# Random Networks Complex Networks, Course 303A, Spring, 2009

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## Random Networks

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Structure Clustering Degree distributions Configuration model Largest component

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## Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = randomly chosen network from this set.
- ▶ To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

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# Some features:

Number of possible edges:

$$0 \le m \le \binom{N}{2} = rac{N(N-1)}{2}$$

- Given *m* edges, there are 
   <sup>N</sup>
   <sup>(N)</sup>
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- Crazy factorial explosion for  $1 \ll m \ll \binom{N}{2}$ .
- Limit of m = 0: empty graph.
- Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph.
- Real world: links are usually costly so real networks are almost always sparse.

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## Given N and m.

- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the  $\binom{N}{2}$  pairs with appropriate probability *p*.
  - Useful for theoretical work.
- 2. Take *N* nodes and add exactly *m* links by selecting edges without replacement.
  - Algorithm: Randomly choose a pair of nodes i and j, i ≠ j, and connect if unconnected; repeat until all m edges are allocated.
  - Best for adding small numbers of links (most cases).
  - 1 and 2 are effectively equivalent for large N

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# A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{2}{N}p\frac{1}{2}N(N-1)=p(N-1).$$

- Which is what it should be...
- If we keep  $\langle k \rangle$  constant then  $p \propto 1/N \to 0$  as  $N \to \infty$ .

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# Random networks: examples

## Next slides: Example realizations of random networks

- ► *N* = 500
- ▶ Vary *m*, the number of edges from 100 to 1000.
- Average degree  $\langle k \rangle$  runs from 0.4 to 4.
- Look at full network plus the largest component.

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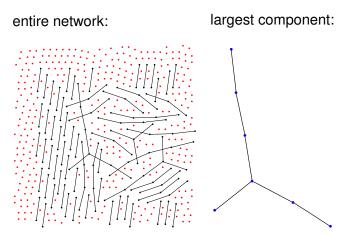
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N = 500, number of edges m = 100average degree  $\langle k \rangle = 0.4$ 

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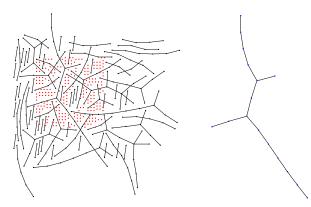
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entire network:



N = 500, number of edges m = 200average degree  $\langle k \rangle = 0.8$ 

largest component:

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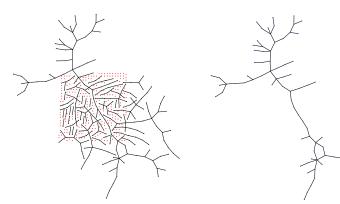
Structure Clustering Degree distributions Configuration model Largest component

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References

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entire network:



N = 500, number of edges m = 230average degree  $\langle k \rangle = 0.92$ 

largest component:

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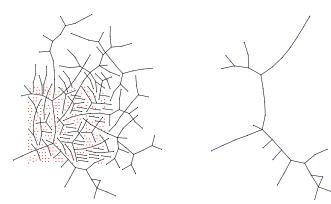
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entire network:



N = 500, number of edges m = 240average degree  $\langle k \rangle = 0.96$ 

largest component:

### Random Networks

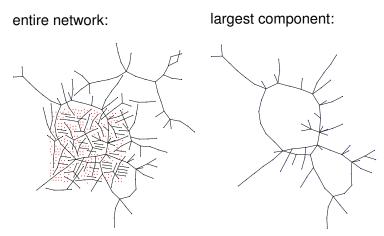
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N = 500, number of edges m = 250average degree  $\langle k \rangle = 1$ 

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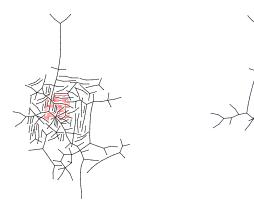
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entire network:



N = 500, number of edges m = 260average degree  $\langle k \rangle = 1.04$ 

largest component:

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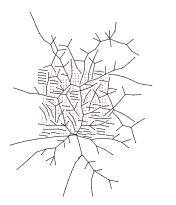
References

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entire network:



largest component:

N = 500, number of edges m = 280average degree  $\langle k \rangle = 1.12$ 

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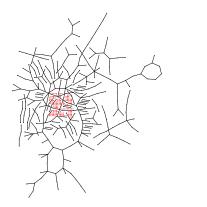
Structure Clustering Degree distributions Configuration model Largest component

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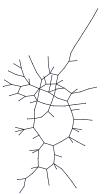
References

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entire network:



largest component:



N = 500, number of edges m = 300average degree  $\langle k \rangle = 1.2$  Random Networks

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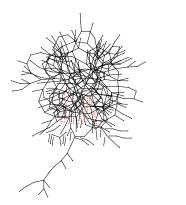
Structure Clustering Degree distributions Configuration model Largest component

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entire network:



largest component:

N = 500, number of edges m = 500average degree  $\langle k \rangle = 2$ 

### Random Networks

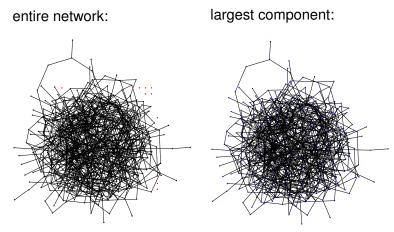
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N = 500, number of edges m = 1000average degree  $\langle k \rangle = 4$ 

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### Random networks: examples for N=500









*m* = 250  $\langle k \rangle = 1$ 



Some visual examples

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m = 230 $\langle k \rangle = 0.92$ 







m = 260 $\langle k \rangle = 1.04$ 

m = 280 $\langle k \rangle = 1.12$ 



m = 300

 $\langle k \rangle = 1.2$ 



m = 500 $\langle k \rangle = 2$ 

m = 1000 $\langle k \rangle = 4$ 

### Random Networks

### Random networks: largest components



m = 100 $\langle k \rangle = 0.4$ 



m = 200 $\langle k \rangle = 0.8$ 





m = 240 $\langle k \rangle = 0.96$ 

m = 250 $\langle k \rangle = 1$ 



 $\begin{array}{l}m=1000\\\langle k\rangle=4\end{array}$ 

#### Random Networks

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m = 260 $\langle k \rangle = 1.04$ 

m = 280 $\langle k \rangle = 1.12$ 

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m = 300 $\langle k \rangle = 1.2$ 

m = 500

 $\langle k \rangle = 2$ 

### Random networks: examples for N=500







m = 250

 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 

m = 250

 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 

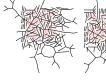


m = 250 $\langle k \rangle = 1$  m = 250 $\langle k \rangle = 1$ 





m = 250 $\langle k \rangle = 1$ 



*m* = 250

 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 

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### Random networks: largest components





m = 250 $\langle k \rangle = 1$  m = 250 $\langle k \rangle = 1$ 





m = 250 $\langle k \rangle = 1$ 

*m* = 250

 $\langle k \rangle = 1$ 







 $\langle k \rangle = 1$ 

m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 

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### Clustering

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### Clustering:

- For method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient (Newman<sup>[1]</sup>):

 $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ 

- Recall: C<sub>2</sub> = probability that two nodes are connected given they have a friend in common.
- For standard random networks, we have simply that

 $C_2 = p$ 

### Random Networks

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### Random Networks

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### **Clustering:**

- So for large random networks (N → ∞), clustering drops to zero.
- Key structural feature of random networks is that they locally look like branching networks (no loops).

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### Degree distribution:

- Recall p<sub>k</sub> = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N − 1 choose k' ways the node can be connected to k of the other N − 1 nodes.
- ► Each connection occurs with probability *p*, each non-connection with probability (1 *p*).
- Therefore have a binomial distribution:

$$P(k;p,N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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### Limiting form of P(k; p, N):

- Our degree distribution:  $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}$
- What happens as  $N \to \infty$ ?
- We must end up with the normal distribution right?
- If p is fixed, then we would end up with a Gaussian with average degree ⟨k⟩ ≃ pN → ∞.
- But we want to keep  $\langle k \rangle$  fixed...
- ▶ So examine limit of P(k; p, N) when  $p \to 0$  and  $N \to \infty$  with  $\langle k \rangle = p(N-1) = \text{constant}$ .

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Substitute  $p = \frac{\langle k \rangle}{N-1}$  into P(k; p, N) and hold k fixed:

$$P(k; p, N) = \binom{N-1}{k} \left(\frac{\langle k \rangle}{N-1}\right)^k \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)(N-2)\cdots(N-k)}{k!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

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$$=\frac{(N-1)(N-2)\cdots(N-k)}{k!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{N^{k}(1-\frac{1}{N})\cdots(1-\frac{k}{N})}{k!N^{k}}\frac{\langle k\rangle^{k}}{(1-\frac{1}{N})^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

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$$=\frac{(N-1)(N-2)\cdots(N-k)}{k!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{\cancel{k}}{\cancel{k!}}\frac{(1-\frac{1}{N})\cdots(1-\frac{k}{N})}{\cancel{k!}}\frac{\langle k\rangle^{k}}{(1-\frac{1}{N})^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

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$$=\frac{(N-1)!}{k!(N-1-k)!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$=\frac{(N-1)(N-2)\cdots(N-k)}{k!}\frac{\langle k\rangle^k}{(N-1)^k}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

$$\simeq \frac{\mathcal{M}^{k}(1-\frac{1}{N})\cdots(1-\frac{k}{N})}{k!\mathcal{M}^{k}}\frac{\langle k\rangle^{k}}{(1-\frac{1}{N})^{k}}\left(1-\frac{\langle k\rangle}{N-1}\right)^{N-1-k}$$

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We are now here:

$$P(k; p, N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k}$$

Now use the excellent result:

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=e^x$$

(Use l'Hôpital's rule to prove.)

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$$P(k;\langle k\rangle) \simeq \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle} \left(1 - \frac{\langle k\rangle}{N-1}\right)^{-k} \to \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle}$$

▶ This is a Poisson distribution ( $\boxplus$ ) with mean  $\langle k \rangle$ .

### Random Networks

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- Generalize to arbitrary degree distribution  $P_k$ .
- Also known as the configuration model<sup>[1]</sup>.
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P<sub>w</sub> and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$ 

- But we'll be more interested in
  - Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
  - Examining mechanisms that lead to networks with certain degree distributions.

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## Coming up:

Example realizations of random networks with power law degree distributions:

- ► *N* = 1000.
- $P_k \propto k^{-\gamma}$  for  $k \ge 1$ .
- Set  $P_0 = 0$  (no isolated nodes).
- Vary exponent  $\gamma$  between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

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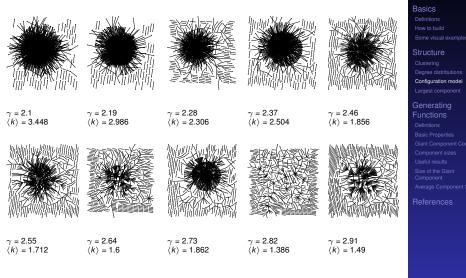
### Random Networks

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# Random networks: examples for N=1000



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### Random Networks

# Random networks: largest components











 $\gamma = 2.1$  $\langle k \rangle = 3.448$ 

 $\gamma = 2.55$ 

(k) = 1.712

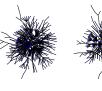
 $\gamma = 2.19$  $\langle k \rangle = 2.986$ 

 $\gamma = 2.64$ 

 $\langle k \rangle = 1.6$ 

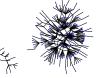


 $\gamma = 2.37$  $\langle k \rangle = 2.504$   $\begin{array}{l} \gamma = 2.46 \\ \langle k \rangle = 1.856 \end{array}$ 









 $\gamma = 2.73$  $\langle k \rangle = 1.862$   $\gamma = 2.82$  $\langle k \rangle = 1.386$   $\gamma = 2.91$  $\langle k \rangle = 1.49$ 

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Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

► Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

$$=e^{-\langle k
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$$=e^{-\langle k
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Random Networks

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$$= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!}$$

$$= \langle \boldsymbol{k} \rangle \boldsymbol{e}^{-\langle \boldsymbol{k} \rangle} \sum_{k=1}^{\infty} \frac{\langle \boldsymbol{k} \rangle^{k-1}}{(k-1)!}$$

$$= \langle k \rangle e^{-\langle k \rangle} \sum_{i=0}^{\infty} \frac{\langle k \rangle^i}{i!} = \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} = \langle k \rangle \checkmark$$

We'll get to a better way of doing this...

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- The variance of degree distributions for random networks turns out to be very important.
- Use calculation similar to one for finding (k) to find the second moment:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Variance is then

 $\sigma^{2} = \langle \boldsymbol{k}^{2} \rangle - \langle \boldsymbol{k} \rangle^{2} = \langle \boldsymbol{k} \rangle^{2} + \langle \boldsymbol{k} \rangle - \langle \boldsymbol{k} \rangle^{2} = \langle \boldsymbol{k} \rangle.$ 

- So standard deviation  $\sigma$  is equal to  $\sqrt{\langle k \rangle}$ .
- Note: This is a special property of Poisson distribution and can trip us up...

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- Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes
- ▶ Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):



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Normalized form:

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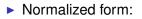
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 $Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_k}{\langle k \rangle}.$ 

 $Q_k \propto k P_k$ 

### Random Networks

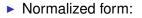
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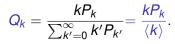
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- For random networks, Q<sub>k</sub> is also the probability that a friend (neighbor) of a random node has k friends.
- Useful variant on  $Q_k$ :

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

- Equivalent to friend having degree k + 1.
- Natural question: what's the expected number of other friends that one friend has?

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Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle \mathbf{k} \rangle_{\mathbf{R}} = \sum_{\mathbf{k}=0}^{\infty} \mathbf{k} \mathbf{R}_{\mathbf{k}} = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k-1}}{\langle k \rangle}$$
$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1)P_{k+1}$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}\left((k+1)^2-(k+1)\right)P_{k+1}$$

(where we have sneakily matched up indices)

$$=rac{1}{\langle k
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- For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

► Therefore:

$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left( \langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

- Again, neatness of results is a special property of the Poisson distribution.
- So friends on average have  $\langle k \rangle$  other friends, and  $\langle k \rangle + 1$  total friends...

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## The edge-degree distribution:

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Therefore:

$$\langle k \rangle_{R} = \frac{1}{\langle k \rangle} \left( \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

- Again, neatness of results is a special property of the Poisson distribution.
- So friends on average have  $\langle k \rangle$  other friends, and  $\langle k \rangle + 1$  total friends...

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## The edge-degree distribution:

- ► Note: our result, \langle k \rangle\_R = \frac{1}{\langle k \rangle} \left( \langle k^2 \rangle \langle k \rangle \right), is true for all random networks, independent of degree distribution.
- For standard random networks, recall

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## Reason #1:

Average # friends of friends per node is

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- Key: Average depends on the 1st and 2nd moments of P<sub>k</sub> and not just the 1st moment.
- ► Three peculiarities:
  - 1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$  but it's actually  $\langle k(k-1) \rangle$ .
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## More on peculiarity #3:

A node's average # of friends: (k)

- Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$
- ► Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left( 1 + \frac{\sigma^2}{\langle k \rangle^2} \right)$$

- So only if everyone has the same degree (variance= σ<sup>2</sup> = 0) can a node be the same as if friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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## (Big) Reason #2:

- ⟨k⟩<sub>R</sub> is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As N → ∞, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- ▶ Defn: Giant component = component that comprises a non-zero fraction of a network as  $N \rightarrow \infty$ .
- Note: Component = Cluster

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## Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring  $\langle k \rangle_R > 1$ .
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = rac{\langle k^2 
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- Again, see that the second moment is an essential part of the story.
- Equivalent statement:  $\langle k^2 \rangle > 2 \langle k \rangle$

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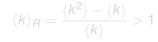
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- Giant component condition (or percolation condition):



- Again, see that the second moment is an essential part of the story.
- Equivalent statement:  $\langle k^2 \rangle > 2 \langle k \rangle$

#### Random Networks

#### Basics Definitions How to build Some visual examples

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Frame 49/89

## Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
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#### Random Networks

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SQC2

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- Equivalent statement: (k<sup>2</sup>) > 2(k)

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SQC2

## Standard random networks:

• Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- ► Therefore when ⟨k⟩ > 1, standard random networks have a giant component.
- When  $\langle k \rangle < 1$ , all components are finite.
- Fine example of a continuous phase transition  $(\boxplus)$ .
- We say  $\langle k \rangle = 1$  marks the critical point of the system.

#### Random Networks

#### Basics Definitions How to build Some visual examples

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### Random Networks

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Frame 50/89

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Random networks with skewed  $P_k$ :

• e.g, if  $P_k = ck^{-\gamma}$  with 2 <  $\gamma$  < 3 then

$$\langle k^2 
angle = c \sum_{k=0}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=0}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma}\Big|_{x=0}^{\infty} = \infty \quad (>\langle k \rangle).$$

- So giant component always exists for these kinds of networks.
- ► Cutoff scaling is k<sup>-3</sup>: if γ > 3 then we have to look harder at ⟨k⟩<sub>R</sub>.

### Random Networks

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### Random Networks

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### Random Networks

#### Basics Definitions How to build Some visual examples

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### And how big is the largest component?

- Define S<sub>1</sub> as the size of the largest component.
- Consider an infinite ER random network with average degree (k).
- ▶ Let's find S<sub>1</sub> with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection:  $\delta = 1 S_1$ .
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.
- ► So

$$\delta = \sum_{k=0}^{\infty} \boldsymbol{P}_k \delta^k$$

Substitute in Poisson distribution...

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SQC2

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References

Carrying on:



$$=e^{-\langle k
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$$= e^{-\langle k 
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▶ Now substitute in  $\delta = 1 - S_1$  and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}$$

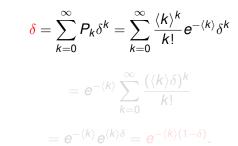
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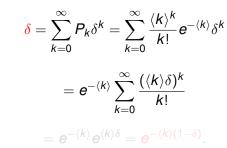
#### Random Networks

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$$\boldsymbol{\delta} = \sum_{k=0}^{\infty} \boldsymbol{P}_k \boldsymbol{\delta}^k = \sum_{k=0}^{\infty} \frac{\langle \boldsymbol{k} \rangle^k}{k!} \boldsymbol{e}^{-\langle \boldsymbol{k} \rangle} \boldsymbol{\delta}^k$$

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We can figure out some limits and details for S<sub>1</sub> = 1 − e<sup>-⟨k⟩S<sub>1</sub></sup>.

First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}$$

• As 
$$\langle k \rangle \rightarrow 0$$
,  $S_1 \rightarrow 0$ .

• As 
$$\langle k \rangle \to \infty$$
,  $S_1 \to 1$ .

▶ Notice that at  $\langle k \rangle = 1$ , the critical point,  $S_1 = 0$ .

- Only solvable for S > 0 when  $\langle k \rangle > 1$
- Really a transcritical bifurcation<sup>[2]</sup>.

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#### Random Networks

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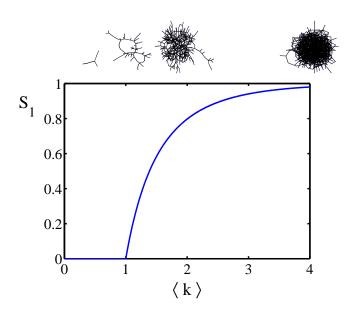
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### Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because  $\langle k \rangle = \langle k \rangle_R$ .
- We need a separate probability δ' for the chance that a node at the end of a random edge is part of the largest component.
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- Idea: Given a sequence a<sub>0</sub>, a<sub>1</sub>, a<sub>2</sub>,..., associate each element with a distinct function or other mathematical object.
- Well-chosen functions allow us to manipulate sequences and retrieve sequence elements.

### Definition:

• The generating function (g.f.) for a sequence  $\{a_n\}$  is

$$F(x) = \sum_{n=0}^{\infty} a_n x^n.$$

- Roughly: transforms a vector in R<sup>∞</sup> into a function defined on R<sup>1</sup>.
- Related to Fourier, Laplace, Mellin, ...

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### Simple example

### Rolling dice:

$$F^{(\Box)}(x) = \sum_{k=1}^{6} p_k x^k = \frac{1}{6} (x + x^2 + x^3 + x^4 + x^5 + x^6).$$

We'll come back to this simple example as we derive various delicious properties of generating functions.

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$$P_k = ce^{-\lambda k}$$

where  $c = 1 - e^{-\lambda}$ .

The generating function for this distribution is

$$F(x) = \sum_{k=0}^{\infty} P_k x^k = \sum_{k=0}^{\infty} c e^{-\lambda k} x^k = \frac{c}{1 - x e^{-\lambda k}}$$

- Notice that  $F(1) = c/(1 e^{-\lambda}) = 1$ .
- For probability distributions, we must always have F(1) = 1 since

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### Generating Functions

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**Component sizes** Average Component Size

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**Basics** 

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Average degree:

$$\langle \mathbf{k} \rangle = \sum_{k=0}^{\infty} \mathbf{k} \mathbf{P}_{\mathbf{k}} = \sum_{k=0}^{\infty} \mathbf{k} \mathbf{P}_{k} x^{k-1} \bigg|_{x=1}$$
$$= \frac{d}{dx} F(x) \bigg|_{x=1} = F'(1)$$

- In general, many calculations become simple, if a little abstract.
- For our exponential example:

$$F'(x) = \frac{(1 - e^{-\lambda})e^{-\lambda}}{(1 - xe^{-\lambda})^2}.$$

So:

$$\langle k \rangle = F'(1) = rac{e^{-\lambda}}{(1-e^{-\lambda})}.$$

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### Useful pieces for probability distributions:

Normalization:

First moment:

$$\langle k \rangle = F'(1)$$

► Higher moments:

 $\langle k^n \rangle = \left( x \frac{\mathrm{d}}{\mathrm{d}x} \right)^n F(x) \Big|_{x=1}$ 

kth element of sequence (general):

$$P_k = \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}x^k} F(x) \Big|_{x=1}$$

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### Useful pieces for probability distributions:

Normalization:

F(1) = 1

First moment:

 $\langle k \rangle = F'(1)$ 

► Higher moments:

 $\langle k^n \rangle = \left( x \frac{\mathrm{d}}{\mathrm{d}x} \right)^n F(x) \Big|_{x=1}$ 

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### Edge-degree distribution

Recall our condition for a giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

- We first need the g.f. for  $R_k$ .
- We'll now use this notation:

 $F_P(x)$  is the g.f. for  $P_k$ .  $F_B(x)$  is the g.f. for  $R_k$ .

Condition in terms of g.f. is:

 $\langle k \rangle_R = F'_R(1) > 1.$ 

▶ Now find how *F<sub>R</sub>* is related to *F<sub>P</sub>*...

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SQC2

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We have

$$F_{R}(x) = \sum_{k=0}^{\infty} \frac{R_{k}x^{k}}{k} = \sum_{k=0}^{\infty} \frac{(k+1)P_{k+1}}{\langle k \rangle} x^{k}.$$

Shift index to j = k + 1 and pull out  $\frac{1}{\langle k \rangle}$ :

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Finally, since  $\langle k \rangle = F'_P(1)$ ,

$$F_R(x) = \frac{F'_P(x)}{F'_P(1)}$$

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# **Basics**

Giant Component Condition

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- Recall giant component condition is  $\langle k \rangle_R = F'_R(1) > 1.$
- Since we have  $F_R(x) = F'_P(x)/F'_P(1)$ ,





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Setting x = 1, our condition becomes



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To figure out the size of the largest component  $(S_1)$ , we need more resolution on component sizes.

### **Definitions:**

- π<sub>n</sub> = probability that a random node belongs to a finite component of size n < ∞.</p>
- ρ<sub>n</sub> = probability a random link leads to a finite subcomponent of size n < ∞.</p>

Local-global connection:

 $P_k, R_k \Leftrightarrow \pi_n, 
ho_n$ neighbors  $\Leftrightarrow$  components

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SQC2

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### G.f.'s for component size distributions:

$$F_{\pi}(x) = \sum_{n=0}^{\infty} \pi_n x^n$$
 and  $F_{\rho}(x) = \sum_{n=0}^{\infty} \rho_n x^n$ 

### The largest component:

Subtle key: F<sub>π</sub>(1) is the probability that a node belongs to a finite component.

• Therefore: 
$$S_1 = 1 - F_{\pi}(1)$$
.

### Our mission, which we accept:

Find the four generating functions

 $F_P, F_R, F_\pi$ , and  $F_\rho$ .

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G.f.'s for component size distributions:

$$F_{\pi}(x) = \sum_{n=0}^{\infty} \pi_n x^n$$
 and  $F_{\rho}(x) = \sum_{n=0}^{\infty} \rho_n x^n$ 

### The largest component:

Subtle key: F<sub>π</sub>(1) is the probability that a node belongs to a finite component.

• Therefore:  $S_1 = 1 - F_{\pi}(1)$ .

### Our mission, which we accept:

Find the four generating functions

 $F_P, F_R, F_\pi$ , and  $F_\rho$ .

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### Sneaky Result 1:

- Consider two random variables U and V whose values may be 0, 1, 2, ...
- ► Write probability distributions as U<sub>k</sub> and U<sub>k</sub> and g.f.'s as F<sub>U</sub> and F<sub>V</sub>.
- SR1: If a third random variable is defined as

$$W = \sum_{i=1}^{V} U^{(i)}$$
 with each  $U^{(i)} \stackrel{d}{=} U$ 

then

$$F_W(x) = F_V(F_U(x))$$

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## Proof of SR1:

### Write probability that variable W has value k as $W_k$ .

 $W_k = \sum_{j=0}^{\infty} V_j \times \Pr(\text{sum of } j \text{ draws of variable } U = k)$ 

$$= \sum_{j=0}^{\infty} V_j \sum_{\substack{\{i_1, i_2, \dots, i_j\} \mid \\ i_1+i_2+\dots+i_j=k}} U_{i_1} U_{i_2} \cdots U_{i_j}$$

$$\therefore F_{W}(x) = \sum_{k=0}^{\infty} W_{k} x^{k} = \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} V_{j} \sum_{\substack{\{i_{1}, i_{2}, \dots, i_{j}\} \mid \\ i_{1}+i_{2}+\dots+i_{j}=k}} U_{i_{1}} U_{i_{2}} \cdots U_{i_{j}} x^{k}$$

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Frame 73/89

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References

With some concentration, observe:

$$F_{W}(x) = \sum_{j=0}^{\infty} V_{j} \sum_{k=0}^{\infty} \underbrace{\sum_{\substack{\{i_{1}, i_{2}, \dots, i_{j}\} \mid \\ i_{1}+i_{2}+\dots+i_{j}=k}} U_{i_{1}} x^{i_{1}} U_{i_{2}} x^{i_{2}} \dots U_{i_{j}} x^{i_{j}}}_{x^{k} \text{ piece of } \left(\sum_{i'=0}^{\infty} U_{i'} x^{i'}\right)^{j}}$$

$$=\sum_{j=0}^{\infty} V_j (F_U(x))^j$$
$$= F_V (F_U(x)) \checkmark$$

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With some concentration, observe:

$$F_{W}(x) = \sum_{j=0}^{\infty} V_{j} \sum_{k=0}^{\infty} \sum_{\substack{\{i_{1},i_{2},...,i_{k}\}|\\i_{1}+i_{2}+...+i_{k}=j}} U_{i_{1}} x^{i_{1}} U_{i_{2}} x^{i_{2}} \cdots U_{i_{j}} x^{i_{j}}}$$

$$x^{k} \text{ piece of } \left(\sum_{i'=0}^{\infty} U_{i'} x^{i'}\right)^{j}$$

$$\left(\sum_{i'=0}^{\infty} U_{i'} x^{i'}\right)^{j} = (F_{U}(x))^{j}$$

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References

- Start with a random variable U with distribution  $U_k$  (k = 0, 1, 2, ...)
- SR2: If a second random variable is defined as

V = U + 1 then  $F_V(x) = xF_U(x)$ 

• Reason:  $V_k = U_{k-1}$  for  $k \ge 1$  and  $V_0 = 0$ .

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Start with a random variable U with distribution  $U_k$  (k = 0, 1, 2, ...)

### SR2: If a second random variable is defined as

$$V = U + 1$$
 then  $|F_V(x) = xF_U(x)|$ 

• Reason: 
$$V_k = U_{k-1}$$
 for  $k \ge 1$  and  $V_0 = 0$ .

$$\therefore F_V(x) = \sum_{k=0}^{\infty} V_k x^k = \sum_{k=1}^{\infty} U_{k-1} x^k$$
$$= x \sum_{j=0}^{\infty} U_j x^j = x F_U(x).$$

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### Generalization of SR2:

▶ (1) If V = U + i then

 $F_V(x) = x^i F_U(x).$ 

▶ (2) If V = U - i then

 $F_V(x) = x^{-i} F_U(x)$ 



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$$=x^{-i}\sum_{k=0}^{\infty}U_kx^k$$

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- Goal: figure out forms of the component generating functions, *F<sub>π</sub>* and *F<sub>ρ</sub>*.
- $\pi_n$  = probability that a random node belongs to a finite component of size *n*

sum of sizes of subcomponents at end of k random links = n - 1

Therefore:



Extra factor of x accounts for random node itself.

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- $\pi_n$  = probability that a random node belongs to a finite component of size *n*

 $= \sum_{k=0}^{\infty} P_k \times \Pr\left(\begin{array}{c} \text{sum of sizes of subcomponents} \\ \text{at end of } k \text{ random links} = n-1 \end{array}\right)$ 

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Therefore:

$$F_{\pi}(x) = \underbrace{x}_{SR2} \underbrace{F_{P}(F_{\rho}(x))}_{SR1}$$

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- *ρ<sub>n</sub>* = probability that a random link leads to a finite subcomponent of size *n*.
- Invoke one step of recursion: ρ<sub>n</sub> = probability that a random node arrived along a random edge is part of a finite subcomponent of size n.

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```

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We now have two functional equations connecting our generating functions:

 $F_{\pi}(x) = xF_{P}(F_{\rho}(x))$  and  $F_{\rho}(x) = xF_{R}(F_{\rho}(x))$ 

- ► Taking stock: We know  $F_P(x)$  and  $F_R(x) = F'_P(x)/F'_P(1)$ .
- We first untangle the second equation to find F<sub>ρ</sub>
- We can do this because it only involves  $F_{\rho}$  and  $F_R$ .
- The first equation then immediately gives us F<sub>π</sub> in terms of F<sub>ρ</sub> and F<sub>R</sub>.

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### Remembering vaguely what we are doing:

Finding  $F_{\pi}$  to obtain the fractional size of the largest component  $S_1 = 1 - F_{\pi}(1)$ .

Set x = 1 in our two equations:

 $F_{\pi}(1) = F_{P}(F_{\rho}(1))$  and  $F_{\rho}(1) = F_{R}(F_{\rho}(1))$ 

Solve second equation numerically for *F<sub>ρ</sub>*(1).
 Plug *F<sub>ρ</sub>*(1) into first equation to obtain *F<sub>π</sub>*(1).

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### Example: Standard random graphs.

• We can show 
$$F_P(x) = e^{-\langle k \rangle (1-x)}$$

 $\therefore F_R(x) = F'_P(x)/F'_P(1) = e^{-\langle k \rangle (1-x)}/e^{-\langle k \rangle (1-x')}|_{x'}$ 

 $=e^{-\langle k\rangle(1-x)}=F_P(x)$  ...aha!

- ► RHS's of our two equations are the same.
   ► So F<sub>π</sub>(x) = F<sub>ρ</sub>(x) = xF<sub>R</sub>(F<sub>ρ</sub>(x)) = xF<sub>R</sub>(F<sub>π</sub>(x))
- Why our dirty (but wrong) trick worked earlier...

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 ► So F<sub>π</sub>(x) = F<sub>ρ</sub>(x) = xF<sub>R</sub>(F<sub>ρ</sub>(x)) = xF<sub>R</sub>(F<sub>π</sub>(x))
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Example: Standard random graphs.

• We can show  $F_P(x) = e^{-\langle k \rangle (1-x)}$ 

$$\therefore F_R(x) = F'_P(x)/F'_P(1) = e^{-\langle k \rangle (1-x)}/e^{-\langle k \rangle (1-x')}|_{x'=1}$$

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Or:  $\langle k 
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We're first after S<sub>1</sub> = 1 − F<sub>π</sub>(1) so set x = 1 and replace F<sub>π</sub>(1) by 1 − S<sub>1</sub>:

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#### Random Networks

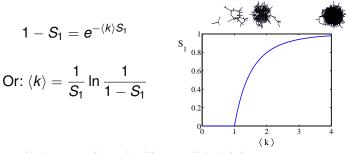
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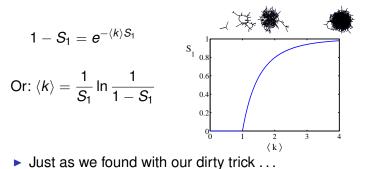
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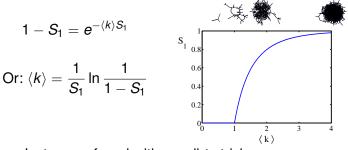
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### Example: Standard random graphs.

• Use fact that  $F_P = F_R$  and  $F_{\pi} = F_{\rho}$ .

Two differentiated equations reduce to only one:

 $F'_{\pi}(x) = F_{P}(F_{\pi}(x)) + xF'_{\pi}(x)F'_{P}(F_{\pi}(x))$ 

Rearrange: 
$$F'_{\pi}(x) = rac{F_{P}(F_{\pi}(x))}{1 - xF'_{P}(F_{\pi}(x))}$$

- Simplify denominator using  $F'_P(x) = \langle k \rangle F_P(x)$
- ► Replace  $F_P(F_\pi(x))$  using  $F_\pi(x) = xF_P(F_\pi(x))$ .
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End result: 
$$\langle n \rangle = F'_{\pi}(1) = \frac{(1-S_1)}{1-\langle k \rangle(1-S_1)}$$

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Our result for standard random networks:

$$\langle n \rangle = F'_{\pi}(1) = rac{(1-S_1)}{1-\langle k \rangle(1-S_1)}$$

- Recall that (k) = 1 is the critical value of average degree for standard random networks.
- Look at what happens when we increase (k) to 1 from below.
- We have  $S_1 = 0$  for all  $\langle k \rangle < 1$  so



- This blows up as  $\langle k \rangle \rightarrow 1$ .
- ► Reason: we have a power law distribution of component sizes at (k) = 1.
- Typical critical point behavior....

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- All nodes are isolated.
- As  $\langle k \rangle \to \infty$ ,  $S_1 \to 1$  and  $\langle n \rangle \to 0$ .
- ▶ No nodes are outside of the giant component.

### Extra on largest component size:

- For  $\langle k \rangle = 1$ ,  $S_1 \sim N^{2/3}$ .
- For  $\langle k \rangle < 1$ ,  $S_1 \sim \log N$ .

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