The Small-World Phenomenon

Complex Networks, Course 295A, Spring, 2008

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Outline

History

Outline

History

An online experiment

Outline

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An online experiment

Previous theoretical work

Outline

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Some problems for sociologists

How are social networks structured?

- How do we define connections?
- How do we measure connections?
- (remote sensing, self-reporting)

Some problems for sociologists

How are social networks structured?

- How do we define connections?
- How do we measure connections?
- (remote sensing, self-reporting)

What about the dynamics of social networks?

- How do social networks evolve?
- How do social movements begin?
- How does collective problem solving work?
- How is information transmitted through social networks?

Social Search

A small slice of the pie:

- Q. Can people pass messages between distant individuals using only their existing social connections?

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Handles:

- The Small World Phenomenon
- or “Six Degrees of Separation.”

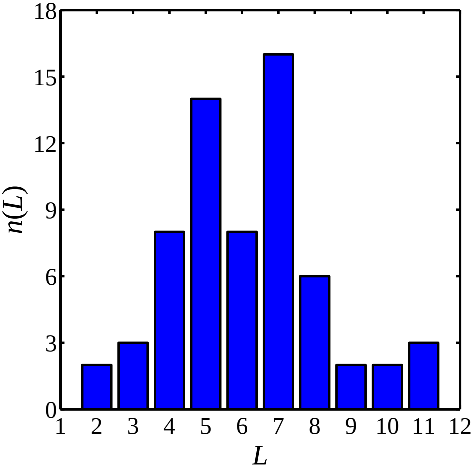
The problem

Stanley Milgram et al., late 1960's:

- Target person worked in Boston as a stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- average chain length $\simeq 6.5$.

The problem

Lengths of successful chains:



From Travers and Milgram (1969) in Sociometry: [4]
"An Experimental Study of the Small World Problem."

The problem

Two features characterize a social 'Small World':

The problem

Two features characterize a social ‘Small World’:

1. Short paths exist

   and

The problem

Two features characterize a social ‘Small World’:

1. Short paths exist

   and

2. People are good at finding them.

Social Search

Milgram's small world experiment with e-mail [2]

Social search—the Columbia experiment

- 60,000+ participants in 166 countries

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- $\Rightarrow$ 384 completed chains (1.6% of all chains).

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- Small changes in attrition rates
  $\Rightarrow$ large changes in completion rates
- e.g., $\searrow$ 15% in attrition rate
  $\Rightarrow$ $\nearrow$ 800% in completion rate

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Geography $\rightarrow$ Work

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Senders of successful messages showed little absolute dependency on

- age, gender

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- age, gender
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Range of completion rates for subpopulations:

30% to 40%

Social search—the Columbia experiment

Nevertheless, some weak discrepencies do exist...

An above average connector:

Norwegian, secular male, aged 30-39, earning over $100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

Social search—the Columbia experiment

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An above average connector:

Norwegian, secular male, aged 30-39, earning over $100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

A below average connector:

Italian, Islamic or Christian female earning less than $2K, with elementary school education and retired, who uses strong ties to family members.

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Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain."

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Why:

- Specificity important

Social search—the Columbia experiment

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choosing recipients because “they have lots of friends” or because they will “likely continue the chain.”

Why:

- Specificity important
- Successful links used relevant information.
  (e.g. connecting to someone who shares same profession as target.)

Social search—the Columbia experiment

Basic results:

- $\langle L \rangle = 4.05$ for all completed chains

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Social search—the Columbia experiment

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- Intra-country chains: $L_* = 5$
- Inter-country chains: $L_* = 7$
- All chains: $L_* = 7$
- Milgram: $L_* \simeq 9$

Previous work—short paths

- Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

N = population size,

d_{AB} = distance between nodes A and B.

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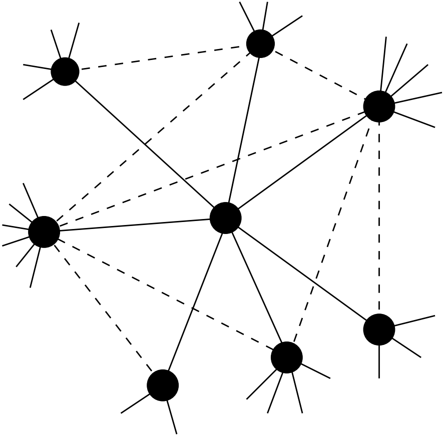
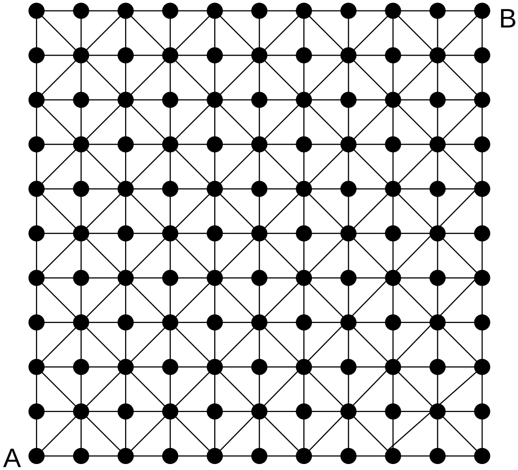
- But: social networks aren't random...

Previous work—short paths



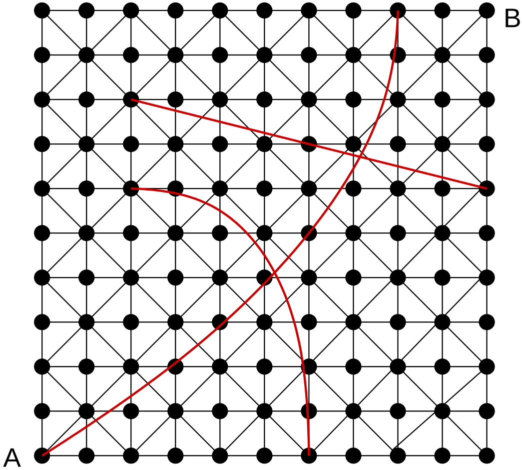
Need “clustering” (your friends are likely to know each other):

Non-randomness gives clustering



$d_{AB} = 10 \rightarrow$ too many long paths.

Randomness + regularity



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall

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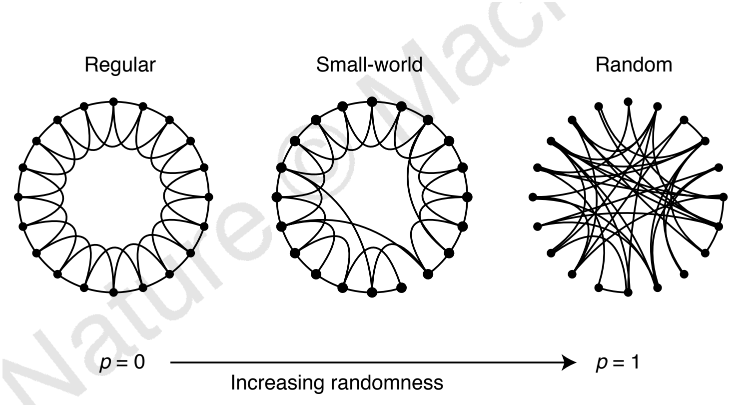
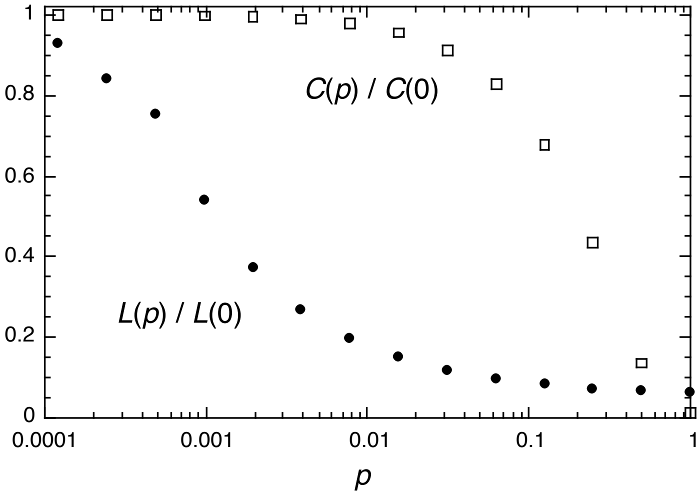
Very weak requirements:

- local regularity + random short cuts

Toy model



The structural small-world property



Previous work—finding short paths

But are these short cuts findable?

Previous work—finding short paths

But are these short cuts findable?

No.

Previous work—finding short paths

But are these short cuts findable?

No.

Nodes cannot find each other quickly
with any local search method.

Previous work—finding short paths

- What can a local search method reasonably use?

Previous work—finding short paths

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- How to find things without a map?

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- Need some measure of distance between friends and the target.

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Some possible knowledge:

- Target's identity
- Friends' popularity
- Friends' identities
- Where message has been

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Jon Kleinberg (Nature, 2000) [3]
"Navigation in a small world."

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Allowed to vary:

1. local search algorithm

Previous work—finding short paths

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   and

2. network structure.

Previous work—finding short paths

Kleinberg's Network:

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1. Start with regular d-dimensional cubic lattice.

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$$p_{ij} \propto {d_{ij}}^{-\alpha}.$$

Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d-dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance *q*.
3. Add *m* short cuts per node.
4. Connect *i* to *j* with probability

$$p_{ij} \propto {d_{ij}}^{-\alpha}.$$

- $\alpha = 0$: random connections.
- α large: reinforce local connections.
- $\alpha = d$: same number of connections at all scales.

Previous work—finding short paths

Theoretical optimal search:

- “Greedy” algorithm.

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Search time grows slowly with system size (like $\log^2 N$).

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Theoretical optimal search:

- "Greedy" algorithm.
- Same number of connections at all scales: $\alpha = d$.

Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.

Previous work—finding short paths

- If networks have hubs can also search well: Adamic et al. (2001) [1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

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- Basic idea: get to hubs first
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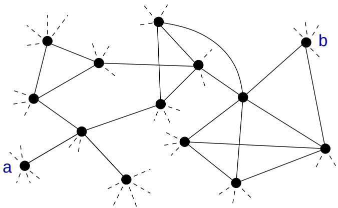
- Basic idea: get to hubs first
  (airline networks).
- But: hubs in social networks are limited.

The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of a is closest to the target b?

What does ‘closest’ mean?

What is ‘social distance’?

The model

One approach: incorporate identity.

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Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Religious beliefs
- Recreational activities.

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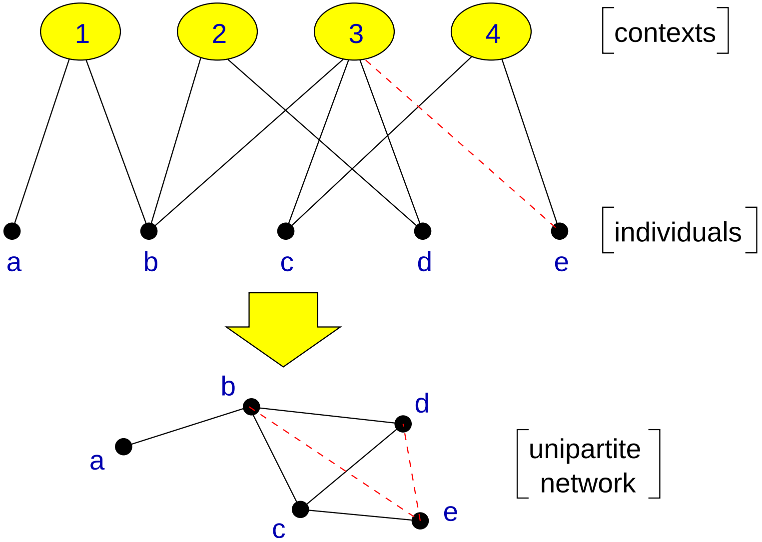
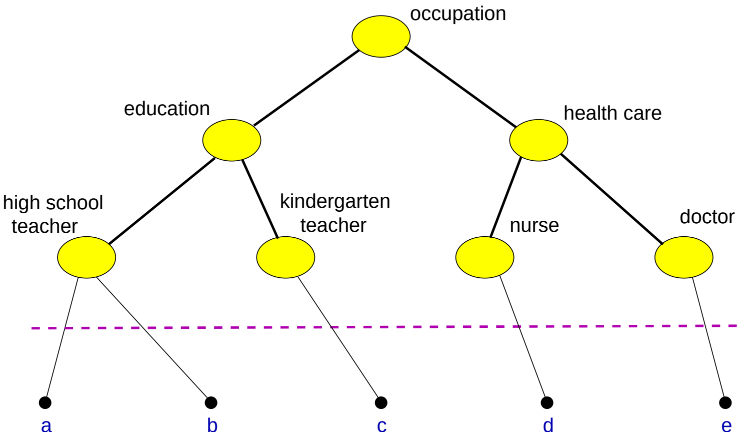
Attributes $\Leftrightarrow$ Contexts $\Leftrightarrow$ Interactions $\Leftrightarrow$ Networks.

Social distance—Bipartite affiliation networks

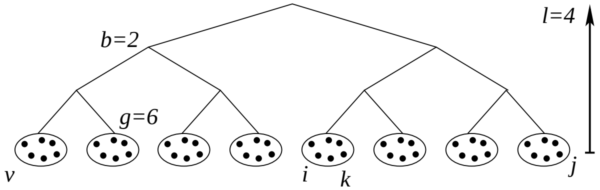


Social distance—Context distance



The model

Distance between two individuals x_{ij} is the height of lowest common ancestor.



$x_{ij} = 3$, $x_{ik} = 1$, $x_{iv} = 4$.

The model

- Individuals are more likely to know each other the closer they are within a hierarchy.

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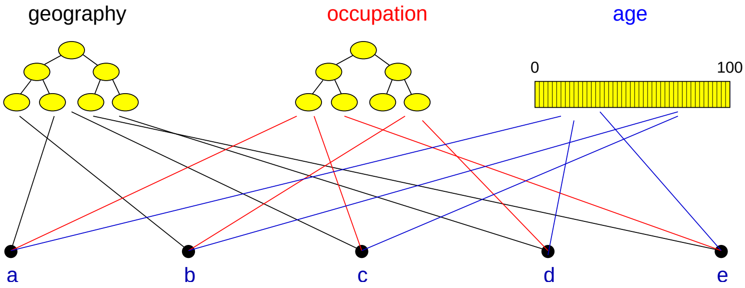
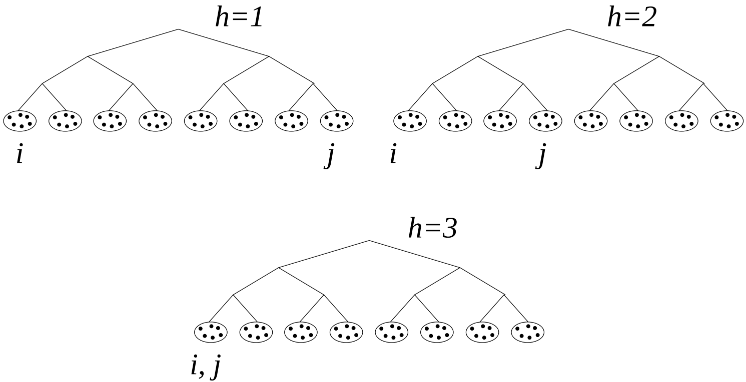
Social distance—Generalized context space



(Blau & Schwartz, Simmel, Breiger)

The model



$\vec{v}_i = [1\ 1\ 1]^T$, $\vec{v}_j = [8\ 4\ 1]^T$

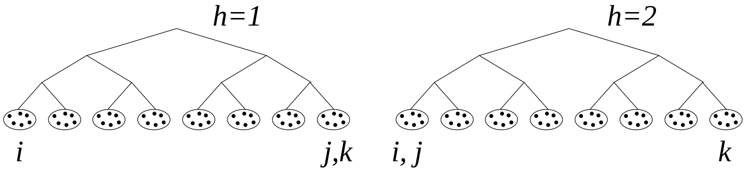
$x^1_{ij} = 4$, $x^2_{ij} = 3$, $x^3_{ij} = 1$.

Social distance:

$$y_{ij} = \min_h x^h_{ij}.$$

The model

Triangle inequality doesn't hold:



$$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$$

The model

- Individuals know the identity vectors of

The model

- Individuals know the identity vectors of
  1. themselves,

The model

- Individuals know the identity vectors of
  1. themselves,
  2. their friends,

The model

- Individuals know the identity vectors of
    1. themselves,
    2. their friends,
       and
    3. the target.

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- Individuals know the identity vectors of
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- Individuals can estimate the social distance between their friends and the target.

The model

- Individuals know the identity vectors of
  1. themselves,
  2. their friends,
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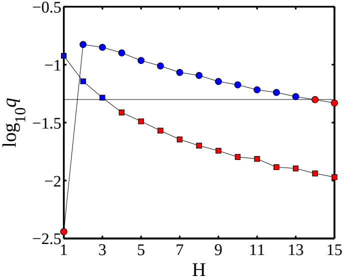
  3. the target.
- Individuals can estimate the social distance between their friends and the target.
- Use a greedy algorithm + allow searches to fail randomly.

The model-results—searchable networks

$\alpha = 0$ versus $\alpha = 2$ for $N \simeq 10^5$:



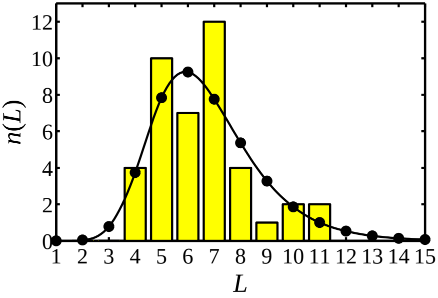
$q \geq r$
$q < r$
$r = 0.05$

q = probability an arbitrary message chain reaches a target.

- A few dimensions help.
- Searchability decreases as population increases.
- Precise form of hierarchy largely doesn't matter.

The model-results

Milgram's Nebraska-Boston data:



Model parameters:

- $N = 10^8$,
- $z = 300$, $g = 100$,
- $b = 10$,
- $\alpha = 1$, $H = 2$;

- $\langle L_{\rm model} \rangle \simeq 6.7$
- $L_{\rm data} \simeq 6.5$

Social search—Data

Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.

Social search—Data

Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Probability of connection as function of real distance $\propto 1/r$.

Social Search—Real world uses

- Tags create identities for objects
- Website tagging: `http://www.del.icio.us`
- (e.g., Wikipedia)
- Photo tagging: `http://www.flickr.com`
- Dynamic creation of metadata plus links between information objects.
- Folksonomy: collaborative creation of metadata

Social Search—Real world uses

Recommender systems:

- Amazon uses people's actions to build effective connections between books.

Social Search—Real world uses

Recommender systems:

- Amazon uses people's actions to build effective connections between books.
- Conflict between 'expert judgments' and tagging of the hoi polloi.

Conclusions

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- Paths are findable if nodes understand how network is formed.
- Importance of identity (interaction contexts).

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