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Properties of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont

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A notable feature of large-scale networks:

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- Graphical renderings are often just a big mess.

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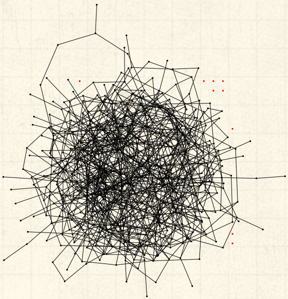

$\Leftarrow$ Typical hairball

- number of nodes $N = 500$
- number of edges $m = 1000$
- average degree $\langle k \rangle = 4$

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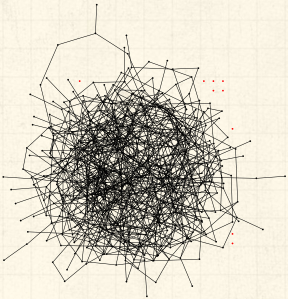

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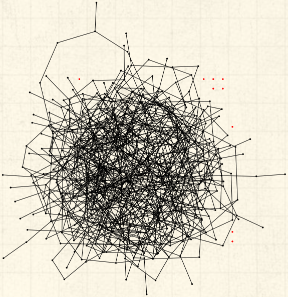

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  said Ponder [Stibbons] —*Making Money*, T. Pratchett.

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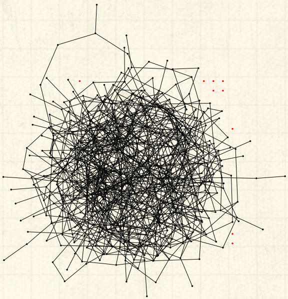

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  said Ponder [Stibbons] —*Making Money*, T. Pratchett.
- We need to extract digestible, meaningful aspects.

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Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
-
- hierarchical scaling
- concurrency
- network distances
- centrality
- multilayerness
- efficiency
- robustness

- Plus coevolution of network structure and processes on networks.

* Degree distribution is the elephant in the room that we are now all very aware of...

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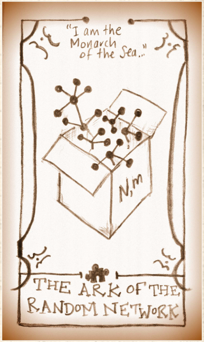
"I am the Monarch of the Sea."
N, m
THE ARK OF THE RANDOM NETWORK

Properties

1. degree distribution P_k

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- hubs may facilitate or impede contagion.

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Note:

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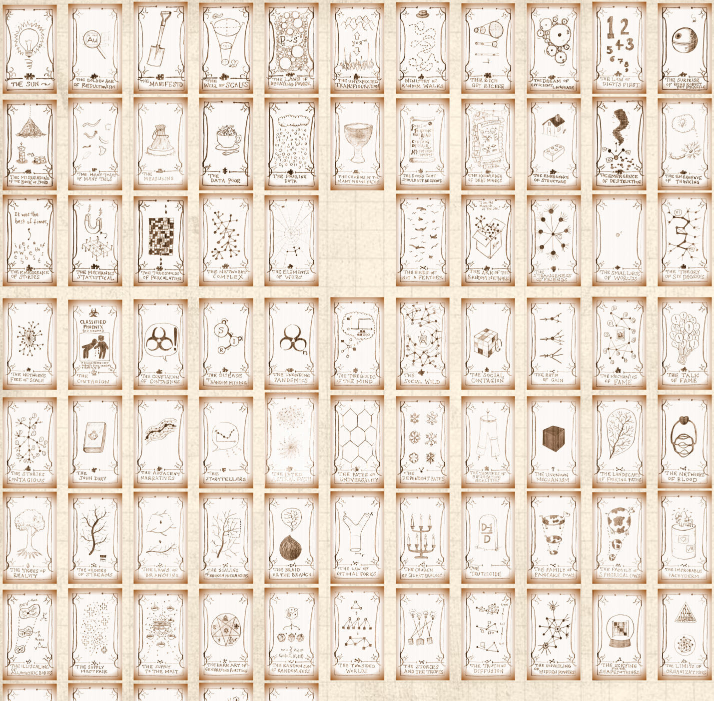
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2. Assortativity/3. Homophily:

- Social networks: Homophily = birds of a feather

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- Disassortative network: high degree nodes connecting to low degree nodes.
  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

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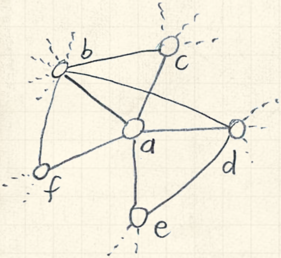

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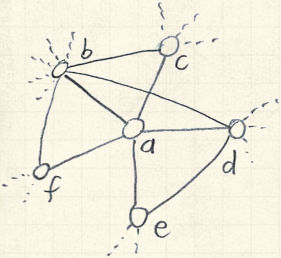

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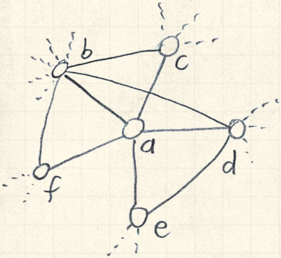

- Your friends tend to know each other.
- Two measures (explained on following slides):
  1. Watts & Strogatz [8]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i$$

  2. Newman [6]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

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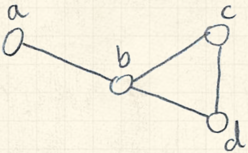
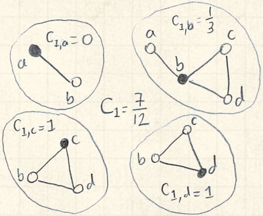


Calculation of C_1:



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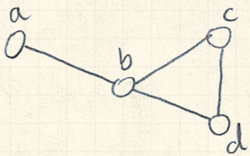

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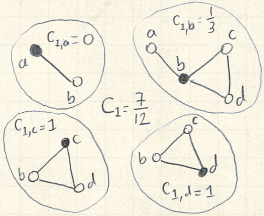

- C_1 is the average fraction of pairs of neighbors who are connected.

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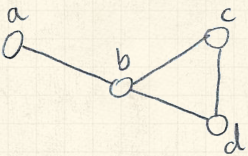

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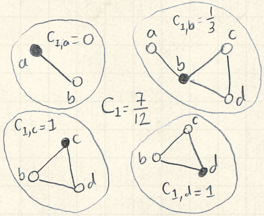

- C_1 is the average fraction of pairs of neighbors who are connected.
- Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

where k_i is node i's degree, and N_i is the set of i's neighbors.

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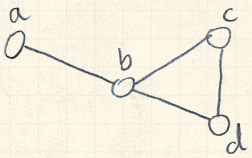

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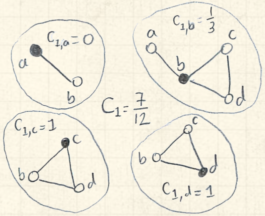

- C_1 is the average fraction of pairs of neighbors who are connected.
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$$\frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

where k_i is node i's degree, and N_i is the set of i's neighbors.

- Averaging over all nodes, we have:

$$C_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

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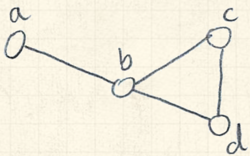
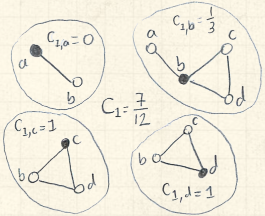


Calculation of C_1:



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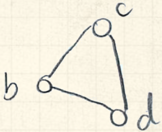
$$C_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i-1)/2} = \left\langle \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i$$

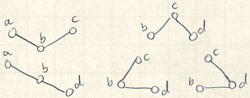
Triples and triangles

Example network:

Triangles:



Triples:



- Nodes i_1, i_2, and i_3 form a **triple** around i_1 if i_1 is connected to i_2 and i_3.

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Triples and triangles

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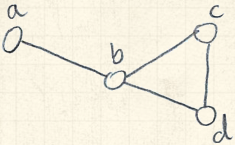

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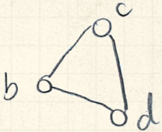

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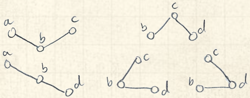

- Nodes i_1, i_2, and i_3 form a triple around i_1 if i_1 is connected to i_2 and i_3.
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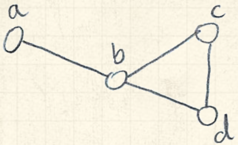

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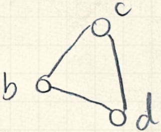

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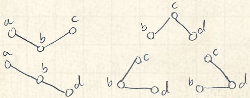

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- Nodes i_1, i_2, and i_3 form a triangle if each pair of nodes is connected
- The definition $C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$ measures the fraction of closed triples

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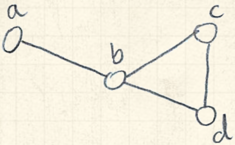

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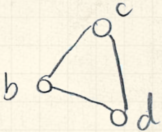

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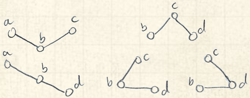

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- The '3' appears because for each triangle, we have 3 closed triples.

Triples and triangles

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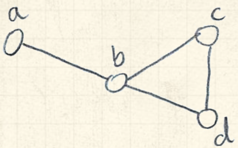

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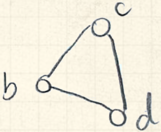

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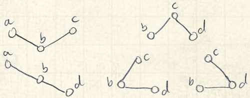

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- The '3' appears because for each triangle, we have 3 closed triples.
- Social Network Analysis (SNA): fraction of transitive triples.

Clustering:

Sneaky counting for undirected, unweighted networks:

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Clustering:

Sneaky counting for undirected, unweighted networks:

- If the path i–j–ℓ exists then $a_{ij}a_{j\ell} = 1$.

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- In general, a path of n edges between nodes i_1 and i_n travelling through nodes i_2, i_3, ...i_{n-1} exists $\iff$ $a_{i_1 i_2} a_{i_2 i_3} a_{i_3 i_4} \cdots a_{i_{n-2} i_{n-1}} a_{i_{n-1} i_n} = 1$.

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- $$\#\text{triples} = \frac{1}{2}\left(\sum_{i=1}^{N} \sum_{\ell=1}^{N} \left[A^2\right]_{i\ell} - \mathrm{Tr} A^2 \right)$$

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- $$\#\text{triangles} = \frac{1}{6}\mathrm{Tr}A^3$$

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Properties

- For sparse networks, C_1 tends to discount highly connected nodes.

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Properties

- For sparse networks, C_1 tends to discount highly connected nodes.
- C_2 is a useful and often preferred variant

Properties

- For sparse networks, C_1 tends to discount highly connected nodes.
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- In general, $C_1 \neq C_2$.

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- In general, $C_1 \neq C_2$.
- C_1 is a global average of a local ratio.

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- C_2 is a ratio of two global quantities.

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5. motifs:

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Properties

5. motifs:

- small, recurring functional subnetworks

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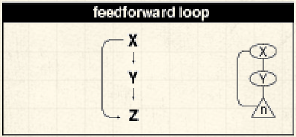
5. motifs:

- small, recurring functional subnetworks
- e.g., Feed Forward Loop:

a



Shen-Orr, Uri Alon, *et al.* [7]

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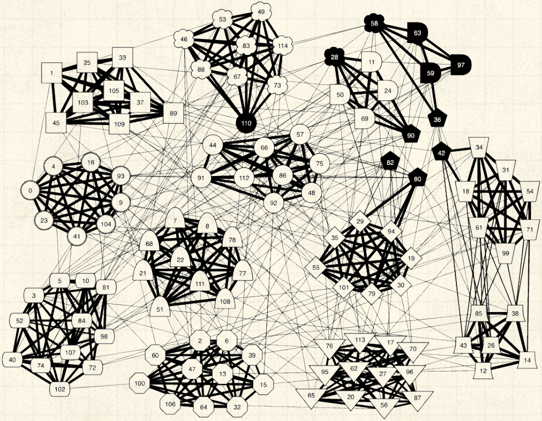
6. modularity and structure/community detection:



Clauset *et al.*, 2006 [2]: NCAA football

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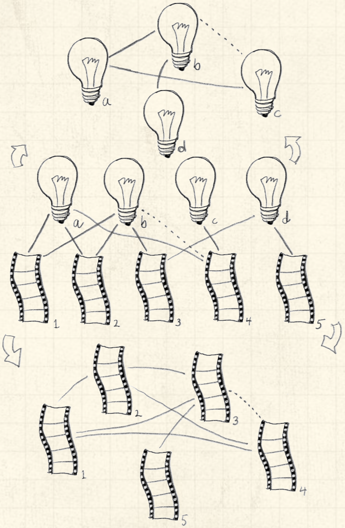
Bipartite/multipartite affiliation structures:



- Many real-world networks have an underlying multi-partite structure.
  - Stories-tropes.
  - Boards and directors.
  - Films-actors-directors.
  - Classes-teachers-students.
  - Upstairs-downstairs.
- Unipartite networks may be induced or co-exist.

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Properties

7. concurrency:

- transmission of a contagious element only occurs during contact

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Properties

7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model

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- dynamic property—static networks are not enough

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- knowledge of previous contacts crucial
- beware cumulated network data

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- Kretzschmar and Morris, 1996 [4]

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- Kretzschmar and Morris, 1996 [4]
- "Temporal networks" become a concrete area of study for Piranha Physicus in 2013.

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Properties

8. Horton-Strahler ratios:

- Metrics for branching networks:

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- Metrics for branching networks:
  - Method for ordering streams hierarchically

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Properties

8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$

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- Metrics for branching networks:
  - Method for ordering streams hierarchically
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9. network distances:

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9. network distances:

(a) shortest path length d_{ij}:

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- Weighted links can be accommodated.

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- network diameter $d_{\max}$:
  Maximum shortest path length between any two nodes.

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- $d_{\mathrm{cl}} = \infty$ only when all nodes are isolated.
- Closeness perhaps compresses too much into one number

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10. centrality:

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10. centrality:

- Many such measures of a node's 'importance.'

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- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg [3])

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Interconnected networks and robustness (two for one deal):

"Catastrophic cascade of failures in interdependent networks" [1].
Buldyrev et al., Nature 2010.

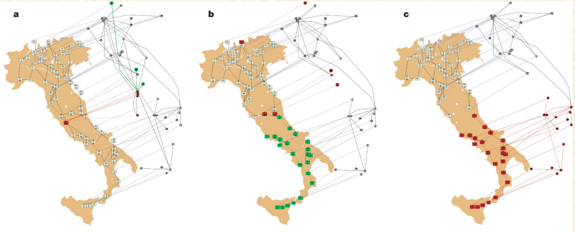

Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003[20]. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a**, One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network) at the next step are marked in green. **b**, Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c**, Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.

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- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

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scale-free-networks,

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Neural reboot (NR):

Mouse

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