295A Complex Networks—Assignment 2
University of Vermont, Spring 2008

Dispersed: Monday, March 3, 2008.
Due: By start of lecture, 9:30 am, Tuesday, March 25, 2008.
Some useful reminders:
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Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2008-01UVM-295/`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

I. Supply networks and allometry:

Consider a set of rectangular areas with side lengths L_1 and L_2 such that $L_1 \propto A^{\gamma_1}$ $L_2 \propto A^{\gamma_2}$ where A is area and $\gamma_1 + \gamma_2 = 1$. Assume $\gamma_1 > \gamma_2$.

Now imagine that material has to be distributed from a central source in each of these areas to sinks distributed with density $\rho(A)$, and that these sinks draw the same amount of material per unit time independent of L_1 and L_2.

1. Find an exact form for how the volume of the most efficient distribution network scales with overall area $A = L_1 L_2$. (Hint: you will have to set up a double integration over the rectangle.)

2. If network volume must remain a constant fraction of overall area, determine the maximal scaling of sink density ρ with A.

II. Size-density law:

In two dimensions, the size-density law for distributed source density $D(\vec{x})$ given a sink density $\rho(\vec{x})$ states that $D \propto \rho^{2/3}$. We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the 2/3 exponent.

1. Repeat this argument for the d-dimensional case and find the general form of the exponent β in $D \propto \rho^{\beta}$.

2. In 1-d, consider a population density $\rho(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. Find the ideal distribution for N sources where N is large.

   One suggestion (this may not work perfectly): Assume sources are located at ae^{bi} where $i = 1, 2, \ldots N$. The end point locations of sources should not matter too much.

   Find an expression for the average distance to the nth source for those sinks closest to that source, and then find a and b (or just one of these parameters) so that this average is a constant.

   Hint: draw yourself a clear picture of what's going on.

   Also: Feel free to do some numerics to see how things work.