Chapter 2: Lecture 7 Linear Algebra, Course 124C, Spring, 2009

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Frame 1/7





- ► Chapter 1 and Chapter 2 (Sections 2.1–2.7)
- ► Chapter 2 is our focus
- ► Knowledge of Chapter 1 as needed for Chapter 2 = solving $A\vec{x} = \vec{b}$.
- Want 'understanding' and 'doing' abilities.



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- ▶ What dimensions of *A* mean:
 - \rightarrow m = number of equations
 - $ightharpoonup n = number of unknowns (x_1, x_2, ...)$
- ► How to draw the row and column pictures.
- Be able to identify row picture (e.g., as representing 2 planes in 3-d).
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Row, Column, & Matrix Pictures of Linear Systems $(A\vec{x} = \vec{b})$

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Solving $A\vec{x} = \vec{b}$ by elimination

Solve four equivalent ways:

- Simultaneous equations (snore)
- 2. Row operations on augmented matrix
 - Systematically transform $A\vec{x} = \vec{b}$ into $U\vec{x} = \vec{c}$
 - Solve by back subsitution
- 3. Row operations with E_{ij} and P_{ij} matrices
- 4. Factor A as A = LU
 - Solve two triangular systems by forward and back substitution
 - First $L\vec{c} = b$ then $U\vec{x} = \vec{c}$.

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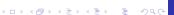
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- ▶ Be able to find the pivots of *A* (they live in *U*)
- Understand how elimination matrices (E_{ij}'s) are constructed from multipliers (I_{ij}'s)
- ▶ Understand how *L* is made up of inverses of elimination matrices

• e.g.:
$$L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} A$$
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- Understand matrix multiplication
- ▶ Understand AB = BA is rarely true

$$(AB)^{-1} = B^{-1}A^{-1}$$



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Inverses

- Understand identity matrix /
- ▶ Understand $AA^{-1} = A^{-1}A = I$
- ▶ Find A^{-¹} with Gauss-Jordan elimination
- Perform row reduction on augmented matrix [A|I]
- ▶ Understand that that finding A^{-1} solves $A\bar{x} = b$ but is often prohibitively expensive to do.
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Matrix algebra

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- Definition: flip entries across main diagonal
- $ightharpoonup A = A^{\mathrm{T}}$: A is symmetric
- ▶ Important property: $(AB)^T = B^T A^T$

Extra pieces

- ▶ If $A\vec{x} = \vec{0}$ has a non-zero solution, A has no inverse
- ▶ If $A\vec{x} = \vec{0}$ has a non-zero solution, then $A\vec{x} = \vec{b}$ always has infinitely many solutions.
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