Overview of Complex Networks

Complex Networks, Course 303A, Spring, 2009

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Overview

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Modelling Complex Networks
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Class Admin

- Office hours:
  - Tuesday 2:30 pm–3:30 pm
  - Thursday 11:30 am–12:30 pm
- Course outline
- Projects

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Basic definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

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Basic definitions

Complex System—Basic ingredients:

- Relationships are nonlinear
- Relationships contain feedback loops
- Complex systems are open (out of equilibrium)
- Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- May result in emergent phenomena

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Basic definitions

Network: (net + work, 1500's)

Noun:

1. Any interconnected group or system
2. Multiple computers and other devices connected together to share information

Verb:

1. To interact socially for the purpose of getting connections or personal advancement
2. To connect two or more computers or other computerized devices

Observation

- Many complex systems can be regarded as complex networks of physical or abstract interactions
- Opens door to mathematical and numerical analysis
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.

Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

Basic definitions

Links = Connections between nodes

- links
  - may be real and fixed (rivers),
  - real and dynamic (airline routes),
  - abstract with physical impact (hyperlinks),
  - or purely abstract (semantic connections between concepts).
- Links may be directed or undirected.
- Links may be binary or weighted.

Basic definitions

Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- k_i = 0,1,2,. . . .
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes as z)

Basic definitions

Adjacency matrix:

- We represent a graph or network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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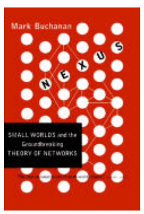
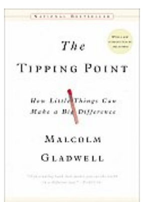


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [7]

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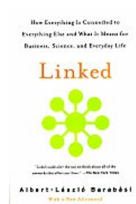
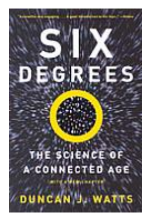


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [17]

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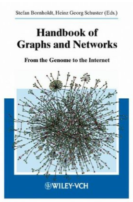
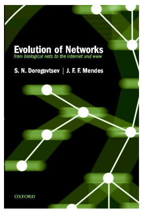
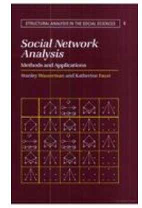


Handbook of Graphs and Networks—editors: Stefan Bornholdt and H. G. Schuster [3]



Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

Books



Social Network Analysis—Stanley Wasserman and Kathleen Faust [16]

Books

Numerous others:

- Complex Social Networks—F. Vega-Redondo [15]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [12]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung

Examples

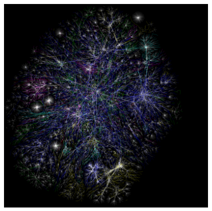
What passes for a complex network?

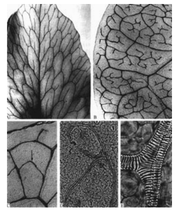
- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...
- Isn't this graph theory?: Yes, but emphasis is on data and mechanistic explanations...

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids





- Distribution (branching) versus redistribution (cyclical)

Examples

Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media

datamining.typepad.com (⊞)

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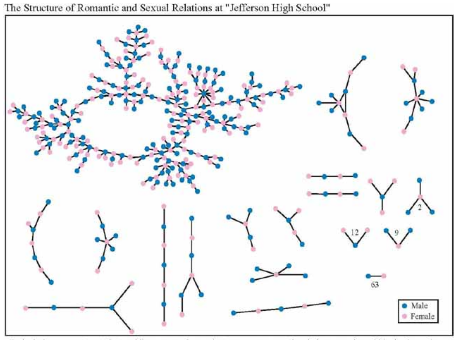
Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- myspace.com (⊞), facebook.com (⊞)

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

- 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

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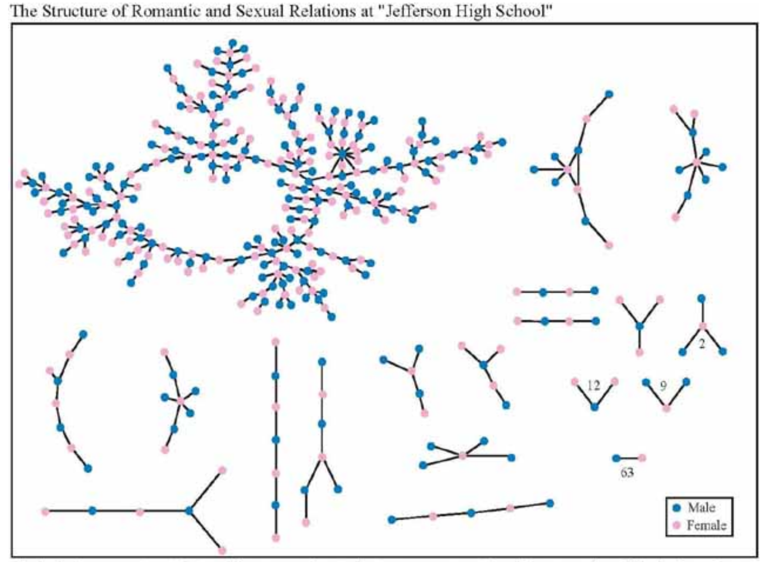
The Structure of Romantic and Sexual Relations at "Jefferson High School"



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Examples

Relational networks

- Consumer purchases (Wal-Mart: $\approx$ 1 petabyte $= 10^{15}$ bytes)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: del.icio.us (⊞), flickr (⊞)

common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

Observations

A notable feature of large-scale networks:

- Graphical renderings of complex networks are often just a big mess.

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Properties

Some key aspects of real complex networks:

- degree distribution
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

- + Coevolution of network structure and processes on networks.

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Properties

1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- k = node degree = number of connections
- ex 1: Erdös-Rényi random networks:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- Distribution is Poisson

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Properties

1. degree distribution P_k

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
- link cost controls skew
- hubs may facilitate or impede contagion

Properties

Note:

- Erdös-Rényi random networks are a *mathematical construct*.
- 'Scale-free' networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.

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Properties

2. assortativity/3. homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[10] similar degree nodes connecting to each other.
  Often social: company directors, coauthors, actors.
- Disassortative network: high degree nodes connecting to low degree nodes.
  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

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Clustering

4. clustering:

- Your friends tend to know each other.
- Two measures:
  1. Watts & Strogatz[19]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i$$

  2. Newman[11]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

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First clustering measure:

- C_1 is the average fraction of pairs of neighbors who are connected.
- Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

  where k_i is node i's degree, and $\mathcal{N}_i$ is the set of i's neighbors.
- Averaging over all nodes, we have

$$C_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

Properties

- For sparse networks, C_1 tends to discount highly connected nodes.
- C_2 is a useful variant
- In general, $C_1 \neq C_2$.

Properties

Triples and triangles

- Nodes i_1, i_2, and i_3 form a triple around i_1 if i_1 is connected to i_2 and i_3.
- Nodes i_1, i_2, and i_3 form a triangle if each pair of nodes is connected
- The definition

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

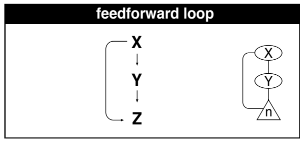
  measures the fraction of closed triples
- Social Network Analysis (SNA): fraction of transitive triples.
- The '3' appears because for each triangle, we have 3 closed triples.

Properties

5. motifs:

- small, recurring functional subnetworks
- e.g., Feed Forward Loop:

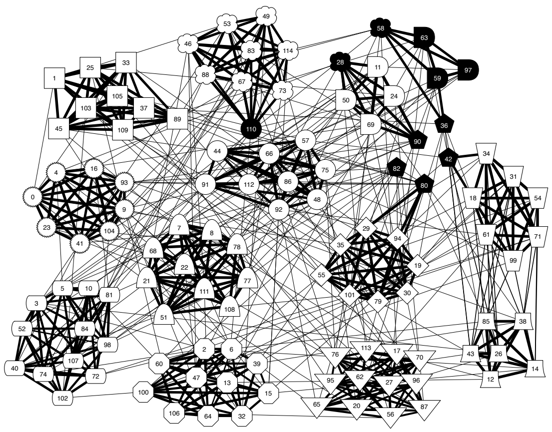
a



Shen-Orr, Uri Alon, *et al.* [13]

Properties

6. modularity and structure/community detection:



Clauset *et al.*, 2006 [5]: NCAA football

Properties

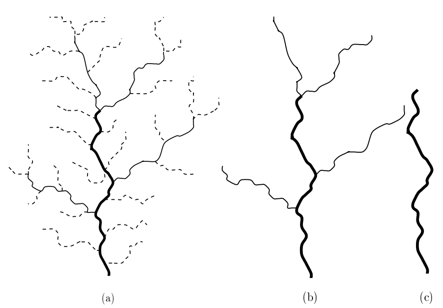
7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [9]

Properties

8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
  - Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_\omega \rangle$



Properties

9. network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
- Good algorithms exist for calculation.
- Weighted links can be accommodated.

Properties

9. network distances:
   - network diameter $d_{\max}$:
     Maximum shortest path length between any two nodes.
   - closeness $d_{\text{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:
     Average 'distance' between any two nodes.
   - Closeness handles disconnected networks ($d_{ij} = \infty$)
   - $d_{\text{cl}} = \infty$ only when all nodes are isolated.
   - Closeness perhaps compresses too much into one number

Properties

10. centrality:
    - Many such measures of a node's 'importance.'
    - ex 1: Degree centrality: k_i.
    - ex 2: Node i's betweenness
      = fraction of shortest paths that pass through i.
    - ex 3: Recursive centrality: Hubs and Authorities (Kleinberg [8])

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Models

Some important models:

1. generalized random networks (touched on in 300)
2. scale-free networks (⊞) (covered in 300)
3. small-world networks (⊞) (covered in 300)
4. statistical generative models (p^*)
5. generalized affiliation networks (partly covered in 300)

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Models

1. generalized random networks:
   - Arbitrary degree distribution P_k.
   - Wire nodes together randomly.
   - Create ensemble to test deviations from randomness.
   - Interesting, applicable, rich mathematically.
   - We will have fun with these guys...

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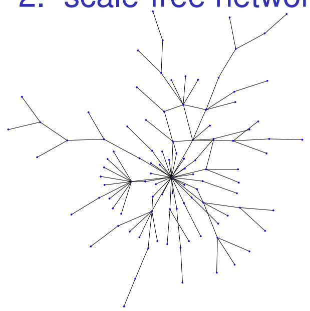
2. ‘scale-free networks’:



$\gamma = 2.5$
$\langle k \rangle = 1.8$
$N = 150$

- Introduced by Barabasi and Albert [1]
- Generative model
- Preferential attachment model with growth:
- P[attachment to node i] $\propto \mathrm{k}_\mathrm{i}^{\alpha}$.
- Produces $P_k \sim k^{-\gamma}$ when $\alpha = 1$.
- Trickiness: other models generate skewed degree distributions.

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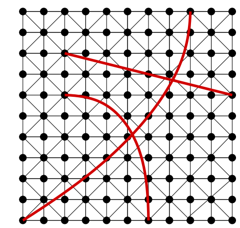
Models

3. small-world networks

- Introduced by Watts and Strogatz [19]

Two scales:

- local regularity (an individual’s friends know each other)
- global randomness (shortcuts).



- Shortcuts allow disease to jump
- Number of infectives increases exponentially in time
- Facilitates synchronization

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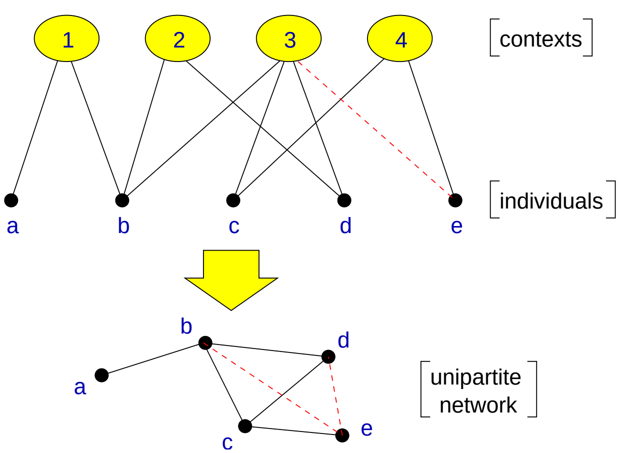
5. generalized affiliation networks



Bipartite affiliation networks: boards and directors, movies and actors.

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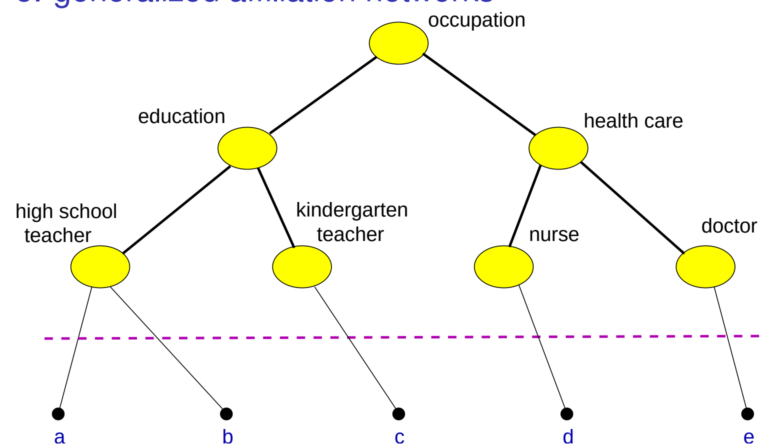
5. generalized affiliation networks



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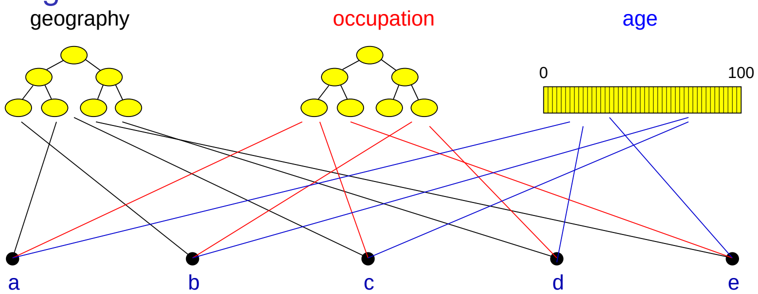

- Blau & Schwartz [2], Simmel [14], Breiger [4], Watts *et al.* [18]

Popularity (according to ISI)

"Collective dynamics of 'small-world' networks" [19]

- Watts and Strogatz
  Nature, 1998
- ≈ 3500 citations (as of Jan 13, 2009)
- 1100 citations in the last year

"Emergence of scaling in random networks" [1]

- Barabási and Albert
  Science, 1999
- ≈ 3472 citations (as of Jan 13, 2009)
- 1172 citations in the last year

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

Nutshell:

Overview Key Points (cont.):

- Obvious connections with the vast extant field of graph theory.
- But focus on dynamics is more of a physics/stat-mech/comp-sci flavor.
- Two main areas of focus:
  1. Description: Characterizing very large networks
  2. Explanation: Micro story ⇒ Macro features
- Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overall structure,...
- Still much work to be done, especially with respect to dynamics... exciting!

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