$.

Social distance:

$$y_{ij} = \min_h x_{ij}^h.$$

The model

Triangle inequality doesn't hold:



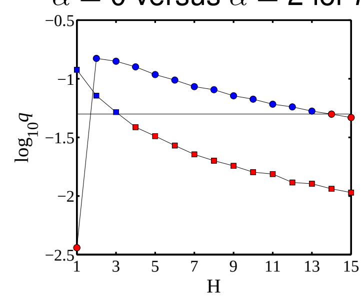
$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$

The model

- Individuals know the identity vectors of
  1. themselves,
  2. their friends,
     and
  3. the target.
- Individuals can estimate the social distance between their friends and the target.
- Use a greedy algorithm + allow searches to fail randomly.

The model-results—searchable networks

$\alpha = 0$ versus $\alpha = 2$ for $N \simeq 10^5$:



$q \geq r$

$q < r$

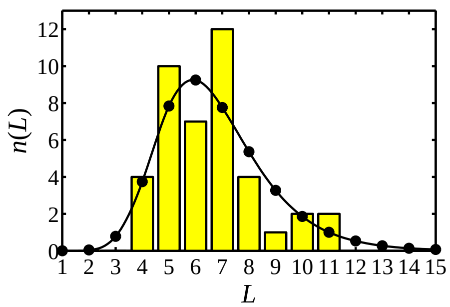
$r = 0.05$

q = probability an arbitrary message chain reaches a target.

- A few dimensions help.
- Searchability decreases as population increases.
- Precise form of hierarchy largely doesn't matter.

The model-results

Milgram's Nebraska-Boston data:



Model parameters:

- $N = 10^8$,
- $z = 300$, $g = 100$,
- $b = 10$,
- $\alpha = 1$, $H = 2$;

- $\langle L_{\text{model}} \rangle \simeq 6.7$
- $L_{\text{data}} \simeq 6.5$

Social search—Data

Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Probability of connection as function of real distance $\propto 1/r$.

Social Search—Real world uses

- Tags create identities for objects
- Website tagging: `http://www.del.icio.us`
- (e.g., Wikipedia)
- Photo tagging: `http://www.flickr.com`
- Dynamic creation of metadata plus links between information objects.
- Folksonomy: collaborative creation of metadata

Social Search—Real world uses

Recommender systems:

- Amazon uses people's actions to build effective connections between books.
- Conflict between 'expert judgments' and tagging of the hoi polloi.

Conclusions

- Bare networks are typically unsearchable.
- Paths are findable if nodes understand how network is formed.
- Importance of identity (interaction contexts).
- Improved social network models.
- Construction of peer-to-peer networks.
- Construction of searchable information databases.

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