

Biological Contagion

Principles of Complex Systems

Course 300, Fall, 2008

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A confusion of contagions:

- ▶ Is Harry Potter some kind of virus?
- ▶ What about the Da Vinci Code?
- ▶ Does Sudoku spread like a disease?
- ▶ Religion?
- ▶ Democracy...?

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Naturomorphisms

- ▶ “The feeling was contagious.”
- ▶ “The news spread like wildfire.”
- ▶ “Freedom is the most contagious virus known to man.”
—Hubert H. Humphrey, Johnson’s vice president
- ▶ “Nothing is so contagious as enthusiasm.”
—Samuel Taylor Coleridge

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Optimism according to Ambrose Bierce: (田)

The doctrine that everything is beautiful, including what is ugly, everything good, especially the bad, and everything right that is wrong. ...

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Optimism according to Ambrose Bierce: (田)

The doctrine that everything is beautiful, including what is ugly, everything good, especially the bad, and everything right that is wrong. ... **It is hereditary, but fortunately not contagious.**

Social contagion

Eric Hoffer, 1902–1983

There is a grandeur in the uniformity of the mass. When a fashion, a dance, a song, a slogan or a joke sweeps like wildfire from one end of the continent to the other, and a hundred million people roar with laughter, sway their bodies in unison, hum one song or break forth in anger and denunciation, there is the overpowering feeling that in this country we have come nearer the brotherhood of man than ever before.

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The spread of fanaticism

Hoffer's acclaimed work:

"The True Believer:

Thoughts On The Nature Of Mass Movements" (1951) [3]

Quotes-aplenty:

- ▶ "We can be absolutely certain only about things we do not understand."
- ▶ "Mass movements can rise and spread without belief in a God, but never without belief in a devil."
- ▶ "Where freedom is real, equality is the passion of the masses. Where equality is real, freedom is the passion of a small minority."

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despair.com

“When people are free to do as they please, they usually imitate each other.”

—Eric Hoffer
“The Passionate State of Mind” [4]

The collective...

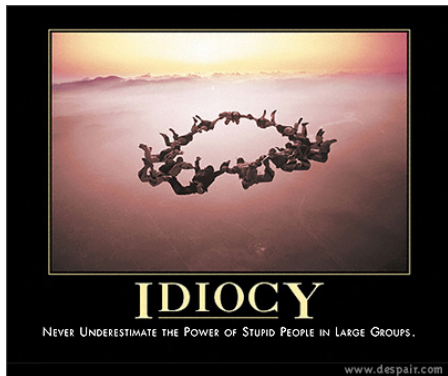
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“Never Underestimate
the Power of Stupid
People in Large
Groups.”

despair.com

Definitions

- ▶ (1) The spreading of a quality or quantity between individuals in a population.
- ▶ (2) A disease itself:
the plague, a blight, the dreaded lurgi, ...
- ▶ from Latin: *con* = 'together with' + *tangere* 'to touch.'
- ▶ Contagion has unpleasant overtones...
- ▶ Just **Spreading** might be a more neutral word
- ▶ But contagion is kind of exciting...

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Examples of non-disease spreading:

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Interesting infections:

- ▶ Spreading of buildings in the US. (田)
- ▶ Spreading of spreading (田).
- ▶ Viral get-out-the-vote video. (田)

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Two main classes of contagion

1. Infectious diseases
2. Social contagion

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1. **Infectious diseases**

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1. **Infectious diseases:**
tuberculosis, HIV, ebola, SARS, influenza, ...
2. **Social contagion**

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Two main classes of contagion

1. **Infectious diseases:**
tuberculosis, HIV, ebola, SARS, influenza, ...
2. **Social contagion:**
fashion, word usage, rumors, riots, religion, ...

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The standard SIR model^[8]

- ▶ The basic model of disease contagion
- ▶ Three states:
 1. S = Susceptible
 2. I = Infective/Infectious
 3. R = Recovered
- ▶ $S(t) + I(t) + R(t) = 1$
- ▶ Presumes random interactions (mass-action principle)
- ▶ Interactions are independent (no memory)
- ▶ Discrete and continuous time versions

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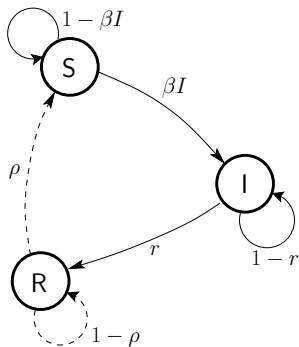
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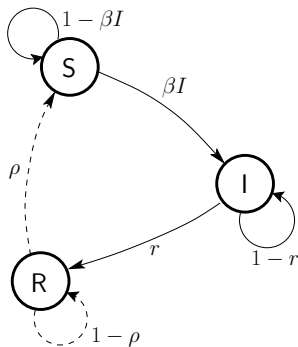
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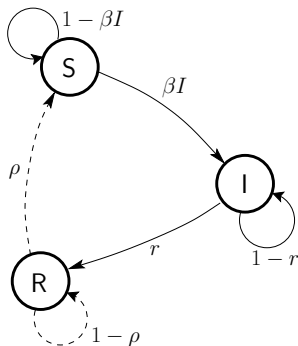


Discrete time automata example:



Transition Probabilities:

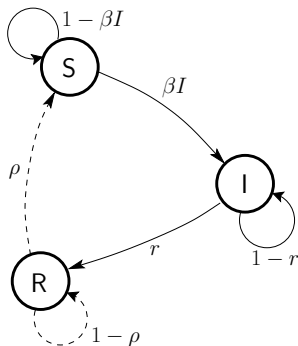
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Transition Probabilities:

β for being infected given contact with infected

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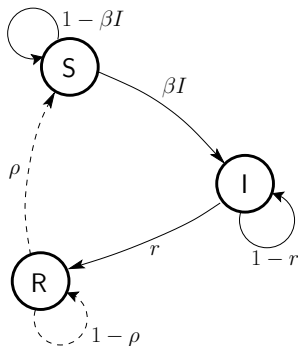


Transition Probabilities:

β for being infected given
contact with infected

r for recovery

Discrete time automata example:



Transition Probabilities:

β for being infected given
contact with infected

r for recovery

ρ for loss of immunity

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Original models attributed to

- ▶ 1920's: Reed and Frost
- ▶ 1920's/1930's: Kermack and McKendrick [5, 7, 6]
- ▶ Coupled differential equations with a mass-action principle

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Independent Interaction models

Differential equations for continuous model

$$\frac{d}{dt}S = -\beta IS + \rho R$$

$$\frac{d}{dt}I = \beta IS - rI$$

$$\frac{d}{dt}R = rI - \rho R$$

β , r , and ρ are now **rates**.

Reproduction Number R_0 :

- ▶ R_0 = expected number of infected individuals resulting from a single initial infective
- ▶ Epidemic threshold: If $R_0 > 1$, 'epidemic' occurs.

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Reproduction Number R_0

Discrete version:

- ▶ Set up: One Infective in a randomly mixing population of Susceptibles
- ▶ At time $t = 0$, single infective random bumps into a Susceptible
- ▶ Probability of transmission = β
- ▶ At time $t = 1$, single Infective remains infected with probability $1 - r$
- ▶ At time $t = k$, single Infective remains infected with probability $(1 - r)^k$

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Reproduction Number R_0

Discrete version:

- ▶ Expected number infected by original Infective:

$$R_0 = \beta + (1 - r)\beta + (1 - r)^2\beta + (1 - r)^3\beta + \dots$$

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- ▶ Expected number infected by original Infective:

$$R_0 = \beta + (1 - r)\beta + (1 - r)^2\beta + (1 - r)^3\beta + \dots$$

$$= \beta \left(1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots \right)$$

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$$= \beta \left(1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots \right)$$

$$= \beta \frac{1}{1 - (1 - r)}$$

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$$= \beta \left(1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots \right)$$

$$= \beta \frac{1}{1 - (1 - r)} = \beta/r$$

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Reproduction Number R_0

Discrete version:

- ▶ Expected number infected by original Infective:

$$R_0 = \beta + (1 - r)\beta + (1 - r)^2\beta + (1 - r)^3\beta + \dots$$

$$= \beta \left(1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots \right)$$

$$= \beta \frac{1}{1 - (1 - r)} = \beta/r$$

For S_0 initial infectives ($1 - S_0 = R_0$ immune):

$$R_0 = S_0\beta/r$$

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Independent Interaction models

For the continuous version

- ▶ Second equation:

$$\frac{d}{dt}I = \beta SI - rI$$

- ▶ Number of infectives grows initially if

$$\beta S(0) - r > 0$$

- ▶ Same story as for discrete model.

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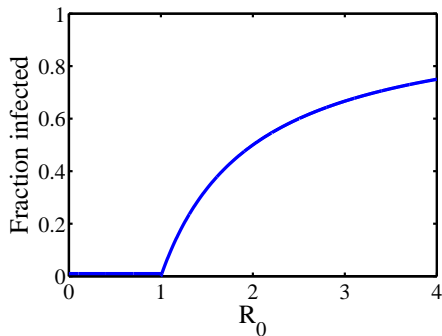
Background

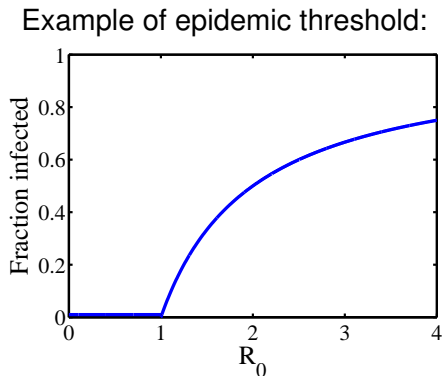
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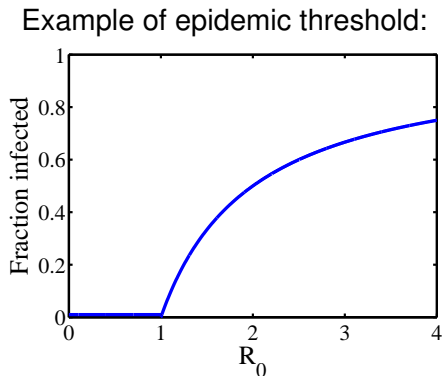
Independent Interaction models

Example of epidemic threshold:





- ▶ Continuous phase transition.



- ▶ Continuous phase transition.
- ▶ Fine idea from a simple model.

Independent Interaction models

Many variants of the SIR model:

- ▶ **SIS**: susceptible-infective-susceptible
- ▶ **SIRS**: susceptible-infective-recovered-susceptible
- ▶ compartment models (age or gender partitions)
- ▶ more categories such as 'exposed' (**SEIRS**)
- ▶ recruitment (migration, birth)

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For novel diseases:

1. Can we predict the size of an epidemic?
2. How important is the reproduction number R_0 ?

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R_0 and variation in epidemic sizes

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R_0 approximately same for all of the following:

- ▶ 1918-19 “Spanish Flu” ~ 500,000 deaths in US
- ▶ 1957-58 “Asian Flu” ~ 70,000 deaths in US
- ▶ 1968-69 “Hong Kong Flu” ~ 34,000 deaths in US
- ▶ 2003 “SARS Epidemic” ~ 800 deaths world-wide

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Size distributions

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Size distributions are important elsewhere:

- ▶ earthquakes (Gutenberg-Richter law)
- ▶ city sizes, forest fires, war fatalities
- ▶ wealth distributions
- ▶ 'popularity' (books, music, websites, ideas)
- ▶ Epidemics?

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- ▶ 'popularity' (books, music, websites, ideas)
- ▶ **Epidemics?**

Power laws distributions are common but not obligatory...

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Really, what about epidemics?

- ▶ Simply hasn't attracted much attention.
- ▶ Data not as clean as for other phenomena.

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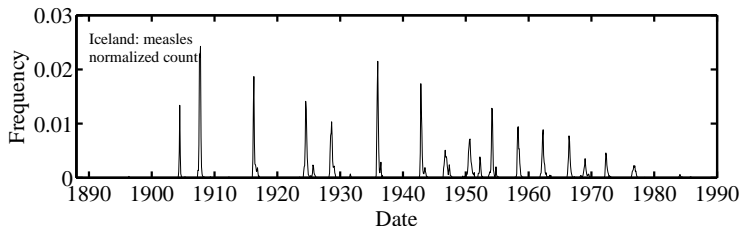
References

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Feeling Ill in Iceland

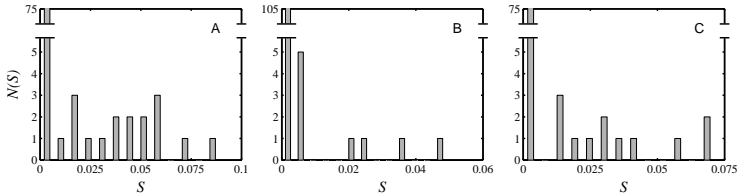
Caseload recorded monthly for range of diseases in
Iceland, 1888-1890



Treat outbreaks separated in time as 'novel' diseases.

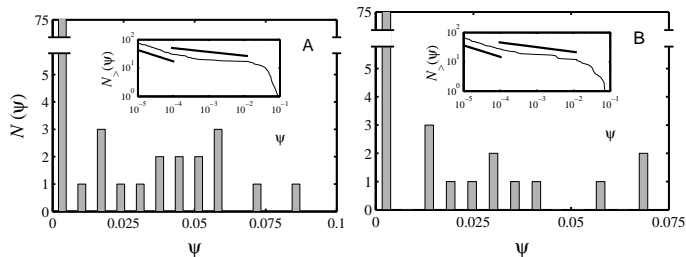
Really not so good at all in Iceland

Epidemic size distributions $N(S)$ for
Measles, Rubella, and Whooping Cough.



Spike near $S = 0$, relatively flat otherwise.

Measles & Pertussis



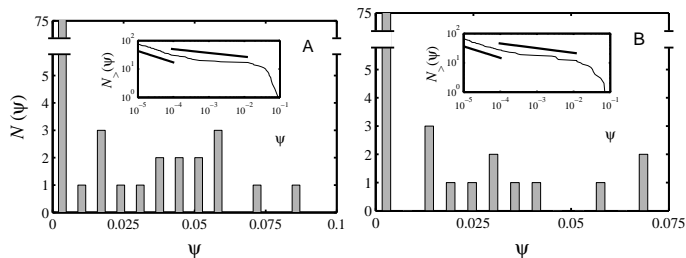
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Insert plots:

Complementary cumulative frequency distributions:

$$N(\psi' > \psi) \propto \psi^{-\gamma+1}$$

Limited scaling with a possible break.

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Power law distributions

Measured values of γ :

- ▶ measles: 1.40 (low Ψ) and 1.13 (high Ψ)
- ▶ pertussis: 1.39 (low Ψ) and 1.16 (high Ψ)

- ▶ Expect $2 \leq \gamma < 3$ (finite mean, infinite variance)
- ▶ When $\gamma < 1$, can't normalize
- ▶ Distribution is quite flat.

Power law distributions

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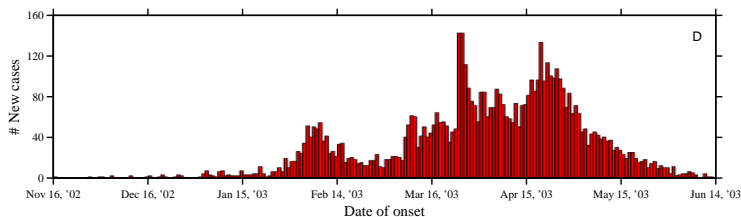
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Resurgence—example of SARS



- ▶ Epidemic slows...
- ▶ Epidemic discovers new 'pools' of susceptibles:
Resurgence.
- ▶ Importance of rare, stochastic events.

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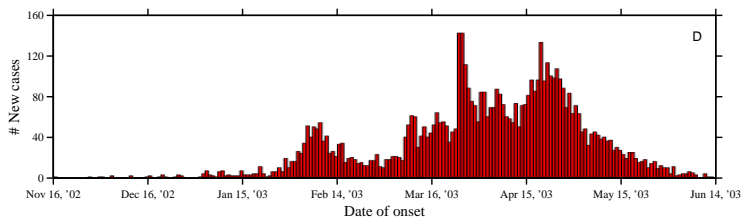
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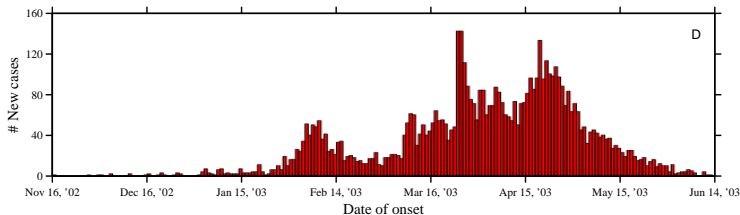
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Resurgence—example of SARS



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then an infective moves to a new context.
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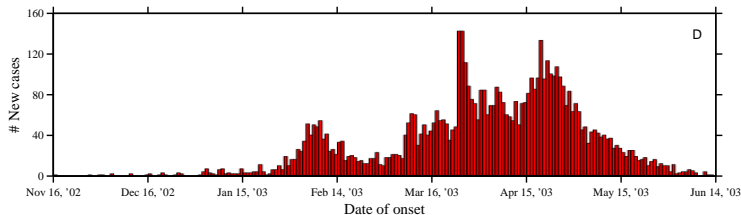
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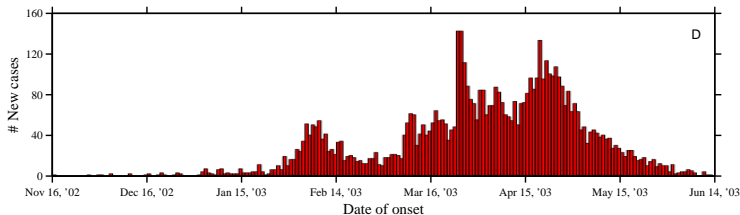
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Resurgence—example of SARS



- ▶ Epidemic slows...
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- ▶ Epidemic discovers new 'pools' of susceptibles:
Resurgence.
- ▶ **Importance of rare, stochastic events.**

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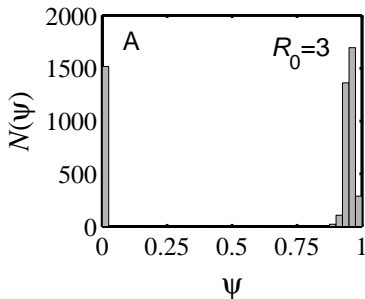
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So... can a simple model produce

1. **broad epidemic distributions**
and
2. **resurgence ?**

Size distributions



Simple models typically produce **bimodal** or **unimodal** size distributions.

- ▶ This **includes** network models:
random, small-world, scale-free, ...
- ▶ Exceptions:
 1. Forest fire models
 2. Sophisticated metapopulation models

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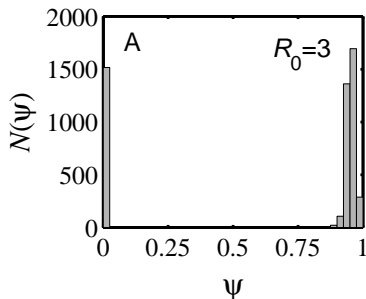
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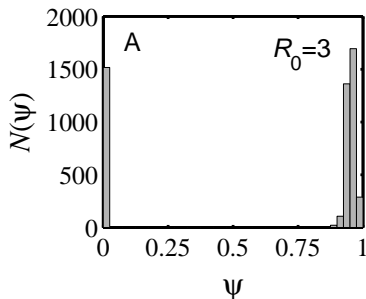
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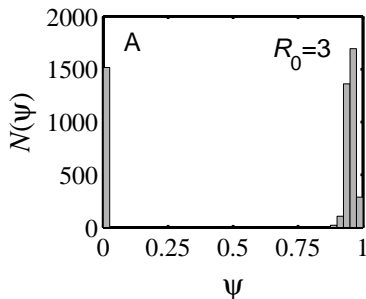
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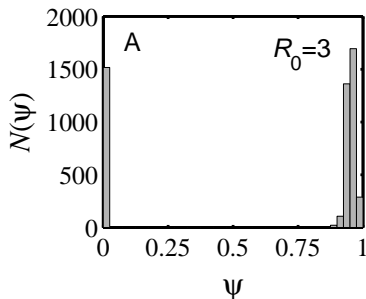
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Burning through the population

Forest fire models: [9]

- ▶ Rhodes & Anderson, 1996
- ▶ The physicist's approach:
“if it works for magnets, it'll work for people...”

A bit of a stretch:

1. Epidemics \equiv forest fires spreading on 3-d and 5-d lattices.
2. Claim Iceland and Faroe Islands exhibit power law distributions for outbreaks.
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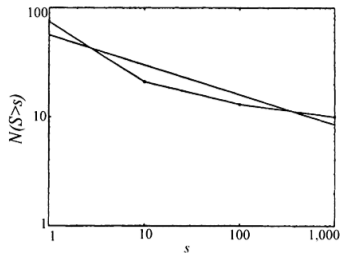
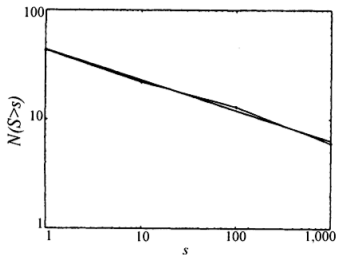
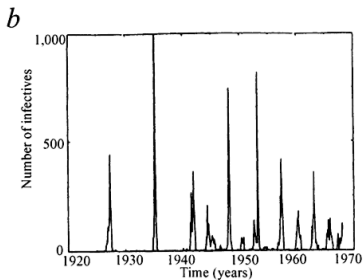
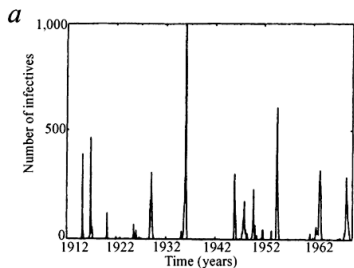
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Size distributions



From Rhodes and Anderson, 1996.

Sophisticated metapopulation models

- ▶ **Community based mixing: Longini (two scales).**
- ▶ Eubank et al.'s EpiSims/TRANSIMS—city simulations.
- ▶ Spreading through countries—Airlines: Germann et al., Corlizza et al.
- ▶ Vital work but perhaps hard to generalize from...
- ▶ ⇒ Create a simple model involving multiscale travel
- ▶ Multiscale models suggested by others but not formalized (Bailey, Cliff and Haggett, Ferguson et al.)

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 - ▶ Multiscale models suggested by others but not formalized (Bailey, Cliff and Haggett, Ferguson et al.)

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Sophisticated metapopulation models

- ▶ Community based mixing: Longini (two scales).
- ▶ Eubank et al.'s EpiSims/TRANSIMS—city simulations.
- ▶ Spreading through countries—Airlines: Germann et al., Corlizza et al.
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- ▶ Very big question: **What is N ?**
- ▶ Should we model SARS in Hong Kong as spreading
- ▶ For simple models, we need to know the final size beforehand...

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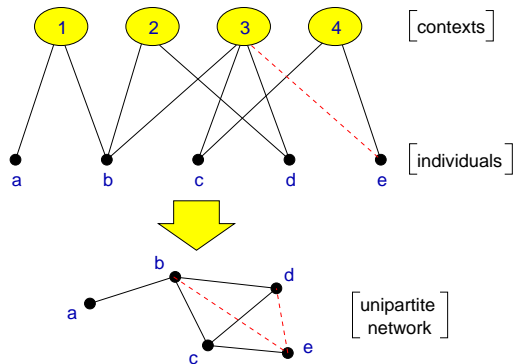
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Contexts and Identities—Bipartite networks



- ▶ boards of directors
- ▶ movies
- ▶ transportation modes (subway)

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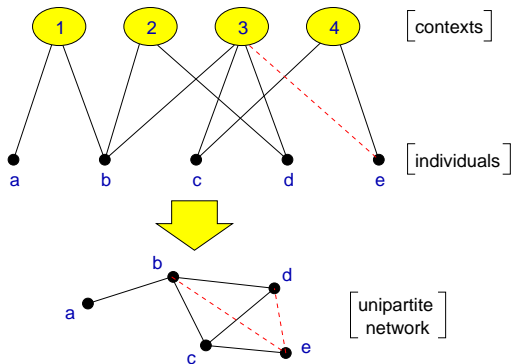
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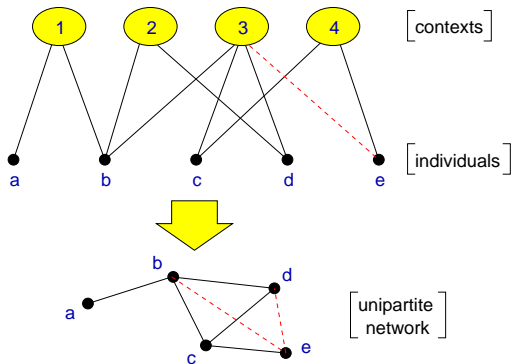
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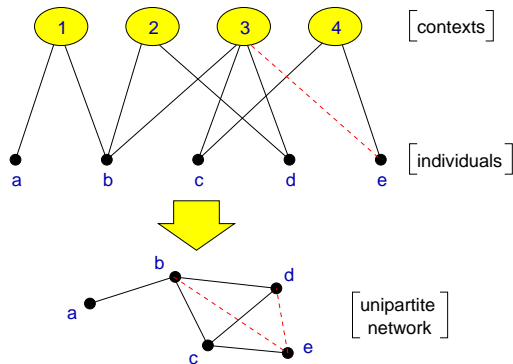
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Idea for social networks: incorporate **identity**.

Identity is formed from attributes such as:

- ▶ Geographic location
- ▶ Type of employment
- ▶ Age
- ▶ Recreational activities

Groups are crucial...

- ▶ formed by people with at least one similar attribute
- ▶ Attributes \leftrightarrow Contexts \leftrightarrow Interactions \leftrightarrow Networks. [11]

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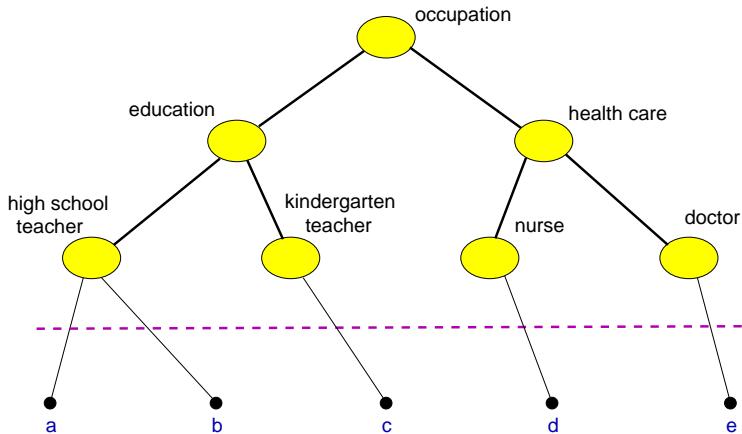
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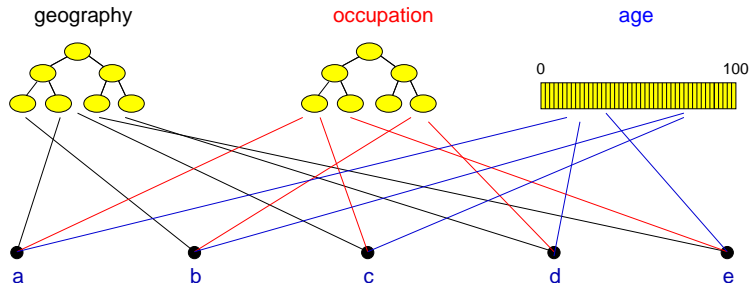
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Infer interactions/network from identities



Distance makes sense in identity/context space.

Generalized context space



(Blau & Schwartz^[1], Simmel^[10], Breiger^[2])

A toy agent-based model

Geography—allow people to move between contexts:

- ▶ Locally: standard SIR model with random mixing
- ▶ discrete time simulation
- ▶ β = infection probability
- ▶ γ = recovery probability
- ▶ P = probability of travel
- ▶ **Movement distance:** $\Pr(d) \propto \exp(-d/\xi)$
- ▶ ξ = typical travel distance

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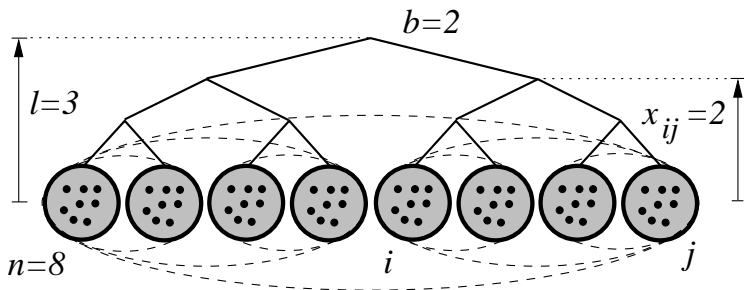
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A toy agent-based model

Schematic:



- ▶ Define P_0 = Expected number of infected individuals **leaving** initially infected context.
- ▶ Need $P_0 > 1$ for disease to spread (independent of R_0).
- ▶ Limit epidemic size by **restricting frequency of travel and/or range**

Model output

- ▶ Define P_0 = Expected number of infected individuals **leaving** initially infected context.
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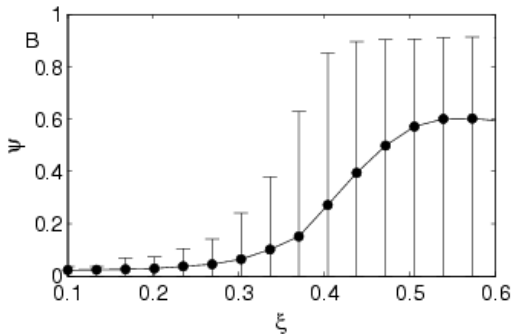
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Model output

Varying ξ :

- ▶ Transition in expected final size based on typical movement distance

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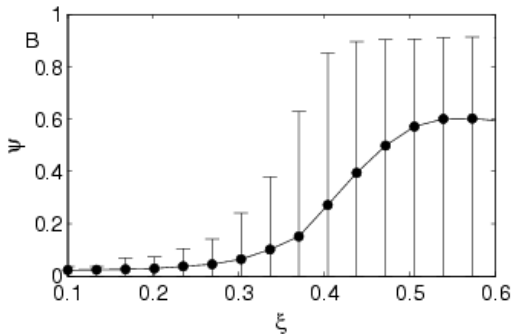
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Model output

Varying ξ :

- ▶ Transition in expected final size based on typical movement distance (**sensible**)

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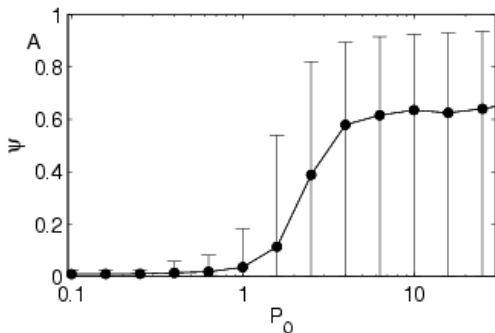
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Model output

Varying P_0 :



- ▶ Transition in expected final size based on typical number of infectives leaving first group
- ▶ Travel advisories: ξ has larger effect than P_0 .

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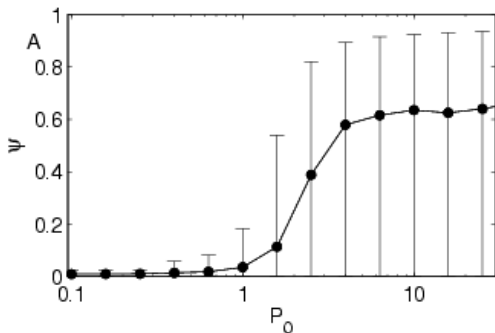
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Varying P_0 :

- ▶ Transition in expected final size based on typical number of infectives leaving first group (**also sensible**)
- ▶ Travel advisories: ξ has larger effect than P_0 .

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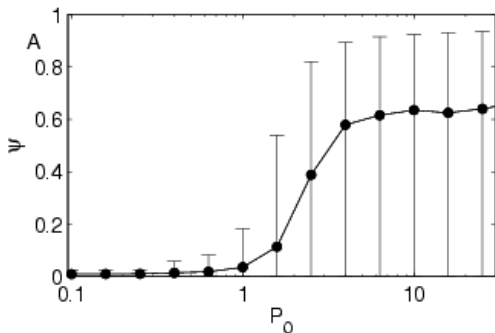
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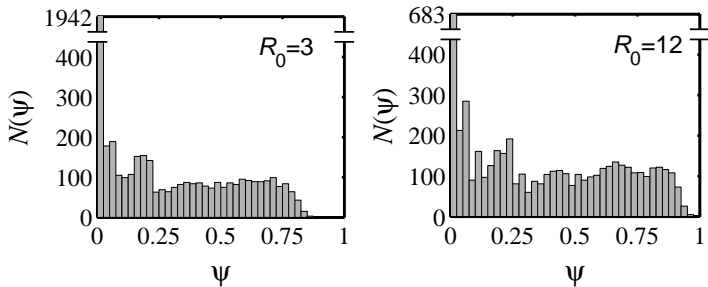
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Example model output: size distributions



- ▶ Flat distributions are possible for certain ξ and P .
- ▶ Different R_0 's may produce similar distributions
- ▶ Same epidemic sizes may arise from different R_0 's

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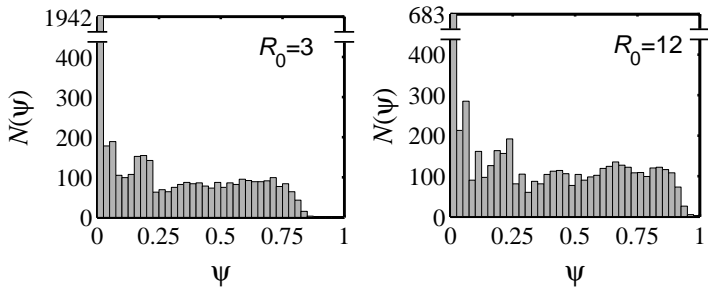
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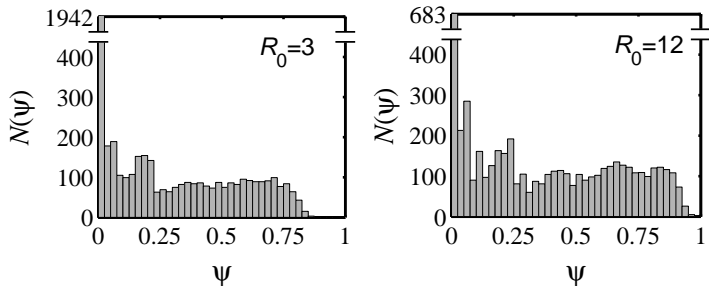
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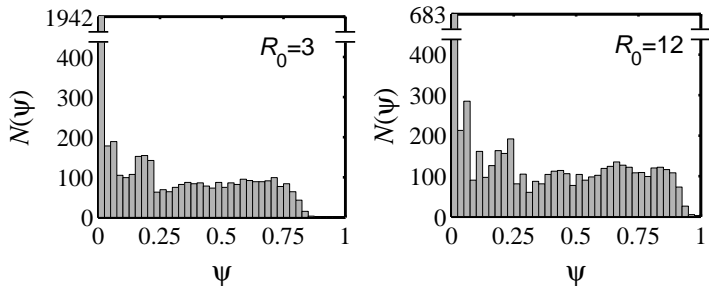
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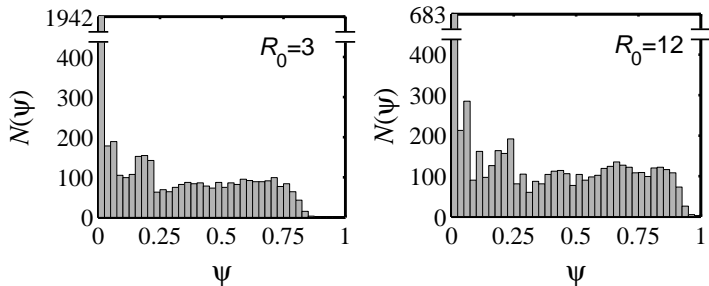
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Example model output: size distributions



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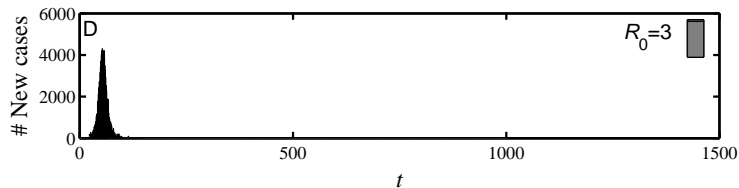
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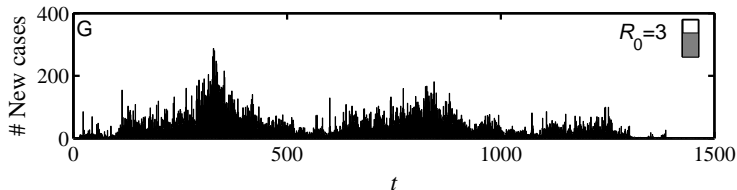
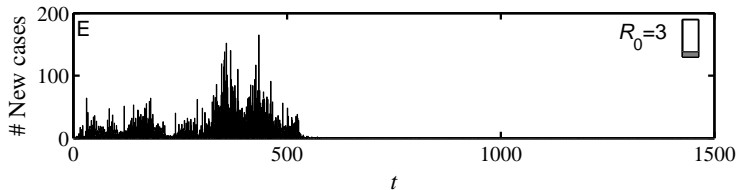
Model output—resurgence

Standard model:



Model output—resurgence

Standard model with transport:



The upshot

Multiscale population structure

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The upshot

Multiscale population structure
+
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- ▶ For this model, epidemic size is highly unpredictable
- ▶ Model is more complicated than SIR but still simple
- ▶ We haven't even included normal social responses such as travel bans and self-quarantine.
- ▶ The reproduction number R_0 is not very useful.
- ▶ R_0 , however measured, is not informative about
 1. how likely the observed epidemic size was,
 2. and how likely future epidemics will be.
- ▶ Problem: R_0 summarises **one** epidemic after the fact and enfold movement, everything.

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- ▶ Disease spread highly sensitive to population structure
- ▶ Rare events may matter enormously
- ▶ More support for controlling population movement

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- ▶ Rare events may matter enormously (e.g., an infected individual taking an international flight)
- ▶ More support for controlling population movement (e.g., travel advisories, quarantine)

Conclusions

What to do:

- ▶ Need to separate movement from disease
- ▶ R_0 needs a friend or two.
- ▶ Need $R_0 > 1$ and $P_0 > 1$ and ξ sufficiently large for disease to have a chance of spreading

More wondering:

- ▶ Exactly how important are rare events in disease spreading?
- ▶ Again, what is N ?

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Attempts to use beyond disease:

- ▶ Adoption of ideas/beliefs (Goffman & Newell, 1964)
- ▶ Spread of rumors (Daley & Kendall, 1965)
- ▶ Diffusion of innovations (Bass, 1969)
- ▶ Spread of fanatical behavior (Castillo-Chávez & Song, 2003)

Simple disease spreading models

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