Chapter 6: Lecture 25 Linear Algebra, Course 124C, Spring, 2009

Prof. Peter Dodds

Department of Mathematics & Statistics University of Vermont



Licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Licenses

The fundamental theorem of linear algebra

Frame 1/11





- ▶ Applies to any $m \times n$ matrix A.
- ▶ Symmetry of A and A^{T} .

Where \vec{x} lives:

- ▶ Row space $C(A^{\mathrm{T}}) \subset R^n$.
- ▶ (Right) Nullspace N(A) ⊂ Rⁿ.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = r
- ▶ Orthogonality: $C(A^1) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{\mathrm{T}}) \subset R^m$.
- ▶ dim C(A) + dim $N(A^{\perp}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{\mathrm{T}}) = R^m$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?

Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
- Symmetry of A and A^T.

The fundamental theorem of linear algebra



- ▶ Applies to any $m \times n$ matrix A.
- ▶ Symmetry of A and A^T.

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ► dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{\mathrm{T}}) \subset R^m$.
- ▶ dim C(A) + dim $N(A^T) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1 Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
- ▶ Symmetry of A and A^T.

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{\mathrm{T}}) \subset R^m$
- ightharpoonup dim C(A) + dim $N(A^{T}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^T) = R^m$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Guess who? Bonus example

Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
- ► Symmetry of *A* and *A*^T.

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{\mathrm{T}}) \subset R^m$.
- ▶ dim C(A) + dim $N(A^{T}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example



- ▶ Applies to any $m \times n$ matrix A.
- ► Symmetry of *A* and *A*^T.

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^T) \subset R^m$.
- ▶ dim C(A) + dim $N(A^{T}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Guess who?
Bonus example

Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
- ▶ Symmetry of A and A^{T} .

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^T) \subset R^m$.
- ▶ dim C(A) + dim $N(A^{T}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Guess who? Bonus example



- ▶ Applies to any $m \times n$ matrix A.
- ▶ Symmetry of A and A^{T} .

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{T}) \subset R^{m}$.
- ▶ dim C(A) + dim $N(A^{T}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Bonus example



- ▶ Applies to any $m \times n$ matrix A.
- ▶ Symmetry of A and A^T.

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{T}) \subset R^{m}$.
- ▶ dim C(A) + dim $N(A^{T}) = r + (m r) = m$
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who? Bonus example

Bonus example 1
Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
- ► Symmetry of *A* and *A*^T.

Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n
- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{T}) \subset R^{m}$.
- ▶ dim C(A) + dim $N(A^{T})$ = r + (m r) = m
- ▶ Orthogonality: $C(A) \otimes N(A^{T}) = R^{m}$

The fundamental theorem of linear algebra

Approximating matrices with SVD

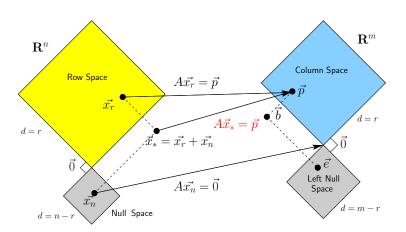
The basic idea Guess who?

Bonus example 1 Bonus example 2





Best solution \vec{x}_* when $\vec{b} = \vec{p} + \vec{e}$:



The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Frame 3/11



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ► The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ► The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- $\blacktriangleright \{\hat{v}_1, \ldots, \hat{v}_r\}$ span Row space
- $\blacktriangleright \{\hat{v}_{r+1}, \ldots, \hat{v}_n\}$ span Null space
- \triangleright { $\hat{u}_1,\ldots,\hat{u}_r$ } span Column space
- \triangleright { \hat{u}_{r+1} \hat{u}_m } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ► The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- $\blacktriangleright \{\hat{v}_1, \ldots, \hat{v}_r\}$ span Row space
- $\triangleright \{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- $ightharpoonup \{\hat{u}_1,\ldots,\hat{u}_r\}$ span Column space
- \blacktriangleright { $\hat{u}_{r+1}, \ldots, \hat{u}_m$ } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^{T} .

Happy bases

- $\triangleright \{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- \triangleright { $\hat{v}_{r+1}, \ldots, \hat{v}_n$ } span Null space
- \triangleright { $\hat{u}_1, \ldots, \hat{u}_r$ } span Golumn space
- \triangleright { \hat{u}_{r+1} \hat{u}_m } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- $\triangleright \{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- \triangleright { $\hat{v}_{r+1}, \ldots, \hat{v}_n$ } span Null space
- \triangleright { $\hat{u}_1, \ldots, \hat{u}_r$ } span Column space
- \triangleright { \hat{u}_{r+1} \hat{u}_m } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- \triangleright { $\hat{v}_1, \ldots, \hat{v}_r$ } span Row space
- $ightharpoonup \{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- \blacktriangleright { $\hat{u}_1,\ldots,\hat{u}_r$ } span Column space
- \triangleright { $\hat{u}_{r+1}, \ldots, \hat{u}_m$ } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- ▶ $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- ▶ $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- ▶ $\{\hat{u}_1, \dots, \hat{u}_r\}$ span Column space
- \blacktriangleright { $\hat{u}_{r+1}, \dots, \hat{u}_m$ } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- ▶ $\{\hat{u}_1, \dots, \hat{u}_r\}$ span Column space
- $ightharpoonup \{\hat{u}_{r+1},\ldots,\hat{u}_m\}$ span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- $\{\hat{u}_1,\ldots,\hat{u}_r\}$ span Column space
- \blacktriangleright { $\hat{u}_{r+1},\ldots,\hat{u}_m$ } span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2





Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- ▶ $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- $\{\hat{u}_1, \dots, \hat{u}_r\}$ span Column space
- $\{\hat{u}_{r+1}, \dots, \hat{u}_m\}$ span Left Null space

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1 Bonus example 2



How $A\vec{x}$ works:

- $A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ *A* is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any A is a diagonal matrix ∑

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1



How $A\vec{x}$ works:

- $\rightarrow A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ *A* is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- ► When viewed the right way, any *A* is a diagonal matrix *S*

The fundamental theorem of linear algebra

matrices with SVD

Guess who?
Bonus example 1

Bonus example 1 Bonus example 2



How $A\vec{x}$ works:

- $\rightarrow A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ *A* is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any A is a diagonal matrix Σ.

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1

Bonus example 1 Bonus example 2



How $A\vec{x}$ works:

- $ightharpoonup A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ A is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any A is a diagonal matrix Σ.

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1



How $A\vec{x}$ works:

- $\rightarrow A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ A is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any A is a diagonal matrix Σ.

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1



Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- ► Truncate series SVD representation of *A*:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- ▶ Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- $ightharpoonup \operatorname{Rank} r = \min(m, n).$
- Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1 Bonus example 2



Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- ▶ Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- Rank $r = \min(m, n)$.
- ▶ Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1 Bonus example 2



Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- ► Truncate series SVD representation of *A*:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- Rank $r = \min(m, n)$.
- ▶ Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus exampl

Bonus example 1
Bonus example 2



Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- ► Truncate series SVD representation of *A*:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- ▶ Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- ▶ Rank $r = \min(m, n)$.
- Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1



Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- ► Truncate series SVD representation of *A*:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- ▶ Rank $r = \min(m, n)$.
- ▶ Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example 1 Bonus example 2



Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- ▶ Rank $r = \min(m, n)$.
- Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea Guess who?

Bonus example

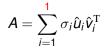


Approximating natrices with SVD

Guess who?

Bonus exampl

Bonus example :







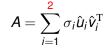


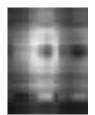
Approximating matrices with SVE

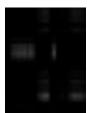
Guess who?

Bonus example

Bonus example :







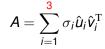


Approximating natrices with SVD

Guess who?

Bonus example

Bonus example :





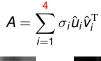




Approximating matrices with SVE

Guess who?

Bonus example







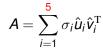


Approximating natrices with SVD

Guess who?

Guess wno?

Bonus example







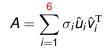


Approximating matrices with SVE

Guess who?

Bonus exampl

Bonus example :







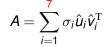


Approximating matrices with SVE

Guess who?

Guess who?

Bonus example





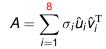




Approximating natrices with SVD

Guess who?

Bonus example









Approximating natrices with SVD

Guess who?

Bonus example









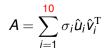


Approximating matrices with SVE

Guess who?

Bonus exampl

Bonus example :







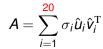


Approximating matrices with SVI

Guess who?

Guess who?

Bonus example 2









Approximating natrices with SVD

Guess who?

Bonus example









Approximating natrices with SVD

Guess who?

Bonus example









Approximating natrices with SVD

Guess who?

Bonus example









Approximating natrices with SVD

Guess who?

Bonus example

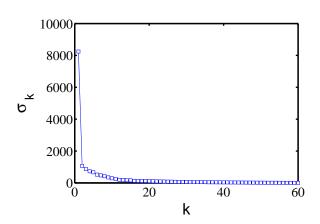








Decay of sigma values: Einstein



The fundamental theorem of linear algebra

Approximating natrices with SVD

The basic idea Guess who?

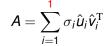
Bonus example 1
Bonus example 2

Frame 8/11



The fundamental algebra

Guess who?







The fundamental algebra

Guess who?





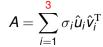




Approximating natrices with SVD

Guess who?

Bonus example 1









Approximating matrices with SVE

Guess who?

Bonus exampl

Bonus example 2









Approximating matrices with SVE

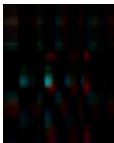
Guess who?

Bonus example

Bonus example 2





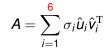




Approximating natrices with SVD

Guess who?

Bonus example







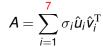


Approximating matrices with SVE

Guess who?

Guess who?

Bonus example 2





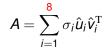




Approximating matrices with SVE

Guess who?

Bonus example 2









Approximating matrices with SVE

Guess who?

Guess who?

Bonus example 2







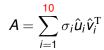


Approximating matrices with SVE

Guess who?

Guess wno?

Bonus example 2





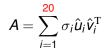




Approximating matrices with SVD

Guess who?

Bonus example





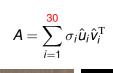




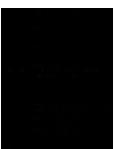
Approximating matrices with SVE

Guess who?

Bonus example





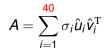




Approximating matrices with SVE

Guess who?

Bonus example







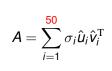


Approximating matrices with SVE

Guess who?

Guess who?

Bonus example 2







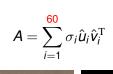


Approximating matrices with SVE

Guess who?

Bonus example

Bonus example 2







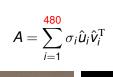


Approximating matrices with SVE

Guess who?

Guess who?

Bonus example :









$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

Approximating natrices with SVD

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who

Bonus example 1



$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who

Bonus example 1



$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who

Bonus example 1



$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who

Bonus example 1



$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea
Guess who?
Bonus example 1

Bonus example 1



$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?
Bonus example 1

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVE

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear

Ch. 6: Lec. 25

Approximating

Guess who?

algebra

Bonus example 1



$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVE

Guess who?

Bonus example 1



$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear

Ch. 6: Lec. 25

Approximating

Guess who?

algebra

Bonus example 1



$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear algebra

Ch. 6: Lec. 25

Approximating matrices with SVE

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear

Ch. 6: Lec. 25

Approximating

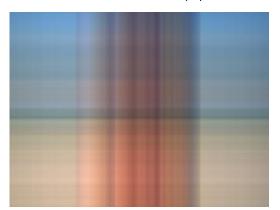
Guess who?

algebra

Bonus example 1



$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

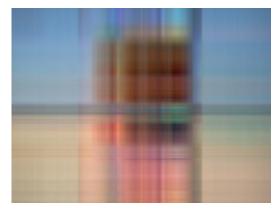
Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

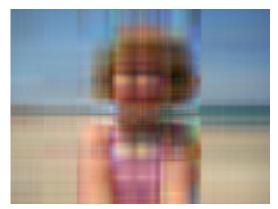
Approximating matrices with SVD

Guess who?

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

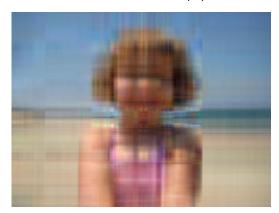
Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

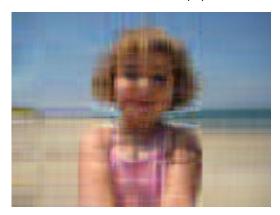
Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

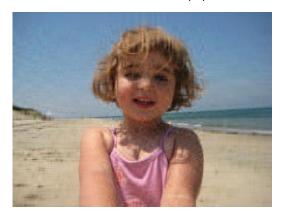
Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental

Ch. 6: Lec. 25

algebra

Approximating natrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear algebra

Ch. 6: Lec. 25

Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear algebra

Ch. 6: Lec. 25

Approximating matrices with SVD

Guess who?

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





The fundamental theorem of linear algebra

Ch. 6: Lec. 25

Approximating matrices with SVD

Guess who?

Bonus example 1
Bonus example 2



