

Social Contagion

Principles of Complex Systems

Course 300, Fall, 2008

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Models

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Granovetter's model
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Examples abound

- ▶ fashion
- ▶ striking
- ▶ smoking (田) [6]
- ▶ residential segregation [15]
- ▶ ipods
- ▶ obesity (田) [5]
- ▶ Harry Potter
- ▶ voting
- ▶ gossip
- ▶ Rubik's cube 
- ▶ religious beliefs
- ▶ **leaving lectures**

SIR and SIRS contagion possible

- ▶ Classes of behavior versus specific behavior: **dieting**

Framingham heart study:

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Evolving network stories:

- ▶ The spread of quitting smoking (田) [6]
- ▶ The spread of spreading (田) [5]

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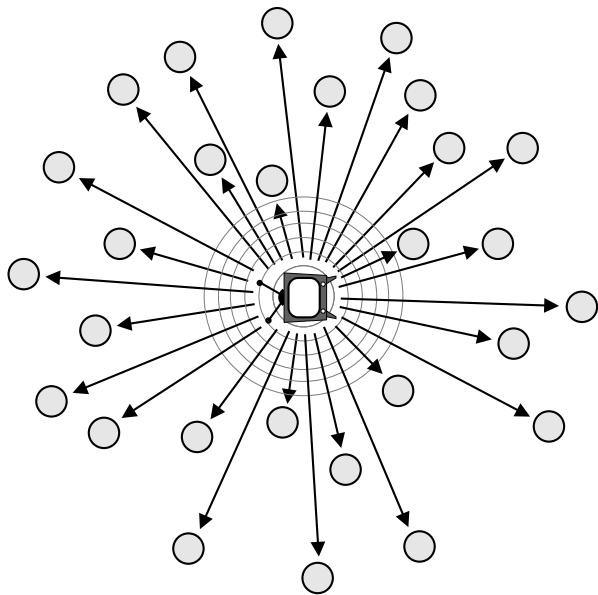
Two focuses for us

- ▶ Widespread media influence
- ▶ Word-of-mouth influence

We need to understand influence

- ▶ Who influences whom? Very hard to measure...
- ▶ What kinds of influence response functions are there?
- ▶ Are some individuals super influencers?
Highly popularized by Gladwell^[8] as 'connectors'
- ▶ The infectious idea of opinion leaders (Katz and Lazarsfeld)^[12]

The hypodermic model of influence

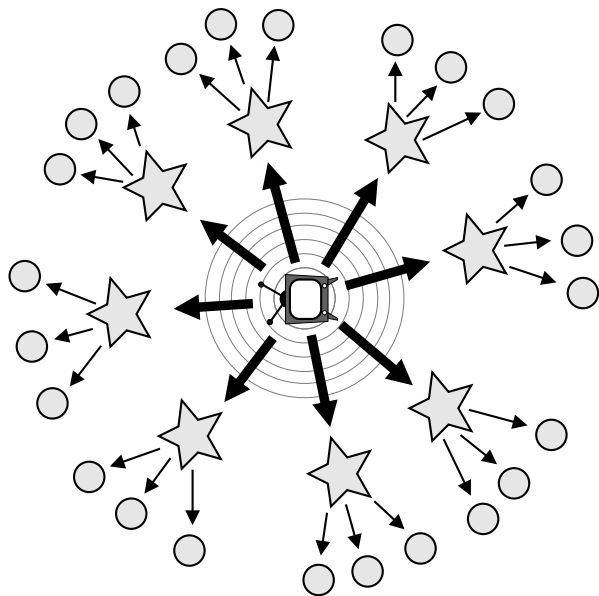


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The two step model of influence [12]

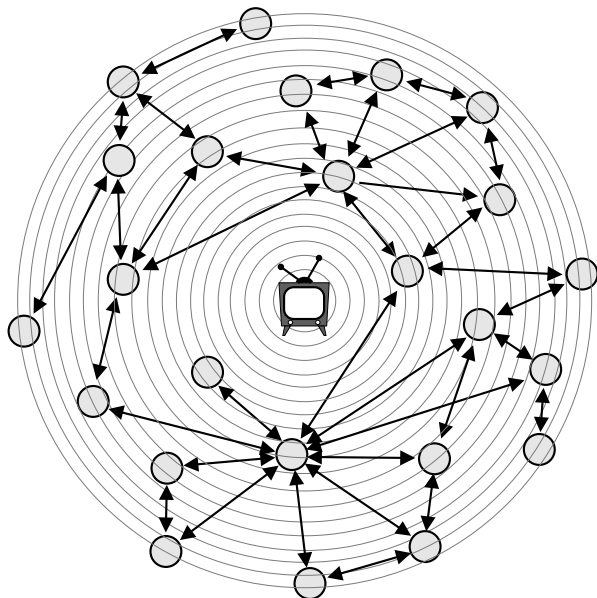


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The general model of influence



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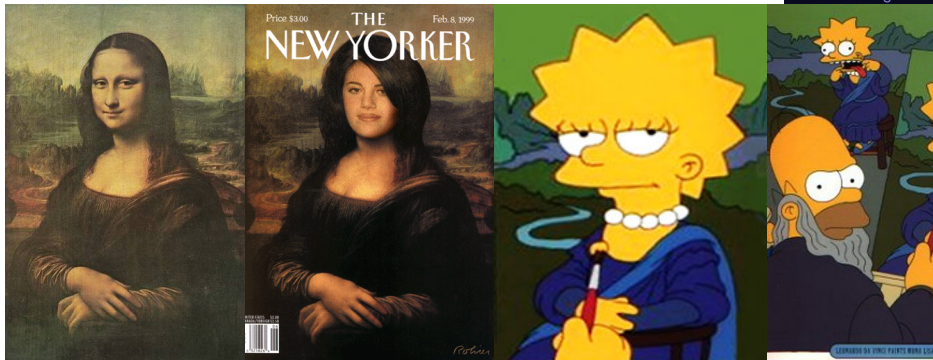
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Why do things spread?

- ▶ Because of system level properties?
- ▶ Or properties of special individuals?
- ▶ Is the match that lights the fire important?
- ▶ Yes. But only because we are narrative-making machines...
- ▶ We like to think things happened for reasons...
- ▶ System/group properties harder to understand
- ▶ Always good to examine what is said before and after the fact...

The Mona Lisa



- ▶ “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- ▶ Not the world’s greatest painting from the start...
- ▶ Escalation through theft, vandalism, **parody**, ...

The completely unpredicted fall of Eastern Europe



Timur Kuran: ^[13, 14] “Now Out of Never: The Element of Surprise in the East European Revolution of 1989”

Messaging with social connections

- ▶ Ads based on message content (e.g., Google and email)
- ▶ Buzz media
- ▶ Facebook's advertising: Beacon (田)

Getting others to do things for you

A very good book: **'Influence'** by Robert Cialdini ^[7]

Six modes of influence

1. **Reciprocation**: *The Old Give and Take... and Take*
2. **Commitment and Consistency**: *Hobgoblins of the Mind*
3. **Social Proof**: *Truths Are Us*
4. **Liking**: *The Friendly Thief*
5. **Authority**: *Directed Deference*
6. **Scarcity**: *The Rule of the Few*

- ▶ **Reciprocation**: Free samples, Hare Krishnas
- ▶ **Commitment and Consistency**: Hazing
- ▶ **Social Proof**: Catherine Genovese, Jonestown
- ▶ **Liking**: Separation into groups is enough to cause problems.
- ▶ **Authority**: Milgram's obedience to authority experiment.
- ▶ **Scarcity**: Prohibition.

Getting others to do things for you

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- ▶ Cialdini's modes are heuristics that help up us get through life.
- ▶ Useful but can be leveraged...

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Other acts of influence

- ▶ Conspicuous Consumption (Veblen, 1912)
- ▶ Conspicuous Destruction (Potlatch)

Some important models

- ▶ Tipping models—Schelling (1971) ^[15, 16, 17]
 - ▶ Simulation on checker boards
 - ▶ Idea of thresholds
 - ▶ Fun with Netlogo and Schelling's model ^[20]...
- ▶ Threshold models—Granovetter (1978) ^[9]
- ▶ Herding models—Bikhchandani, Hirschleifer, Welch (1992) ^[1, 2]
 - ▶ Social learning theory, Informational cascades,...

Thresholds

- ▶ Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- ▶ 'Others' may be everyone in a population, an individual's close friends, any reference group.
- ▶ Response can be probabilistic or deterministic.
- ▶ Individual thresholds can vary
- ▶ Assumption: order of others' adoption does not matter... **(unrealistic)**.
- ▶ Assumption: level of influence per person is uniform **(unrealistic)**.

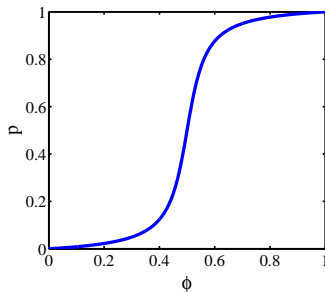
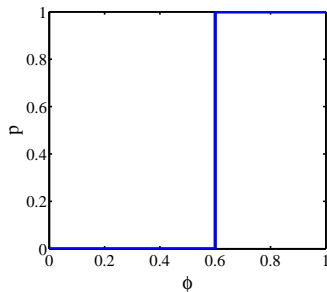
Some possible origins of thresholds:

- ▶ **Desire to coordinate**, to conform.
- ▶ **Lack of information**: impute the worth of a good or behavior based on degree of adoption (social proof)
- ▶ Economics: **Network effects** or **network externalities**
- ▶ Externalities = Effects on others not directly involved in a transaction
- ▶ Examples: telephones, fax machine, Facebook, operating systems
- ▶ An individual's utility increases with the adoption level among peers and the population in general

Granovetter's Threshold model—definitions

- ▶ ϕ^* = threshold of an individual.
- ▶ $f(\phi_*)$ = distribution of thresholds in a population.
- ▶ $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)d\phi'_*$
- ▶ ϕ_t = fraction of people 'rioting' at time step t .

Threshold models



- ▶ Example threshold influence response functions: **deterministic** and **stochastic**
- ▶ ϕ = fraction of contacts 'on' (e.g., rioting)
- ▶ Two states: S and I.

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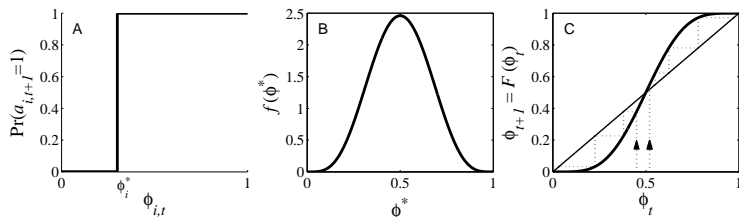
- ▶ At time $t + 1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.



$$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*) d\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$

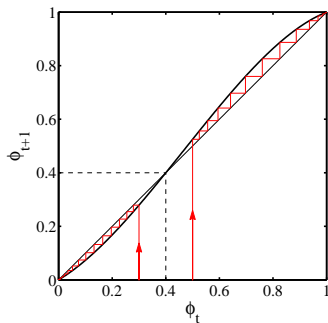
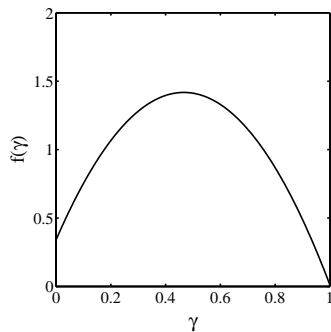
- ▶ \Rightarrow Iterative maps of the unit interval $[0, 1]$.

Action based on perceived behavior of others.



- ▶ Two states: S and I.
- ▶ ϕ = fraction of contacts 'on' (e.g., rioting)
- ▶ Discrete time update (strong assumption!)
- ▶ This is a **Critical mass model**

Threshold models

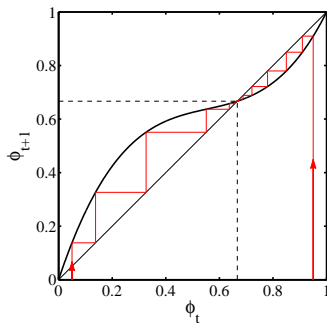
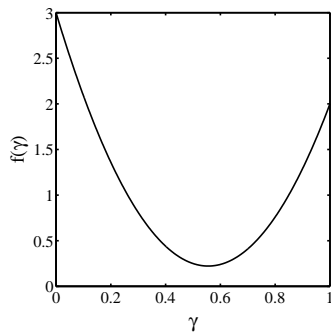


- ▶ Another example of critical mass model...

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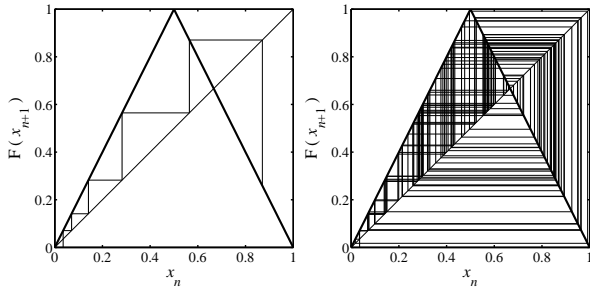
- ▶ Example of single stable state model

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes \Rightarrow large global changes

Threshold models

Chaotic behavior possible [11, 10]



- ▶ Period doubling arises as map amplitude r is increased.
- ▶ Synchronous update assumption is crucial

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Many years after Granovetter and Soong's work:

“A simple model of global cascades on random networks”

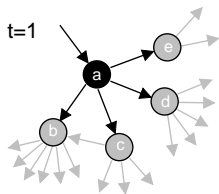
D. J. Watts. Proc. Natl. Acad. Sci., 2002^[19]

- ▶ Mean field model \rightarrow network model
- ▶ Individuals now have a limited view of the world

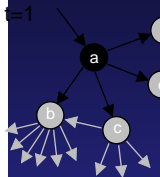
Threshold model on a network

- ▶ Interactions between individuals now represented by a network
- ▶ Network is **sparse**
- ▶ Individual i has k_i contacts
- ▶ Influence on each link is **reciprocal** and of **unit weight**
- ▶ Each individual i has a fixed threshold ϕ_i
- ▶ Individuals repeatedly poll contacts on network
- ▶ Synchronous, discrete time updating
- ▶ Individual i becomes active when fraction of active contacts $a_i \geq \phi_i k_i$
- ▶ Individuals remain active when switched (no recovery = SI model)

Threshold model on a network



- ▶ All nodes have threshold $\phi = 0.2$.



The Cascade Condition:

If one individual is initially activated, what is the probability that an activation will spread over a network?

What features of a network determine whether a cascade will occur or not?

First study random networks:

- ▶ Start with N nodes with a degree distribution p_k
- ▶ Nodes are randomly connected (carefully so)
- ▶ Aim: Figure out when activation will propagate
- ▶ Determine a **cascade condition**

Follow active links

- ▶ An active link is a link connected to an activated node.
- ▶ If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- ▶ We need to understand which nodes can be activated when only one of their neighbors becomes active.

The most gullible

Vulnerables:

- ▶ We call individuals who can be activated by just one contact being active **vulnerables**
- ▶ The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

- ▶ Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- ▶ For global cascades on random networks, must have a *global cluster of vulnerables* ^[19]
- ▶ **Cluster of vulnerables = critical mass**
- ▶ Network story: 1 node \rightarrow critical mass \rightarrow everyone.

Cascade condition

Back to following a link:

- ▶ Link from leads to a node with probability $\propto kP_k$.
- ▶ Follows from links being random + having k chances to connect to a node with degree k .
- ▶ Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle = z$$

- ▶ So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

Cascade condition

Next: Vulnerability of linked node

- ▶ Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_* = 0}^{1/k} f(\phi'_*) d\phi'_*$$

- ▶ If linked node is **vulnerable**, it produces **$k - 1$ new** outgoing active links
- ▶ If linked node is **not vulnerable**, it produces **no** active links.

Putting things together:

- ▶ Expected number of active edges produced by an active edge =



$$\sum_{k=1}^{\infty} \underbrace{(k-1)\beta_k \frac{kP_k}{z}}_{\text{success}} + \underbrace{0(1-\beta_k) \frac{kP_k}{z}}_{\text{failure}}$$



$$= \sum_{k=1}^{\infty} (k-1)k\beta_k P_k / z$$

So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} k(k-1)\beta_k P_k/z \geq 1.$$

- ▶ β_k = probability a degree k node is vulnerable.
- ▶ P_k = probability a node has degree k .

Two special cases:

- ▶ (1) Simple disease-like spreading succeeds: $\beta_k = \beta$



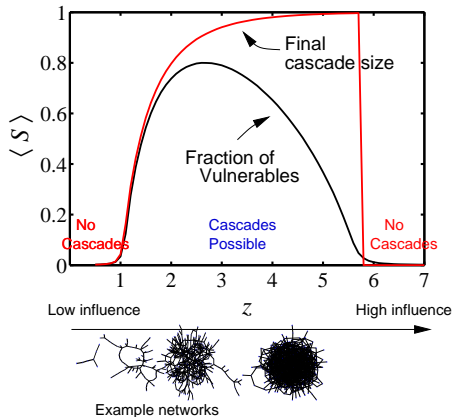
$$\beta \sum_{k=1}^{\infty} k(k-1)P_k/z \geq 1.$$

- ▶ (2) Giant component exists: $\beta = 1$



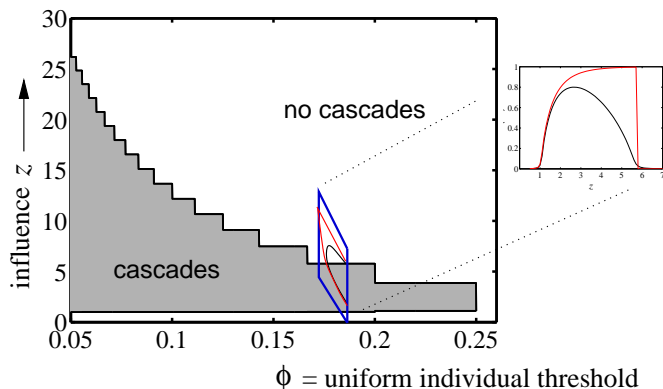
$$\sum_{k=1}^{\infty} k(k-1)P_k/z \geq 1.$$

Cascades on random networks



- ▶ Cascades occur only if size of max vulnerable cluster > 0 .
- ▶ System may be 'robust-yet-fragile'.
- ▶ 'Ignorance' facilitates spreading.

Cascade window for random networks



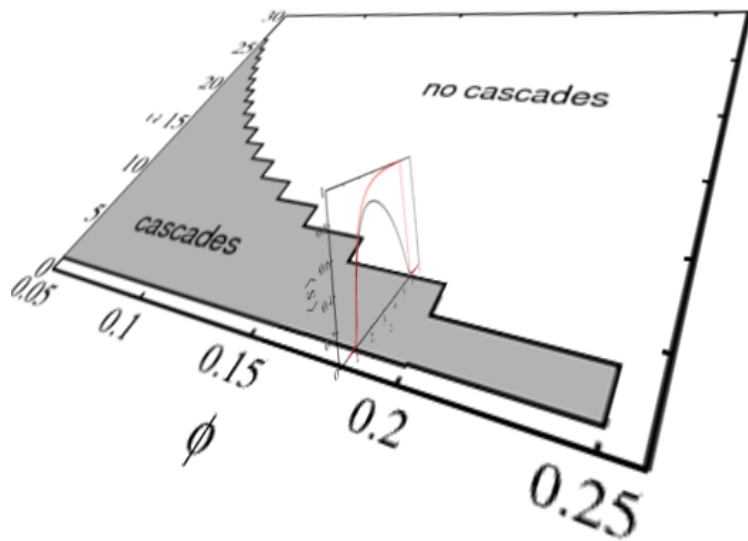
- ▶ 'Cascade window' widens as threshold ϕ decreases.
- ▶ Lower thresholds enable spreading.

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Cascade window for random networks



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Cascade window—summary

For our simple model of a uniform threshold:

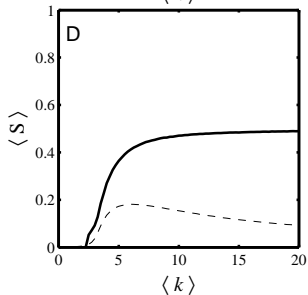
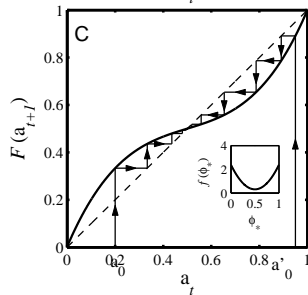
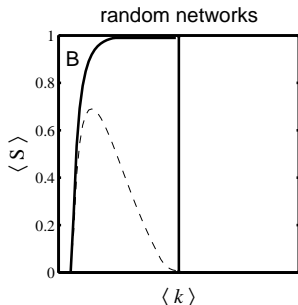
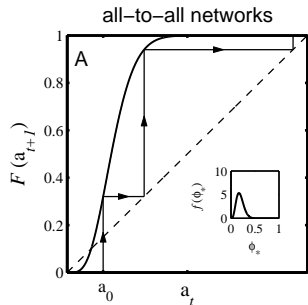
1. **Low** $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. **High** $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. **Intermediate** $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in **“Cascade window.”**

All-to-all versus random networks

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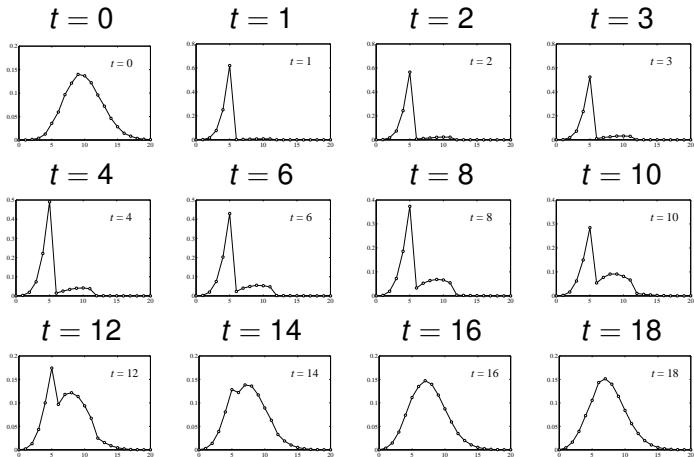


Early adopters—degree distributions

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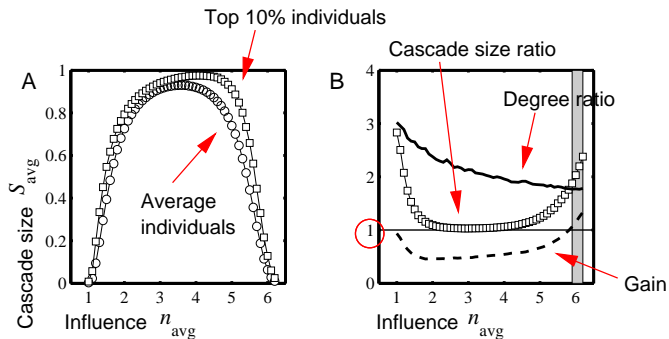
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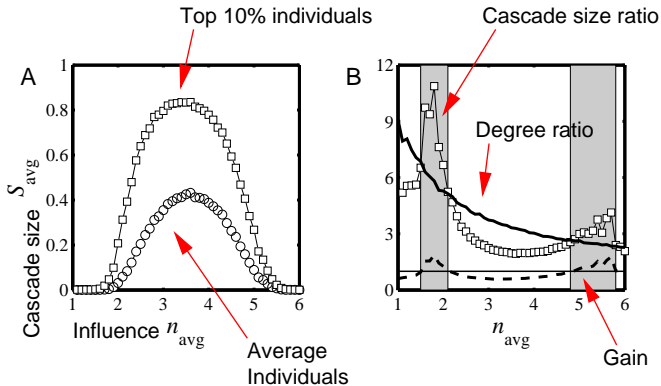
$P_{k,t}$ versus k

The multiplier effect:



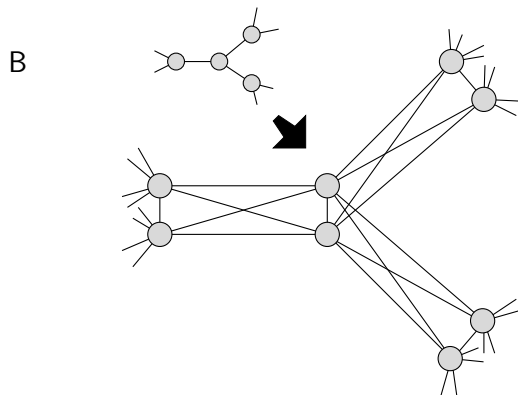
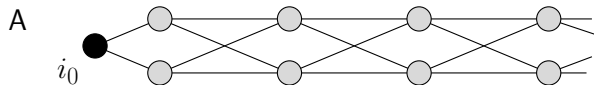
- ▶ Fairly uniform levels of individual influence.
- ▶ Multiplier effect is mostly below 1.

The multiplier effect:



- ▶ Skewed influence distribution example.

Special subnetworks can act as triggers



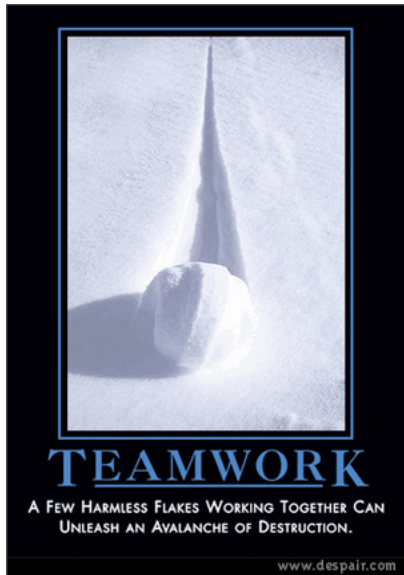
- ▶ $\phi = 1/3$ for all nodes

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The power of groups...



“A few harmless flakes working together can unleash an avalanche of destruction.”

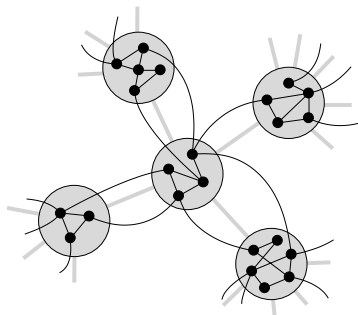
- ▶ Assumption of sparse interactions is good
- ▶ Degree distribution is (generally) key to a network's function
- ▶ Still, random networks don't represent all networks
- ▶ Major element missing: **group structure**

Group structure—Ramified random networks

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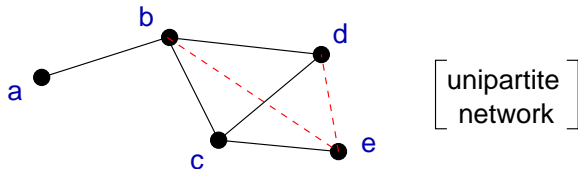
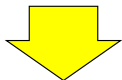
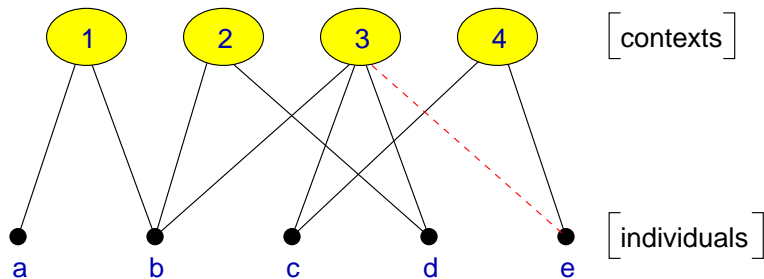
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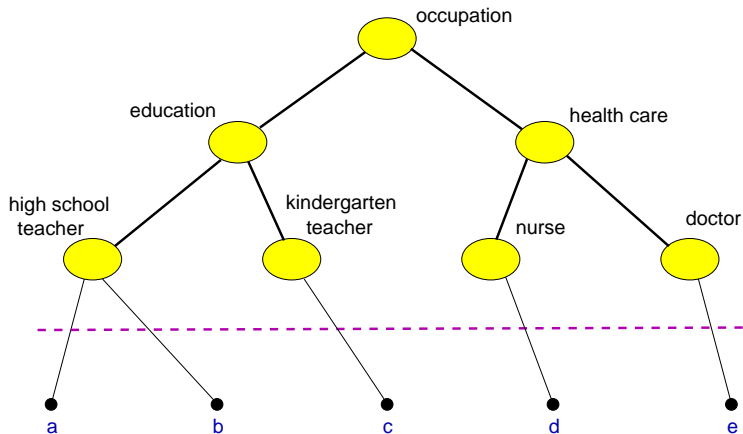


p = intergroup connection probability
 q = intragroup connection probability.

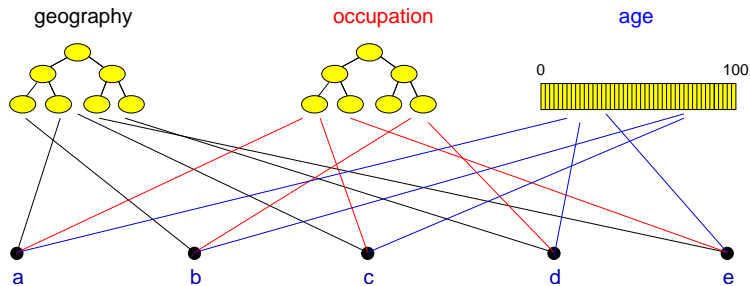
Bipartite networks



Context distance



Generalized affiliation model



(Blau & Schwartz, Simmel, Breiger)

Generalized affiliation model networks with triadic closure

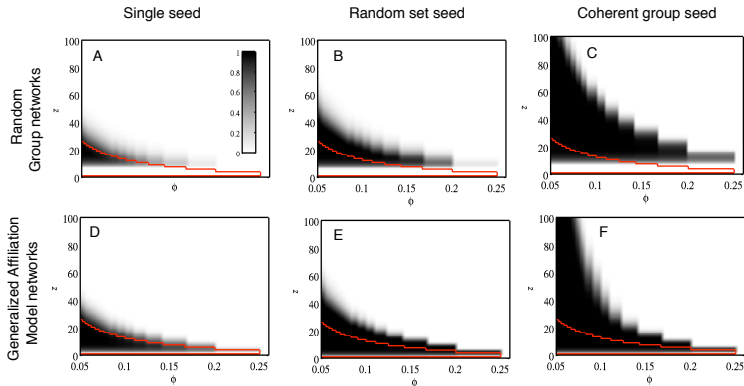
- ▶ Connect nodes with probability $\propto \exp^{-\alpha d}$
where
 α = homophily parameter
and
 d = distance between nodes (height of lowest common ancestor)
- ▶ τ_1 = intergroup probability of friend-of-friend connection
- ▶ τ_2 = intragroup probability of friend-of-friend connection

Cascade windows for group-based networks

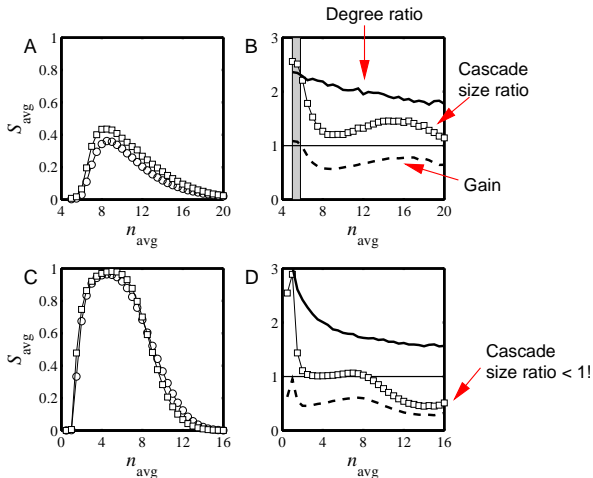
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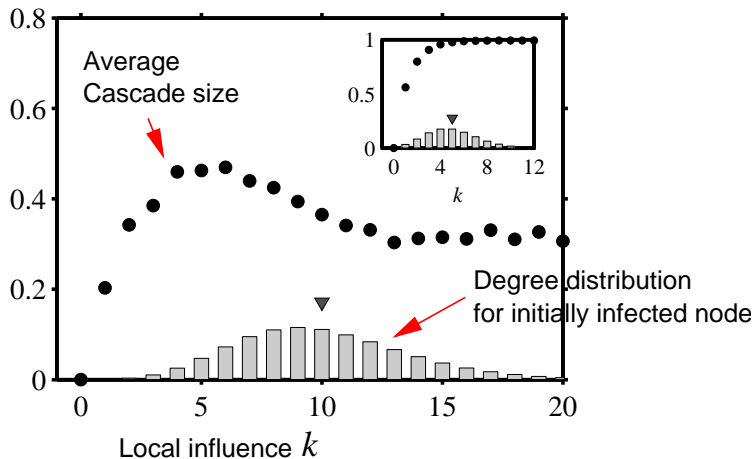


Multiplier effect for group-based networks:



► Multiplier almost always below 1.

Assortativity in group-based networks



- ▶ The most connected nodes aren't always the most 'influential.'
- ▶ **Degree assortativity** is the reason.

Summary

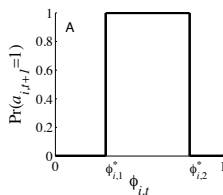
- ▶ 'Influential vulnerables' are key to spread.
- ▶ Early adopters are mostly vulnerables.
- ▶ Vulnerable nodes important but not necessary.
- ▶ Groups may greatly facilitate spread.
- ▶ Seems that cascade condition is a global one.
- ▶ Most extreme/unexpected cascades occur in highly connected networks
- ▶ 'Influentials' are posterior constructs.
- ▶ Many potential influentials exist.

Implications

- ▶ Focus on **the influential vulnerables**.
- ▶ Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- ▶ Only **simple ideas** can spread by word-of-mouth.
(Idea of opinion leaders spreads well...)
- ▶ Want enough individuals who will adopt and display.
- ▶ Displaying can be **passive** = free (yo-yo's, fashion), or **active** = harder to achieve (political messages).
- ▶ Entities can be novel or designed to combine with others, e.g. block another one.

Chaotic contagion:

- ▶ What if individual response functions are not monotonic?
- ▶ Consider a simple deterministic version:
- ▶ Node i has an 'activation threshold' $\phi_{i,1}$
... and a 'de-activation threshold' $\phi_{i,2}$
- ▶ Nodes like to imitate but only up to a limit—they don't want to be like everyone else.

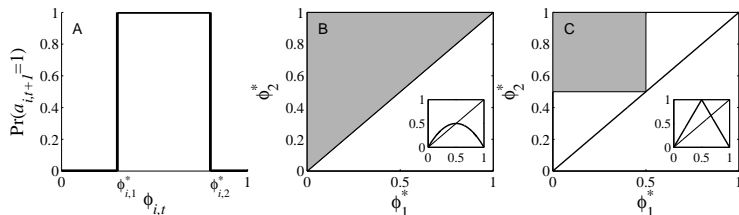


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Two population examples:



- ▶ Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- ▶ Insets show composite response function averaged over population.
- ▶ We'll consider plot C's example: [the tent map](#).

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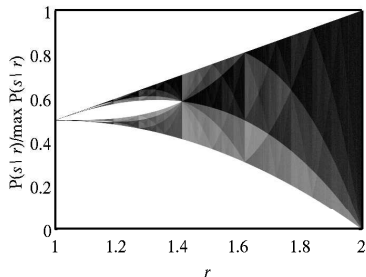
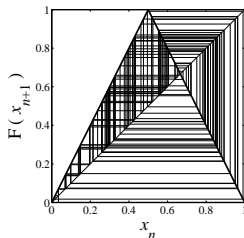
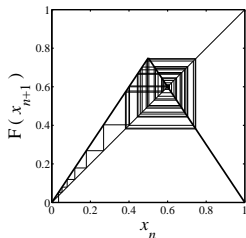
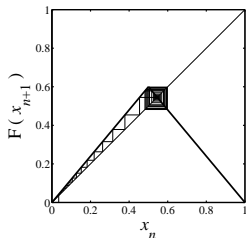
Definition of the tent map:

$$F(x) = \begin{cases} rx & \text{for } 0 \leq x \leq \frac{1}{2}, \\ r(1-x) & \text{for } \frac{1}{2} \leq x \leq 1. \end{cases}$$

- ▶ The usual business: look at how F iteratively maps the unit interval $[0, 1]$.

The tent map

Effect of increasing r from 1 to 2.



Orbit diagram:

Chaotic behavior increases as map slope r is increased.

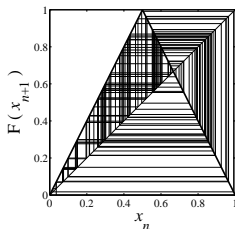
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Chaotic behavior

Take $r = 2$ case:



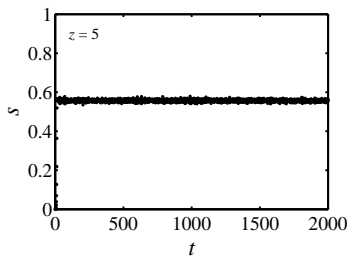
- ▶ What happens if nodes have limited information?
- ▶ As before, allow interactions to take place on a sparse random network.
- ▶ Vary average degree $z = \langle k \rangle$, a measure of information

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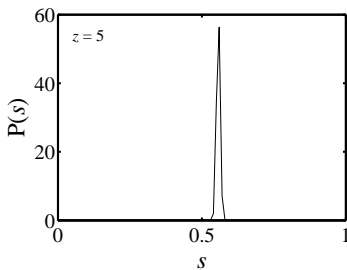
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Invariant densities—stochastic response functions



activation time series



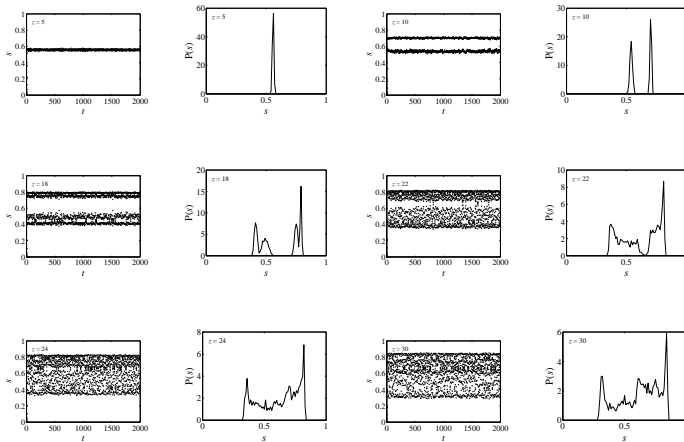
activation density

Invariant densities—stochastic response functions

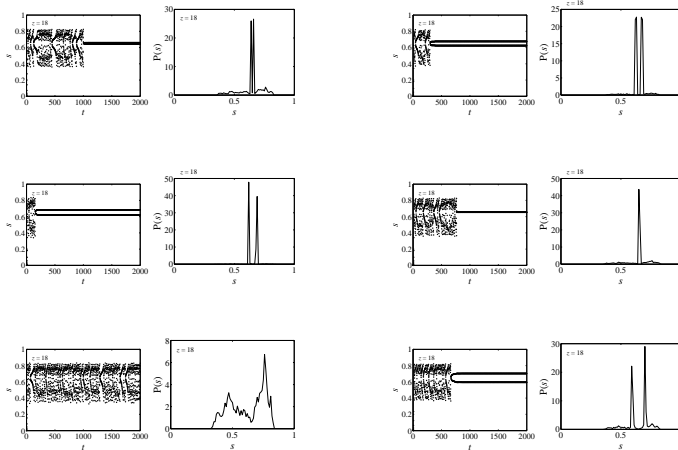
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Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$

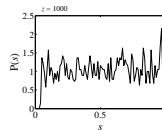
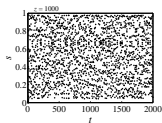
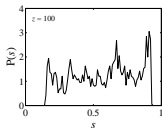
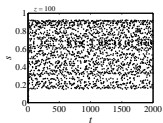


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Invariant densities—stochastic response functions



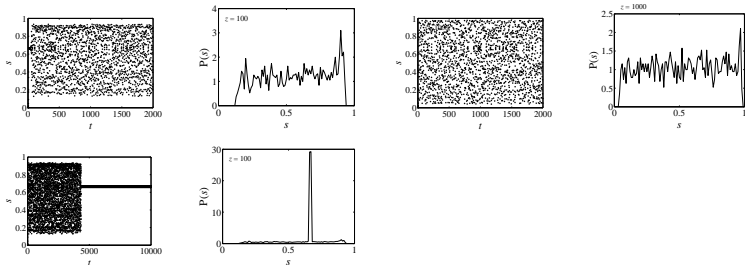
Trying out higher values of $\langle k \rangle$...

Invariant densities—deterministic response functions

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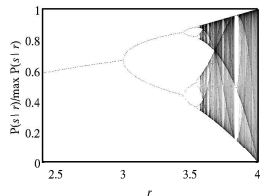
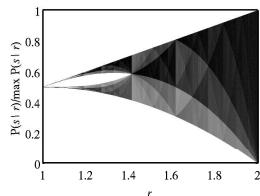
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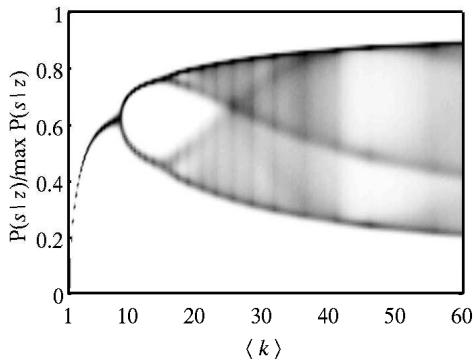


Trying out higher values of $\langle k \rangle$...

Connectivity leads to chaos:



Stochastic response functions:



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Chaotic behavior in coupled systems

Coupled maps are well explored
(Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

► \mathcal{N}_i = neighborhood of node i

1. Node states are **continuous**
2. Increase δ and neighborhood size $|\mathcal{N}|$
⇒ synchronization

But for contagion model:

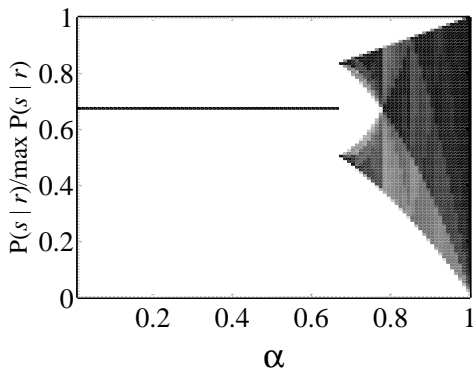
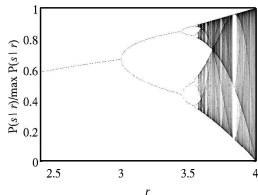
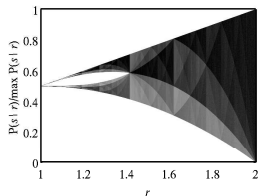
1. Node states are **binary**
2. **Asynchrony remains** as connectivity increases

Bifurcation diagram: Asynchronous updating




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