

The Small-World Phenomenon

Complex Networks, Course 295A, Spring, 2008

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History

An online
experiment

Previous
theoretical work

An improved
model

References

Frame 1/47



Outline

History

An online experiment

Previous theoretical work

An improved model

References

History

An online
experiment

Previous
theoretical work

An improved
model

References

Some problems for sociologists

How are social networks structured?

- ▶ How do we define connections?
- ▶ How do we measure connections?
- ▶ (remote sensing, self-reporting)

What about the dynamics of social networks?

- ▶ How do social networks evolve?
- ▶ How do social movements begin?
- ▶ How does collective problem solving work?
- ▶ How is information transmitted through social networks?

History

An online
experiment

Previous
theoretical work

An improved
model

References

History

An online
experiment

Previous
theoretical work

An improved
model

References

A small slice of the pie:

- ▶ **Q.** Can people pass messages between distant individuals using only their existing social connections?
- ▶ **A.** Apparently yes...

Handles:

- ▶ The Small World Phenomenon
- ▶ or “Six Degrees of Separation.”

History

An online
experiment

Previous
theoretical work

An improved
model

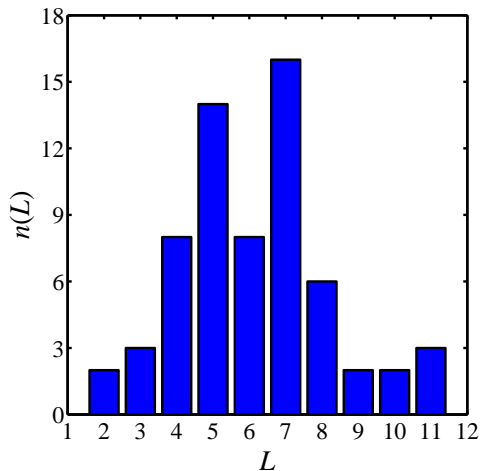
References

Stanley Milgram et al., late 1960's:

- ▶ Target person worked in Boston as a stockbroker.
- ▶ 296 senders from Boston and Omaha.
- ▶ 20% of senders reached target.
- ▶ average chain length $\simeq 6.5$.

The problem

Lengths of successful chains:



From Travers and
Milgram (1969) in
Sociometry:^[4]
“An Experimental
Study of the Small
World Problem.”

History

An online
experiment

Previous
theoretical work

An improved
model

References

History

An online
experiment

Previous
theoretical work

An improved
model

References

Two features characterize a social 'Small World':

1. Short paths exist
and
2. People are good at finding them.

Milgram's small world experiment with e-mail [2]

History

An online experiment

Previous theoretical work

An improved model

References

Events and News
Duncan J. Watts's new book is out now!

Project Information
In the Press
Description
Procedures
Security and Privacy
Articles/References
Results

Research Team
Duncan J. Watts
Peter Dodds
Roby Muhamad

Web Development
Peter Hausel

Vijay (Delhi, India) worked at an engineering firm with

home
my small world
chat
FAQ
related links

login

sign up

The **SMALL WORLD** project is an online experiment to test the idea that any two people in the world can be connected via 'six degrees of separation'.

Your objective is to get a message to a "target person", somewhere in the world, by forwarding the message to a friend of yours--someone who is "closer" to the target than you are. (If you happen know the target, you can of course send it to them)

If we have asked you to participate (you would have received a message from a friend of yours), you should **continue** the chain.

If you are just visiting us, sign up to start a new chain.

Prema (Berkeley, USA) goes to school in California and plays soccer with

Alice (New York, USA)

Chrisis (Berkeley, USA) whose best friend from high school

William (New York, NY) is studying medicine with

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Social search—the Columbia experiment

History

An online
experiment

Previous
theoretical work

An improved
model

References

- ▶ 60,000+ participants in 166 countries
- ▶ 18 targets in 13 countries including
 - ▶ a professor at an Ivy League university,
 - ▶ an archival inspector in Estonia,
 - ▶ a technology consultant in India,
 - ▶ a policeman in Australia,
and
 - ▶ a veterinarian in the Norwegian army.
- ▶ 24,000+ chains

Social search—the Columbia experiment

History

An online
experiment

Previous
theoretical work

An improved
model

References

- ▶ Milgram's participation rate was roughly 75%
- ▶ Email version: Approximately 37% participation rate.
- ▶ Probability of a chain of length 10 getting through:

$$.37^{10} \simeq 5 \times 10^{-5}$$

- ▶ \Rightarrow 384 completed chains (1.6% of all chains).

Social search—the Columbia experiment

History

An online
experiment

Previous
theoretical work

An improved
model

References

- ▶ Motivation/Incentives/Perception matter.
- ▶ If target *seems* reachable
⇒ participation more likely.
- ▶ Small changes in attrition rates
⇒ large changes in completion rates
- ▶ e.g., ↘ 15% in attrition rate
⇒ ↗ 800% in completion rate

Social search—the Columbia experiment

Successful chains disproportionately used

- ▶ weak ties (Granovetter)
- ▶ professional ties (34% vs. 13%)
- ▶ ties originating at work/college
- ▶ target's work (65% vs. 40%)

... and disproportionately avoided

- ▶ hubs (8% vs. 1%) (+ no evidence of funnels)
- ▶ family/friendship ties (60% vs. 83%)

Geography → Work

History

An online
experiment

Previous
theoretical work

An improved
model

References

Social search—the Columbia experiment

Senders of successful messages showed
little absolute dependency on

- ▶ age, gender
- ▶ country of residence
- ▶ income
- ▶ religion
- ▶ relationship to recipient

Range of completion rates for subpopulations:

30% to 40%

History

An online
experiment

Previous
theoretical work

An improved
model

References

Social search—the Columbia experiment

Nevertheless, some weak discrepancies do exist...

An above average connector:

Norwegian, secular male, aged 30-39, earning over \$100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

A below average connector:

Italian, Islamic or Christian female earning less than \$2K, with elementary school education and retired, who uses strong ties to family members.

History

An online
experiment

Previous
theoretical work

An improved
model

References

Social search—the Columbia experiment

History

An online
experiment

Previous
theoretical work

An improved
model

References

Mildly bad for continuing chain:

choosing recipients because “they have lots of friends” or because they will “likely continue the chain.”

Why:

- ▶ Specificity important
- ▶ Successful links used relevant information.
(e.g. connecting to someone who shares same profession as target.)

Social search—the Columbia experiment

History

An online
experiment

Previous
theoretical work

An improved
model

References

Basic results:

- ▶ $\langle L \rangle = 4.05$ for all completed chains
- ▶ L_* = Estimated 'true' median chain length (zero attrition)
- ▶ Intra-country chains: $L_* = 5$
- ▶ Inter-country chains: $L_* = 7$
- ▶ All chains: $L_* = 7$
- ▶ Milgram: $L_* \simeq 9$

Previous work—short paths

History

An online
experiment

Previous
theoretical work

An improved
model

References

- ▶ Connected **random networks** have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

N = population size,

d_{AB} = distance between nodes A and B .

- ▶ **But: social networks aren't random...**

Previous work—short paths

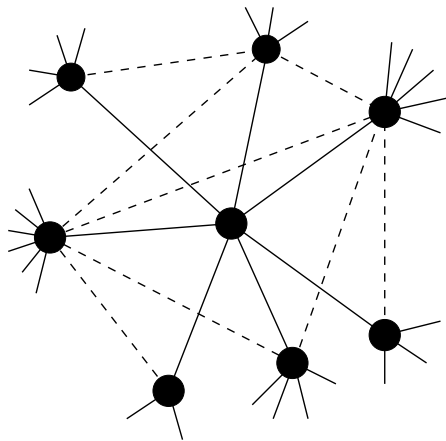
History

An online
experiment

Previous
theoretical work

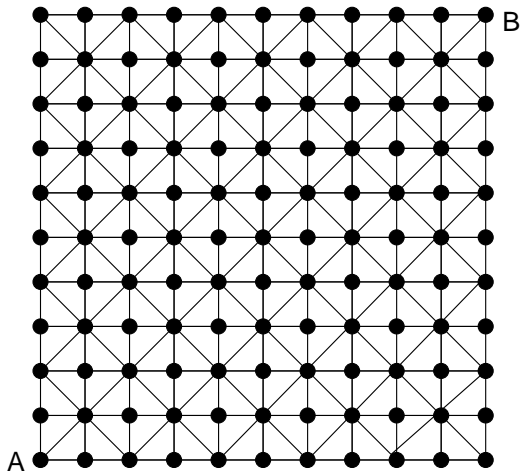
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model

References



Need “clustering” (your friends are likely to know each other):

Non-randomness gives clustering



$d_{AB} = 10 \rightarrow$ too many long paths.

History

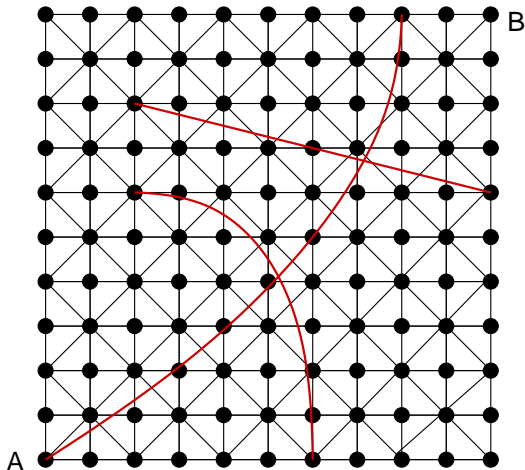
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Previous theoretical work

An improved model

References

Randomness + regularity



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall

History

An online experiment

Previous theoretical work

An improved model

References

Small-world networks

Introduced by

Watts and Strogatz (Nature, 1998) [5]

“Collective dynamics of ‘small-world’ networks.”

Small-world networks were found everywhere:

- ▶ neural network of *C. elegans*,
- ▶ semantic networks of languages,
- ▶ actor collaboration graph,
- ▶ food webs,
- ▶ social networks of comic book characters,...

Very weak requirements:

- ▶ local regularity + random short cuts

History

An online
experiment

Previous
theoretical work

An improved
model

References

Toy model

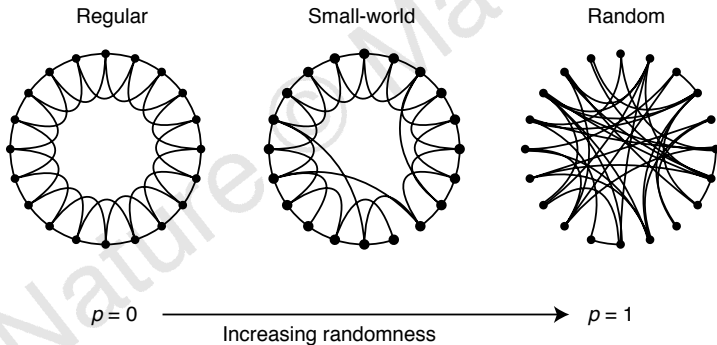
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An online experiment

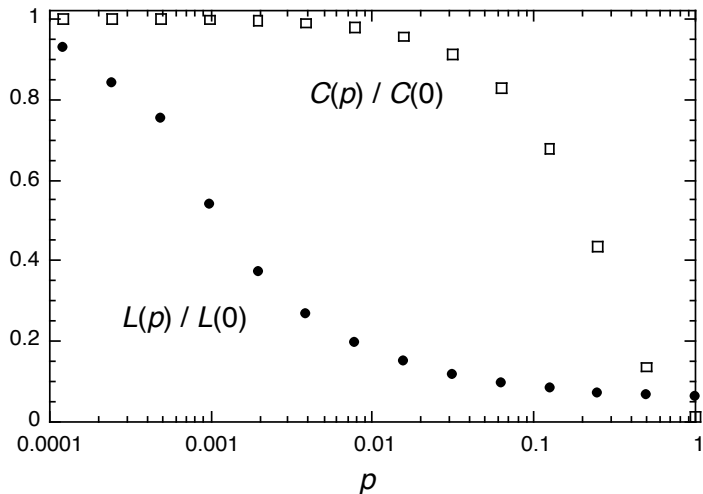
Previous theoretical work

An improved model

References



The structural small-world property



History

An online
experiment

Previous
theoretical work

An improved
model

References

Previous work—finding short paths

History

An online
experiment

Previous
theoretical work

An improved
model

References

But are these short cuts findable?

No.

Nodes **cannot** find each other quickly
with **any local search method**.

Previous work—finding short paths

- ▶ What can a local search method reasonably use?
- ▶ How to find things without a map?
- ▶ Need some measure of distance between friends and the target.

Some possible knowledge:

- ▶ Target's identity
- ▶ Friends' popularity
- ▶ Friends' identities
- ▶ Where message has been

History

An online
experiment

Previous
theoretical work

An improved
model

References

Previous work—finding short paths

Jon Kleinberg (Nature, 2000) ^[3]
“Navigation in a small world.”

Allowed to vary:

1. local search algorithm
and
2. network structure.

History

An online
experiment

Previous
theoretical work

An improved
model

References

Previous work—finding short paths

Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q .
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto d_{ij}^{-\alpha}.$$

- ▶ $\alpha = 0$: random connections.
- ▶ α large: reinforce local connections.
- ▶ $\alpha = d$: same number of connections at all scales.

History

An online
experiment

Previous
theoretical work

An improved
model

References

Previous work—finding short paths

History

An online
experiment

Previous
theoretical work

An improved
model

References

Theoretical optimal search:

- ▶ “Greedy” algorithm.
- ▶ Same number of connections at all scales: $\alpha = d$.

Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.

Previous work—finding short paths

History

An online
experiment

Previous
theoretical work

An improved
model

References

- ▶ If networks have **hubs** can also search well: Adamic et al. (2001)^[1]

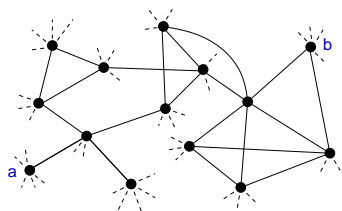
$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

- ▶ Basic idea: get to hubs first (airline networks).
- ▶ **But: hubs in social networks are limited.**

The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of **a** is closest to the target **b**?

What does 'closest' mean?

What is 'social distance'?

History

An online
experiment

Previous
theoretical work

An improved
model

References

One approach: incorporate **identity**.

Identity is formed from attributes such as:

- ▶ Geographic location
- ▶ Type of employment
- ▶ Religious beliefs
- ▶ Recreational activities.

Groups are formed by people with at least one similar attribute.

Attributes \Leftrightarrow Contexts \Leftrightarrow Interactions \Leftrightarrow Networks.

History

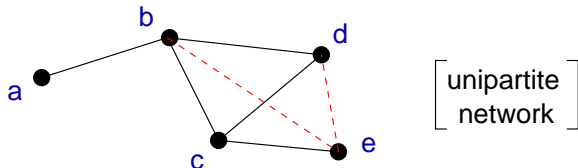
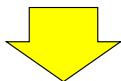
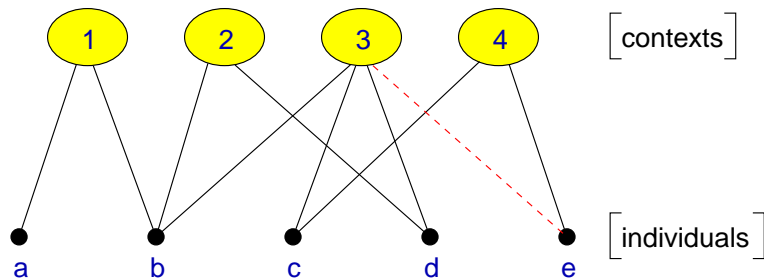
An online
experiment

Previous
theoretical work

An improved
model

References

Social distance—Bipartite affiliation networks



History

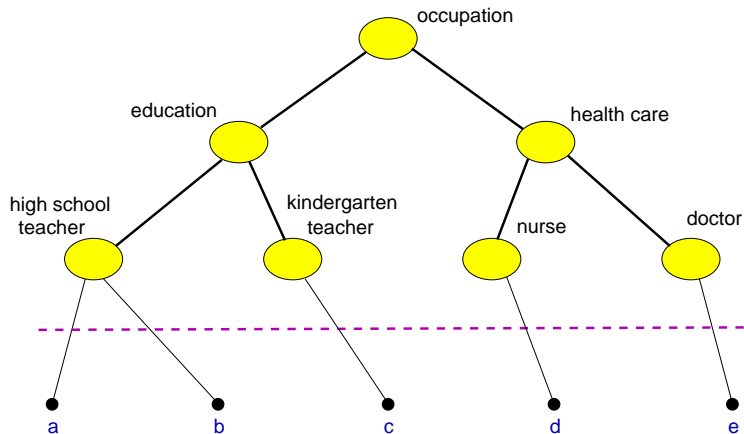
An online experiment

Previous theoretical work

An improved model

References

Social distance—Context distance



History

An online
experiment

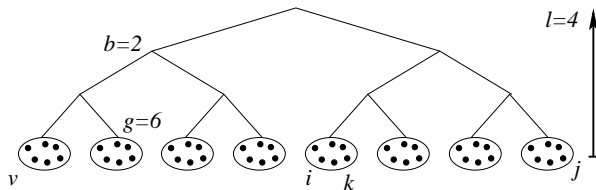
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theoretical work

An improved
model

References

The model

Distance between two individuals x_{ij} is the height of lowest common ancestor.



$$x_{ij} = 3, x_{ik} = 1, x_{iv} = 4.$$

History

An online experiment

Previous theoretical work

An improved model

References

History

An online
experiment

Previous
theoretical work

An improved
model

References

- ▶ Individuals are more likely to know each other the closer they are within a hierarchy.
- ▶ Construct z connections for each node using

$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$

- ▶ $\alpha = 0$: random connections.
- ▶ α large: local connections.

Social distance—Generalized context space

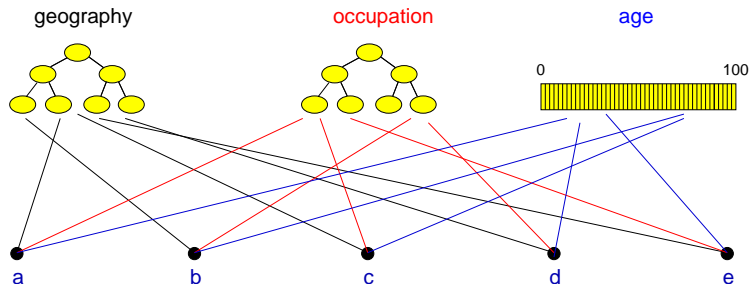
History

An online
experiment

Previous
theoretical work

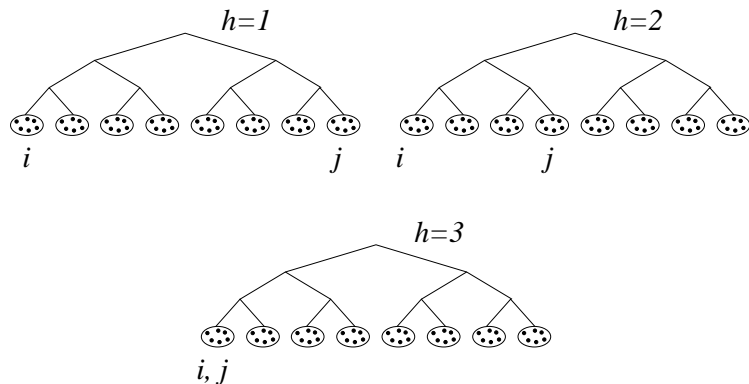
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References



(Blau & Schwartz, Simmel, Breiger)

The model



$$\vec{v}_i = [1 \ 1 \ 1]^T, \vec{v}_j = [8 \ 4 \ 1]^T$$

$$x_{ij}^1 = 4, x_{ij}^2 = 3, x_{ij}^3 = 1.$$

Social distance:

$$y_{ij} = \min_h x_{ij}^h.$$

History

An online experiment

Previous theoretical work

An improved model

References

History

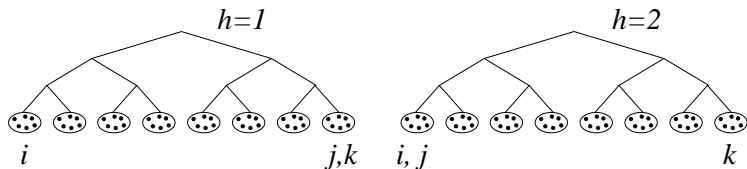
An online
experiment

Previous
theoretical work

An improved
model

References

Triangle inequality doesn't hold:



$$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$$

History

An online
experiment

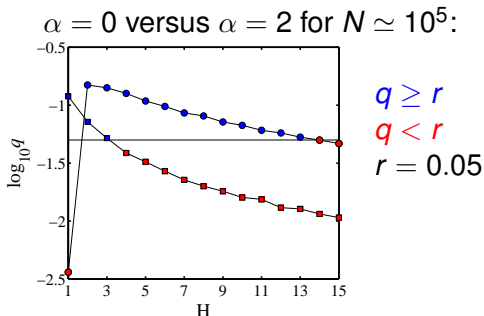
Previous
theoretical work

An improved
model

References

- ▶ Individuals know the identity vectors of
 1. themselves,
 2. their friends,
and
 3. the target.
- ▶ Individuals can estimate the social distance between their friends and the target.
- ▶ Use a greedy algorithm + allow searches to fail randomly.

The model-results—searchable networks



q = probability an arbitrary message chain reaches a target.

- ▶ A few dimensions help.
- ▶ Searchability decreases as population increases.
- ▶ Precise form of hierarchy largely doesn't matter.

History

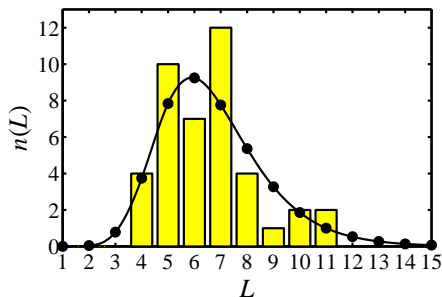
An online
experiment

Previous
theoretical work

An improved
model

References

Milgram's Nebraska-Boston data:



Model parameters:

- ▶ $N = 10^8$,
- ▶ $z = 300, g = 100$,
- ▶ $b = 10$,
- ▶ $\alpha = 1, H = 2$;

- ▶ $\langle L_{\text{model}} \rangle \simeq 6.7$
- ▶ $L_{\text{data}} \simeq 6.5$

History

An online
experiment

Previous
theoretical work

An improved
model

References

History

An online
experiment

Previous
theoretical work

An improved
model

References

Adamic and Adar (2003)

- ▶ For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- ▶ Probability of connection as function of real distance $\propto 1/r$.

Social Search—Real world uses

- ▶ Tags create identities for objects
- ▶ Website tagging: <http://www.del.icio.us>
- ▶ (e.g., Wikipedia)
- ▶ Photo tagging: <http://www.flickr.com>
- ▶ Dynamic creation of metadata plus links between information objects.
- ▶ Folksonomy: collaborative creation of metadata

History

An online
experiment

Previous
theoretical work

An improved
model

References

History

An online
experiment

Previous
theoretical work

An improved
model

References

Recommender systems:

- ▶ Amazon uses people's actions to build effective connections between books.
- ▶ Conflict between 'expert judgments' and tagging of the hoi polloi.

- ▶ Bare networks are typically unsearchable.
- ▶ Paths are findable if nodes understand how network is formed.
- ▶ Importance of identity (interaction contexts).
- ▶ Improved social network models.
- ▶ Construction of peer-to-peer networks.
- ▶ Construction of searchable information databases.

History

An online
experiment

Previous
theoretical work

An improved
model

References

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History

An online
experiment

Previous
theoretical work

An improved
model

References

Frame 46/47


History

An online
experiment

Previous
theoretical work

An improved
model

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