Overview of Complex Networks

Complex Networks, Course 303A, Spring, 2009

Prof. Peter Dodds

Department of Mathematics & Statistics
University of Vermont

Outline

Class Admin

- Office hours:
  - Tuesday 2:30 pm–3:30 pm
  - Thursday 11:30 am–12:30 pm
- Course outline
- Projects

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Basic definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Basic definitions

Complex System—Basic ingredients:

- Relationships are nonlinear
- Relationships contain feedback loops
- Complex systems are open (out of equilibrium)
- Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- May result in emergent phenomena

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Network: (net + work, 1500's)

Noun:

1. Any interconnected group or system
2. Multiple computers and other devices connected together to share information

Verb:

1. To interact socially for the purpose of getting connections or personal advancement
2. To connect two or more computers or other computerized devices

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- Many complex systems can be regarded as complex networks of physical or abstract interactions
- Opens door to mathematical and numerical analysis
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Links = Connections between nodes

- links
  - may be real and fixed (rivers),
  - real and dynamic (airline routes),
  - abstract with physical impact (hyperlinks),
  - or purely abstract (semantic connections between concepts).
- Links may be directed or undirected.
- Links may be binary or weighted.

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Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- $k_i = 0, 1, 2, \ldots$.
- Notation: the average degree of a network = $\langle k \rangle$

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Basic definitions

Adjacency matrix:

- We represent a graph or network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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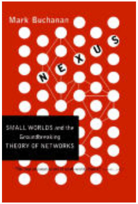
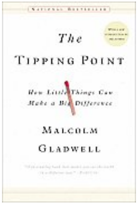


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [7]

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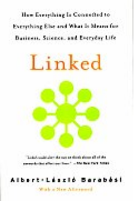
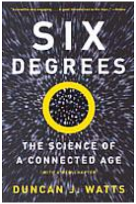


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [17]

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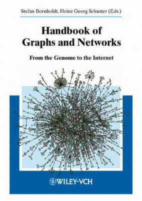
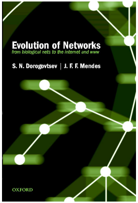


Handbook of Graphs and Networks—editors: Stefan Bornholdt and H. G. Schuster [3]



Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

Books

Social Network Analysis—Stanley Wasserman and Kathleen Faust [16]

Books

Numerous others:

- Complex Social Networks—F. Vega-Redondo [15]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [12]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung

Examples

What passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...
- Isn't this graph theory?: Yes, but emphasis is on data and mechanistic explanations...

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Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids

- Distribution (branching) versus redistribution (cyclical)

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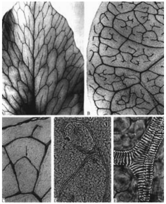

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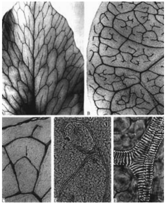

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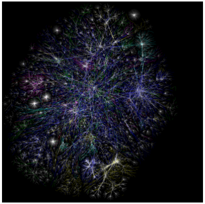

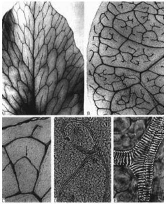

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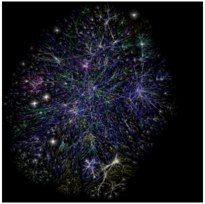

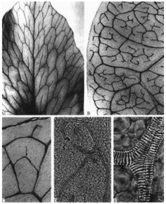

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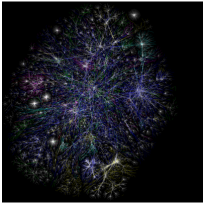

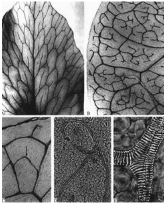

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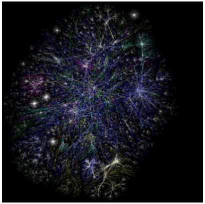

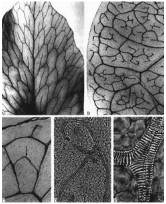

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- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
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datamining.typepad.com (⊞)

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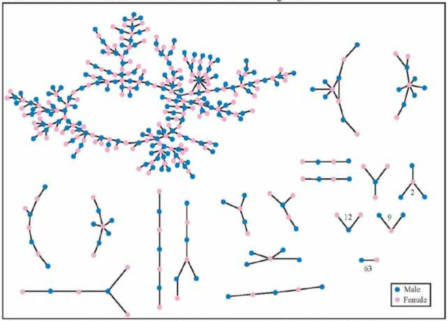
Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- myspace.com (⊞), facebook.com (⊞)

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

- 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

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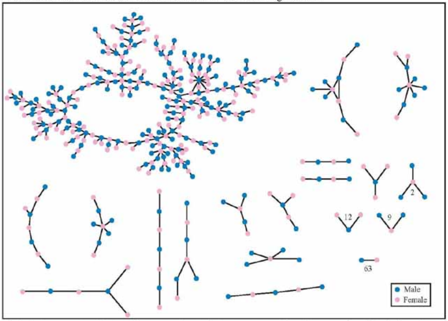

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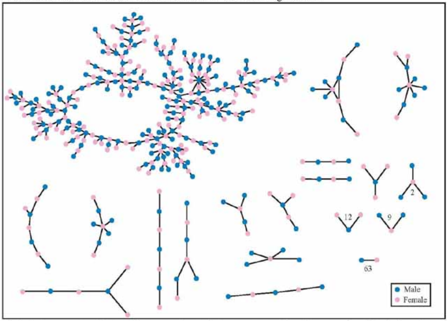

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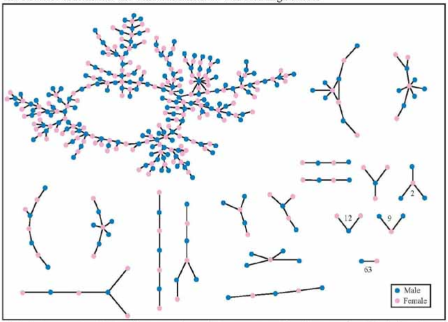

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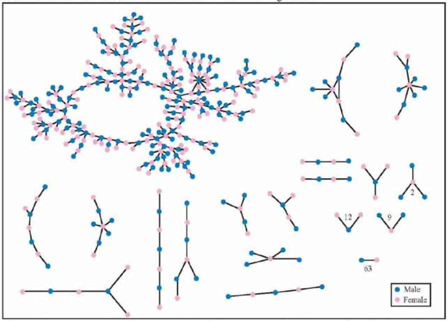

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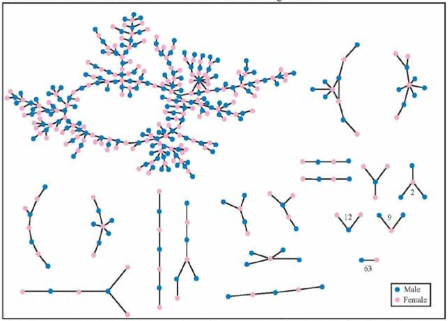

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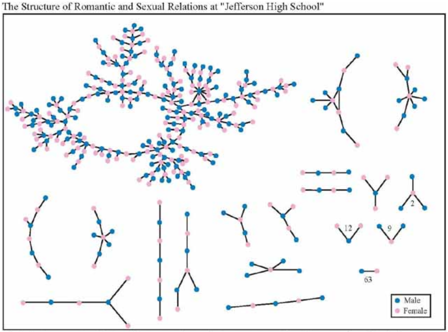

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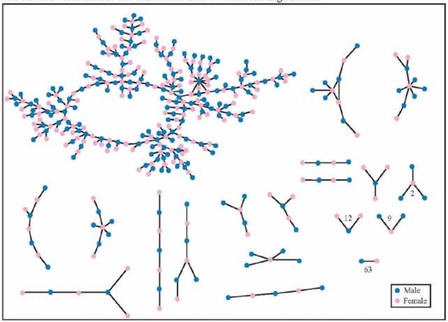

Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

- ‘Remotely sensed’ by: email activity, instant messaging, phone logs (*cough*).

Examples

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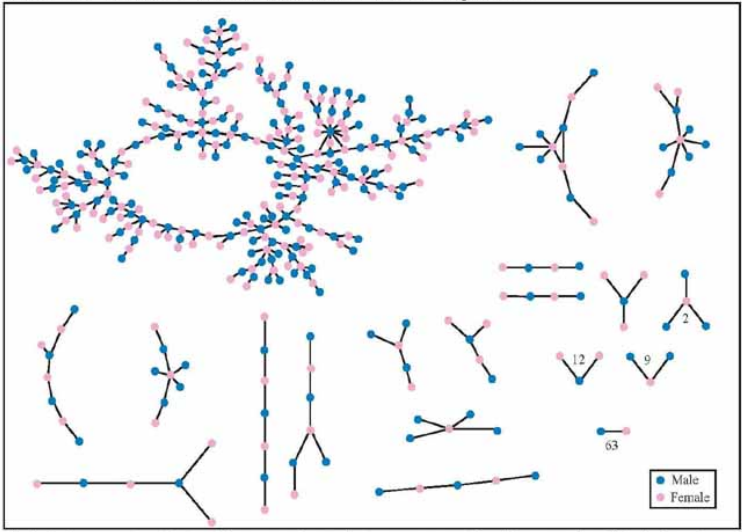

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Relational networks

- Consumer purchases
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
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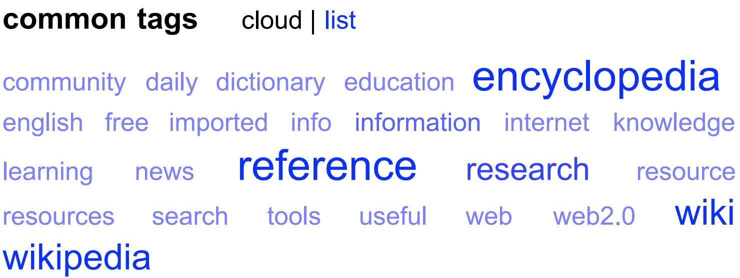

Observations

A notable feature of large-scale networks:

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Properties

Some key aspects of real complex networks:

- degree distribution
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

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1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- k = node degree = number of connections
- ex 1: Erdős-Rényi random networks:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- Distribution is Poisson

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- ‘Scale-free’ networks are growing networks that form according to a plausible mechanism.
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2. assortativity/3. homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[10] similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.

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  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

Clustering

4. clustering:

- Your friends tend to know each other.
- Two measures:
  1. Watts & Strogatz[19]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

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where k_i is node i's degree, and $\mathcal{N}_i$ is the set of i's neighbors.

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Properties

Triples and triangles

- Nodes i_1, i_2, and i_3 form a triple around i_1 if i_1 is connected to i_2 and i_3.
- Nodes i_1, i_2, and i_3 form a triangle if each pair of nodes is connected
- The definition

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

measures the fraction of closed triples

- Social Network Analysis (SNA): fraction of transitive triples.
- The '3' appears because for each triangle, we have 3 closed triples.

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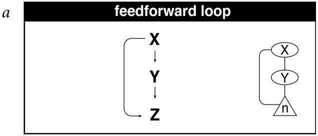

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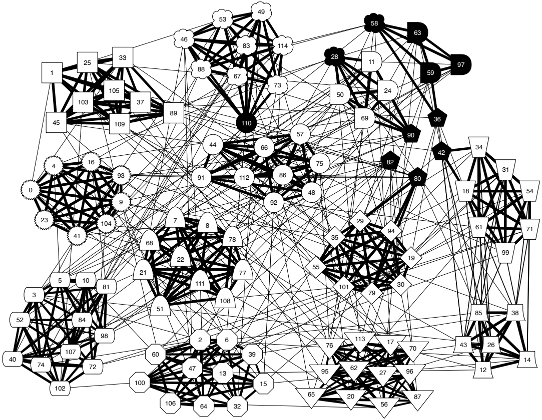
Properties

6. modularity and structure/community detection:



Clauset *et al.*, 2006 [5]: NCAA football

Properties

7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [9]

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8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
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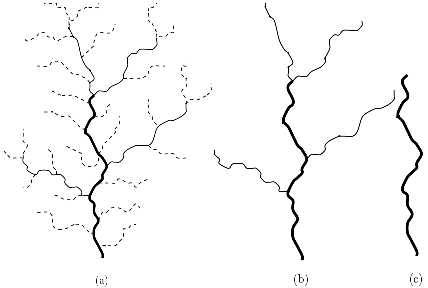

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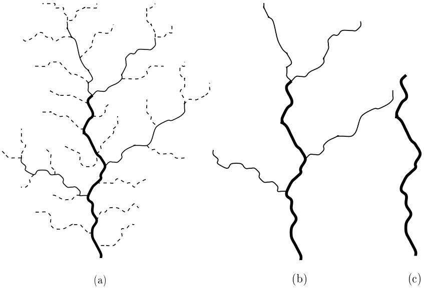

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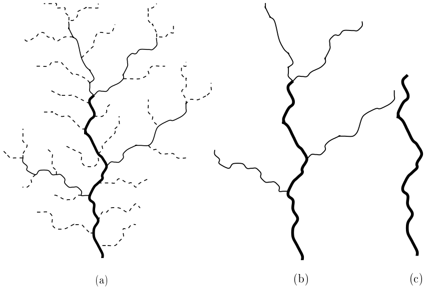

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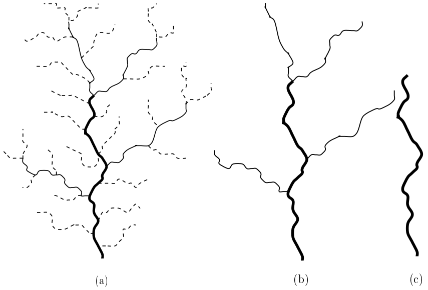

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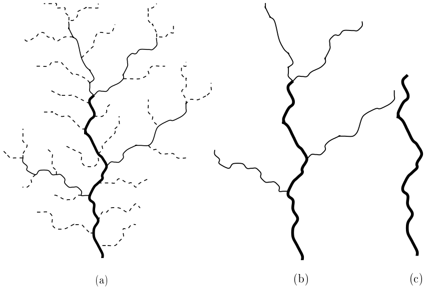

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9. network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
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- network diameter $d_{\max}$:
  Maximum shortest path length between any two nodes.
- closeness $d_{\mathrm{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:
  Average 'distance' between any two nodes.
- Closeness handles disconnected networks ($d_{ij} = \infty$)
- $d_{\mathrm{cl}} = \infty$ only when all nodes are isolated.
- Closeness perhaps compresses too much into one number

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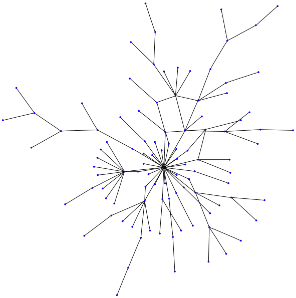

γ = 2.5
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$N = 150$

- Introduced by Barabasi and Albert [1]
- Generative model
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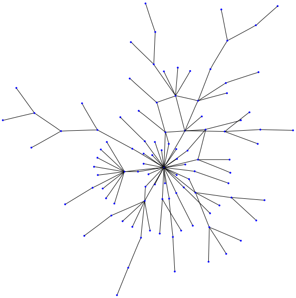

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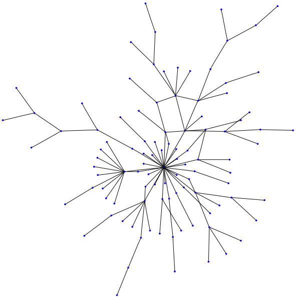

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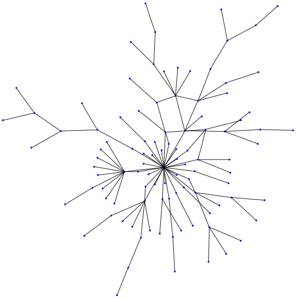

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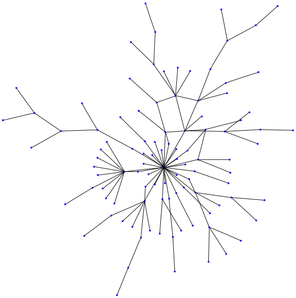

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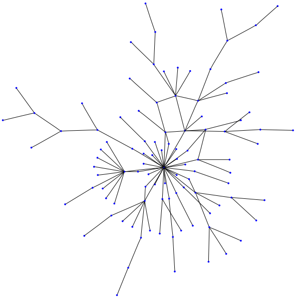

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  - global randomness (shortcuts).
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- Number of infectives increases exponentially in time
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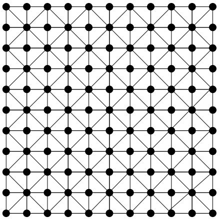

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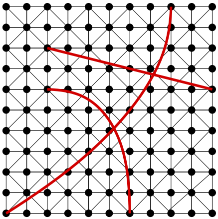

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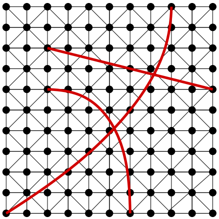

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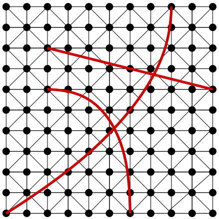

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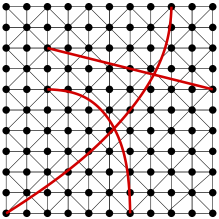

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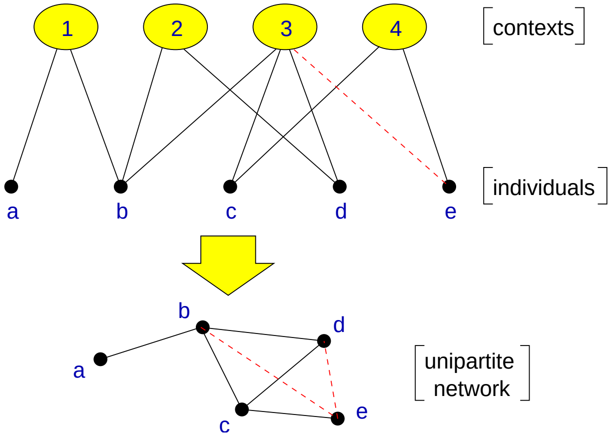

Bipartite affiliation networks: boards and directors, movies and actors.

Models

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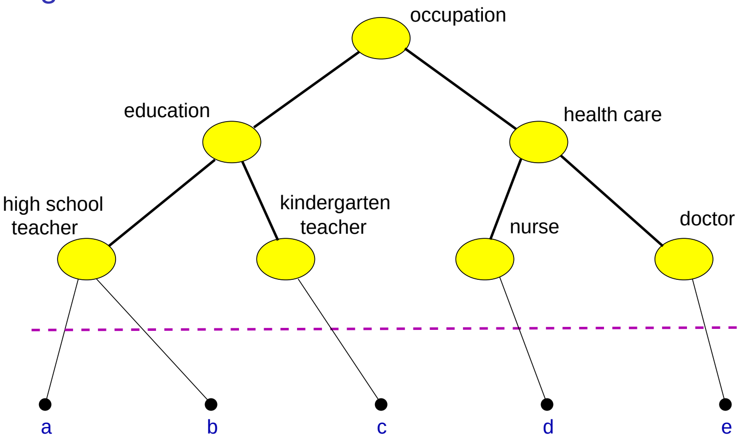

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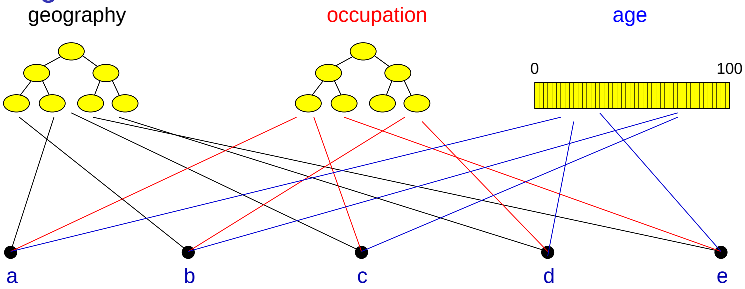

- Blau & Schwartz [2], Simmel [14], Breiger [4], Watts *et al.* [18]

Popularity (according to ISI)

"Collective dynamics of 'small-world' networks"[19]

- Watts and Strogatz
  Nature, 1998
- ≈ 3500 citations (as of Jan 13, 2009)
- 1100 citations in the last year

"Emergence of scaling in random networks"[1]

- Barabási and Albert
  Science, 1999
- ≈ 3472 citations (as of Jan 13, 2009)
- 1172 citations in the last year

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
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- Three main (blurred) categories:
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