Applications of Random Networks

Complex Networks, Course 303A, Spring, 2009

Prof. Peter Dodds

Department of Mathematics & Statistics
University of Vermont

Outline

More on building random networks

- Problem: How much of a real network's structure is non-random?
- Key elephant in the room: the degree distribution P_k.
- First observe departure of P_k from a Poisson distribution.
- Next: measure the departure of a real network with a degree frequency N_k from a random network with the same degree frequency.
- Degree frequency N_k = observed frequency of degrees for a real network.
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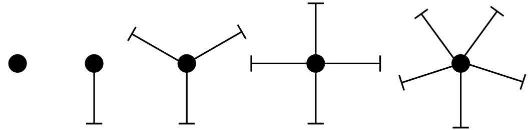
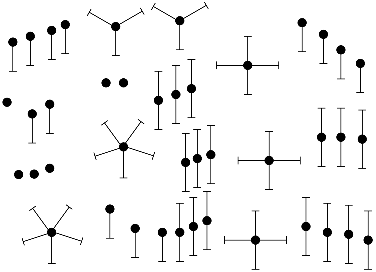
Building random networks: Stubs

Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):





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- Must have an even number of stubs.
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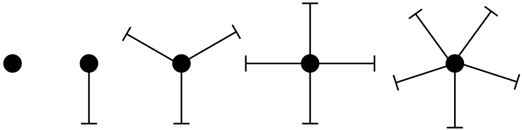

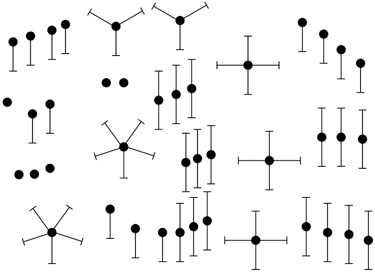

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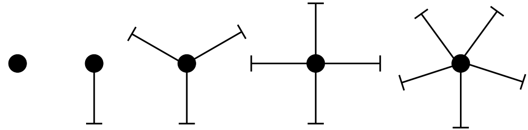

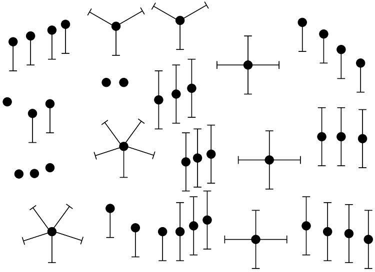

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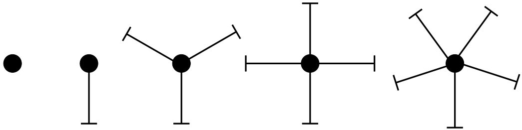

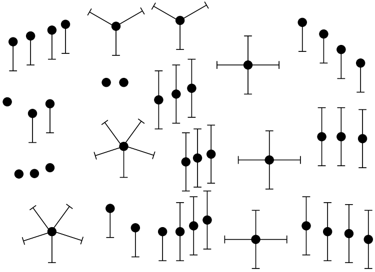

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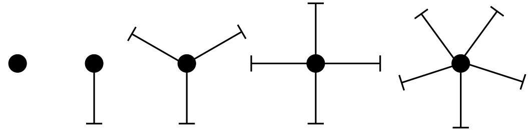

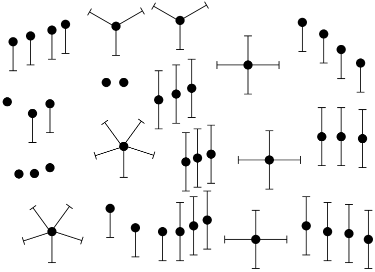

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Building random networks: First rewiring

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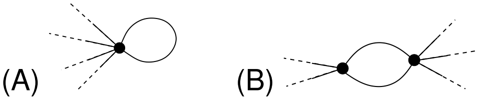
- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



- Being careful: we can't change the degree of any node, so we can't simply move links around.
- Simplest solution: randomly rewire two edges at a time.

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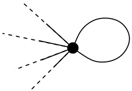
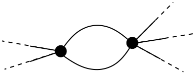
(A)  (B) 

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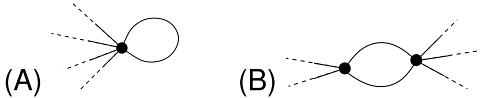

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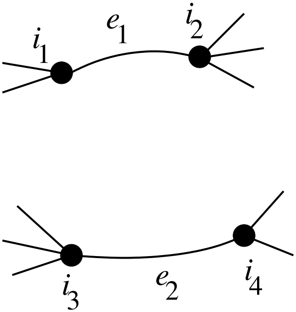
General random rewiring algorithm



- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.
- Rewire one end of each edge.
- Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.
- Same as finding on/off/on/off 4-cycles. and rotating them.

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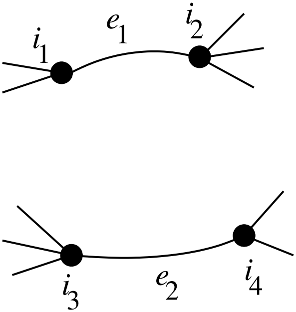

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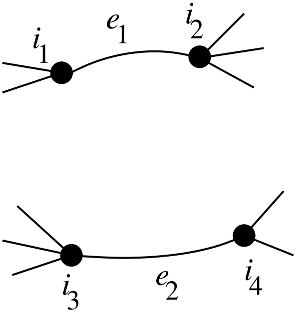

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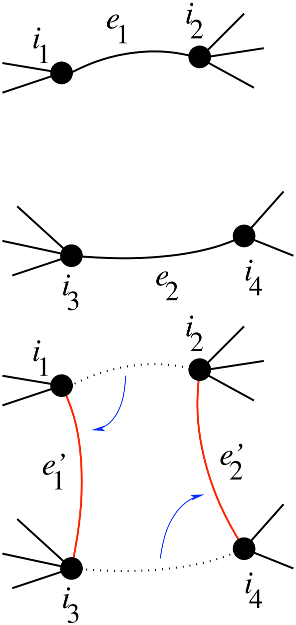

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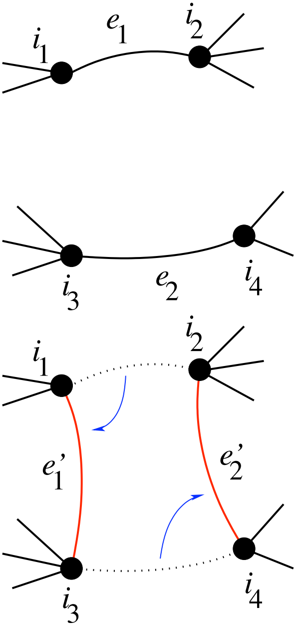

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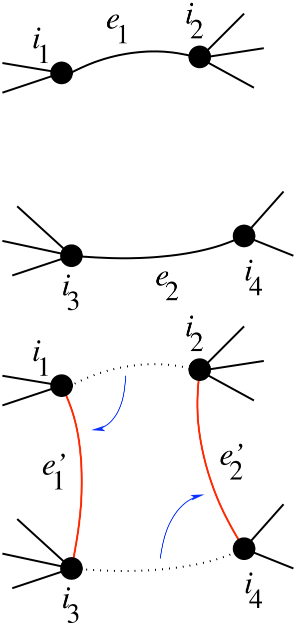

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Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

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- Randomize network wiring by applying rewiring algorithm liberally.
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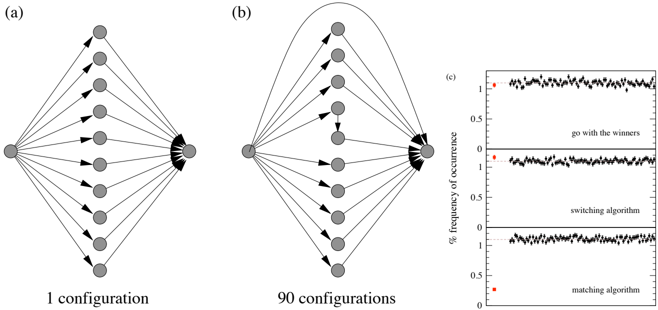

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- What if we have P_k instead of N_k?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k.
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Analysis of real networks
How to build revisited
Motifs

References

Network motifs

- Idea of motifs [2] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
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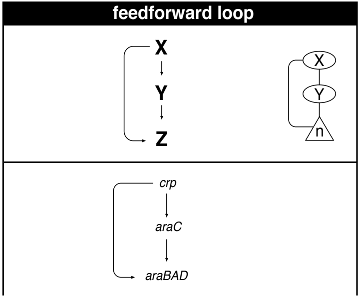

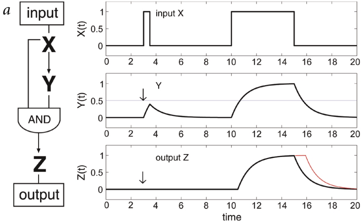

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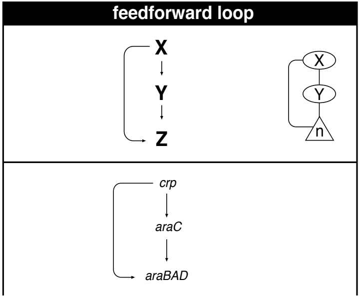

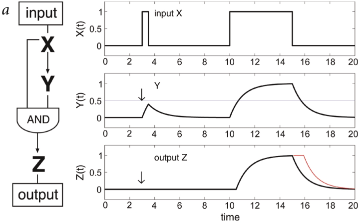

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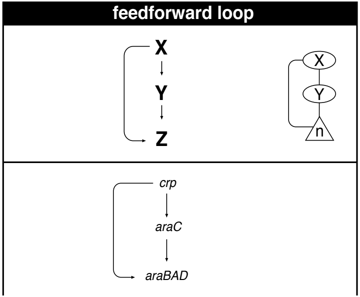

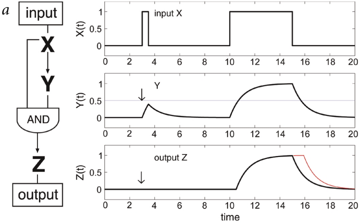

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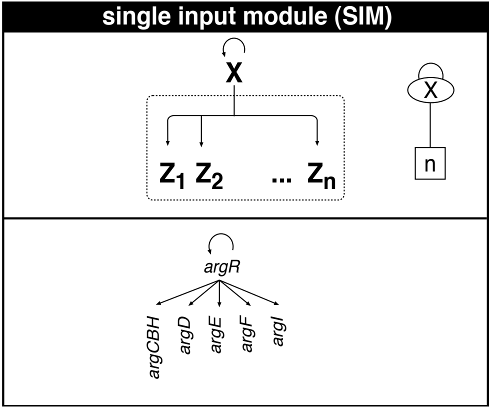

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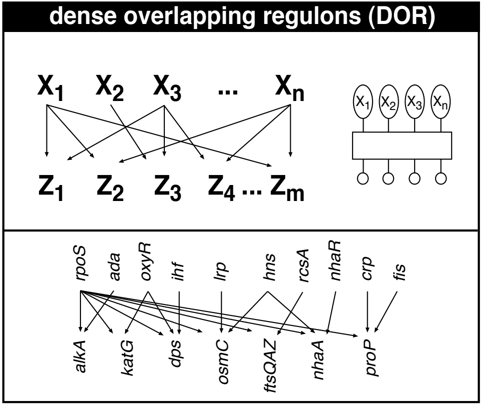

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[1] R. Milo, N. Kashtan, S. Itzkovitz, M. E. J. Newman, and U. Alon.
On the uniform generation of random graphs with prescribed degree sequences, 2003. pdf (⊞)

[2] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon.
Network motifs in the transcriptional regulation network of *Escherichia coli*.
Nature Genetics, pages 64–68, 2002. pdf (⊞)