

System Robustness

Last updated: 2025/10/28, 19:37:18 EDT

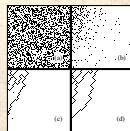
Principles of Complex Systems, Vols. 1, 2, 3D, 4 Fourever, V for Vendetta

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Institute
University of Vermont | Santa Fe Institute



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System Robustness
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Robustness

HOT theory

Random forests

Self-Organized Criticality

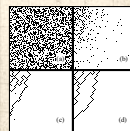
COLD theory

Network robustness

Extras

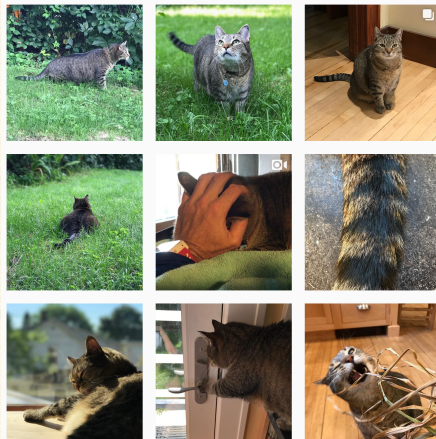
References

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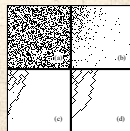
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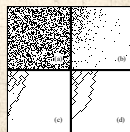
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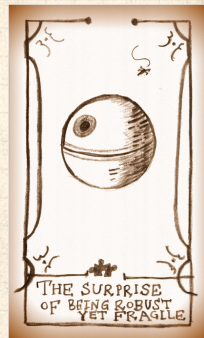
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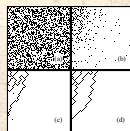
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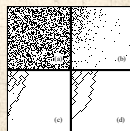
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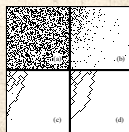


Many complex systems are prone to cascading catastrophic failure:





Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

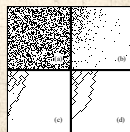




Many complex systems are prone to cascading catastrophic failure: **exciting!!!**



Blackouts





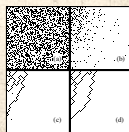
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Blackouts



Disease outbreaks





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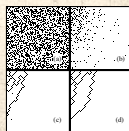
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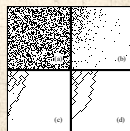
Wildfires





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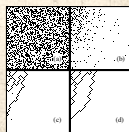
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-  Disease outbreaks
-  Wildfires
-  Earthquakes





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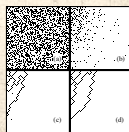
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-  Earthquakes
-  Organisms, individuals and societies





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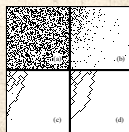
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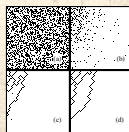
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- Blackouts
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- Earthquakes
- Organisms, individuals and societies
- Ecosystems
- Cities
- Myths: Achilles.



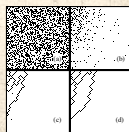


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But complex systems also show persistent **robustness**



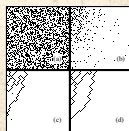


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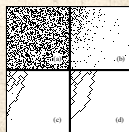


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-  Myths: Achilles.

 But complex systems also show persistent **robustness** (not as exciting but important...)

 Robustness and Failure may be a power-law story...



Our emblem of Robust-Yet-Fragile:

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Robustness

HOT theory

Random forests

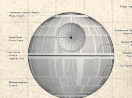
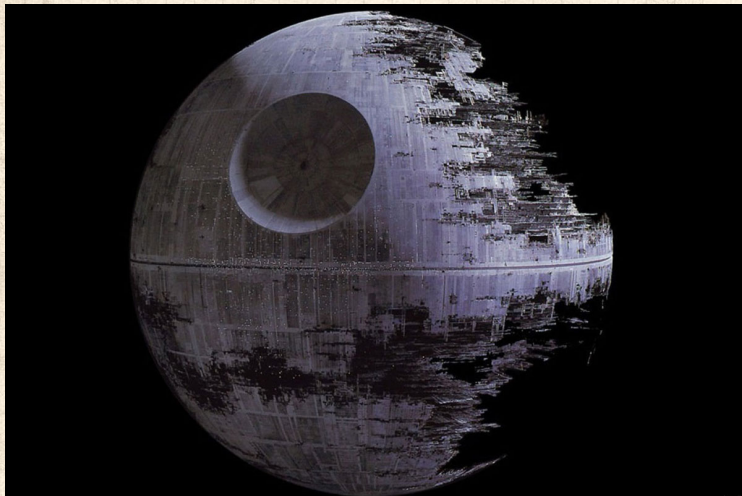
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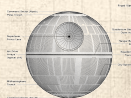
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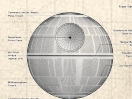
References

“Trouble ...”  





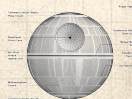
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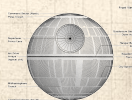
1. **Evolutionary processes**





System robustness may result from

1. Evolutionary processes
2. Engineering/Design



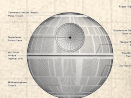


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Idea: Explore systems optimized to perform under uncertain conditions.





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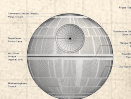


Idea: Explore systems optimized to perform under uncertain conditions.



The handle:

‘Highly Optimized Tolerance’ (HOT) [4, 5, 6, 10]





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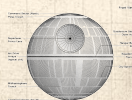


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The catchphrase: Robust-Yet-Fragile





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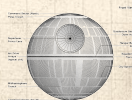
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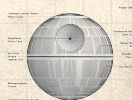
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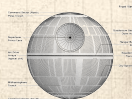
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Great abstracts of the world #73: “There aren’t any.” [7]

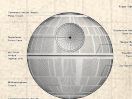


Features of HOT systems: [5, 6]



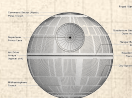
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 High performance and robustness



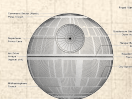
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-  Designed/evolved to handle known stochastic environmental variability



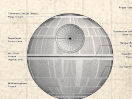
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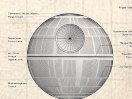
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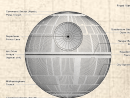
Features of HOT systems: [5, 6]

- High performance and robustness
- Designed/evolved to handle known stochastic environmental variability
- Fragile** in the face of unpredicted environmental signals
- Highly specialized, low entropy configurations
- Power-law distributions appear (of course...)



HOT combines things we've seen:

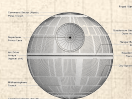
 Variable transformation



HOT combines things we've seen:

 Variable transformation

 Constrained optimization

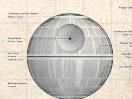


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 Need power law transformation between variables:
 $(Y = X^{-\alpha})$



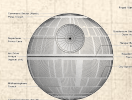
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MIWO is good

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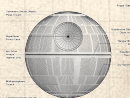
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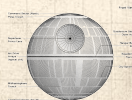
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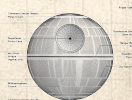
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X has a characteristic size but Y does not



Robustness

Forest fire example: ^[5]

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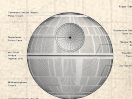
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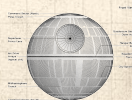
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¹This is bad notation. Would be better to have $N = L \times L$

Forest fire example: ^[5]

 Square $N \times N$ grid¹

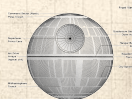


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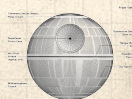
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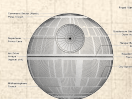
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- 🧱 Sites contain a tree with probability $\rho = \text{density}$
- 🧱 Sites are empty with probability $1 - \rho$
- 🧱 Fires start at location (i, j) according to some distribution P_{ij}



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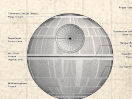
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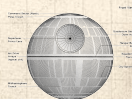
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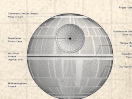
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- ☐ Fires spread from tree to tree (nearest neighbor only)
- ☐ Connected clusters of trees burn completely
- ☐ Empty sites block fire
- ☐ **Best case scenario:**
Build firebreaks to maximize average # trees left intact given one spark



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Robustness

Forest fire example: [5]

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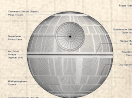
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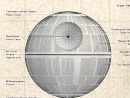
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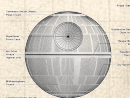
Build a forest by adding one tree at a time



Forest fire example: [5]

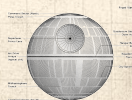
 Build a forest by adding one tree at a time

 Test D ways of adding one tree



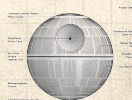
Forest fire example: [5]

-  Build a forest by adding one tree at a time
-  Test D ways of adding one tree
-  $D =$ design parameter



Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree
- $D = \text{design parameter}$
- Average over $P_{ij} = \text{spark probability}$



Forest fire example: [5]

 Build a forest by adding one tree at a time

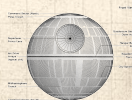
 Test D ways of adding one tree

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 Average over P_{ij} = spark probability

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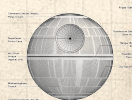
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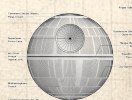
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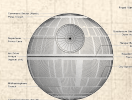
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 $f(c)$ = distribution of fire sizes c (= cost)

 Yield = $Y = \rho - \langle c \rangle$



Specifics:



$$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$

where

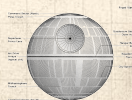
$$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$



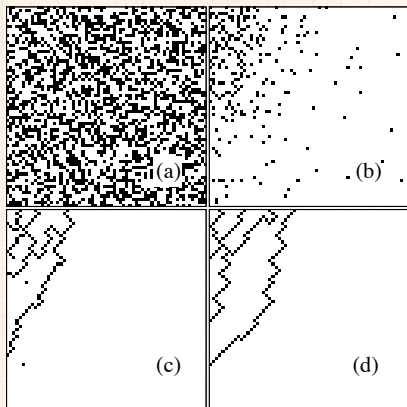
In the original work, $b_y > b_x$



Distribution has more width in y direction.



HOT Forests [5]



$$N = 64$$

(a) $D = 1$

(b) $D = 2$

(c) $D = N$

(d) $D = N^2$



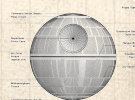
P_{ij} has an
asymmetric,
offset normal
decay



White square
= tree



Black square =
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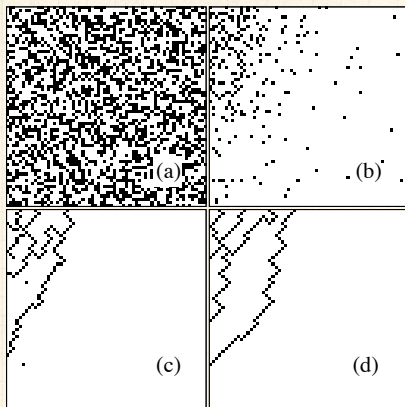
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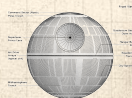
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Optimized forests do well on average



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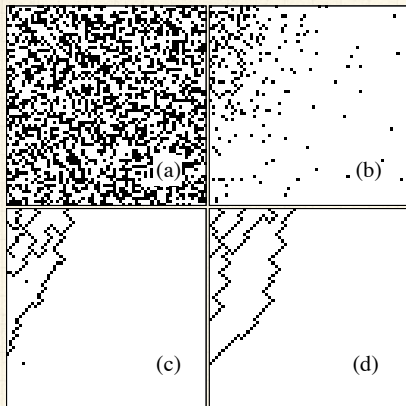
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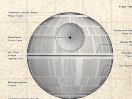
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Optimized forests do well on average



But rare, extreme events occur



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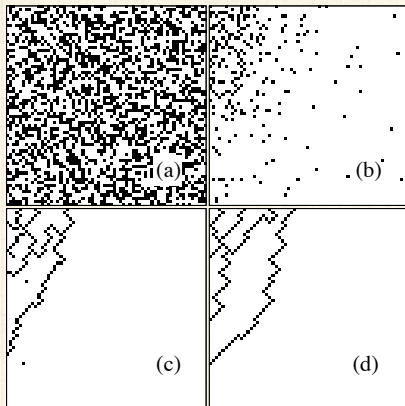
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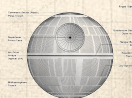
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Optimized forests do well on average (robustness)



But rare, extreme events occur



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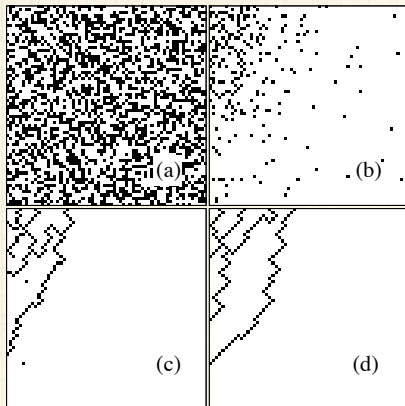
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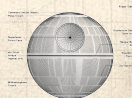
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Optimized forests do well on average (robustness)



But rare, extreme events occur (fragility)



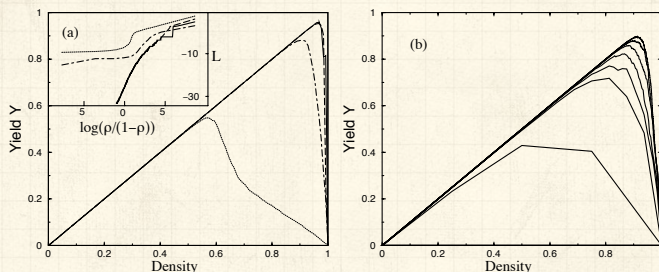
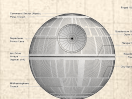


FIG. 2. Yield vs density $Y(\rho)$: (a) for design parameters $D = 1$ (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with $N = 64$, and (b) for $D = 2$ and $N = 2, 2^2, \dots, 2^7$ running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions $L = \log[\langle f \rangle / (1 - \langle f \rangle)]$, on a scale which more clearly differentiates between the curves.

[5]





$Y =$ 'the average density of trees left unburned in a configuration after a single spark hits.' [5]

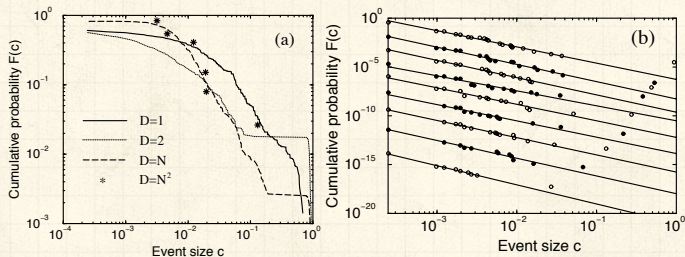
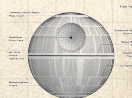
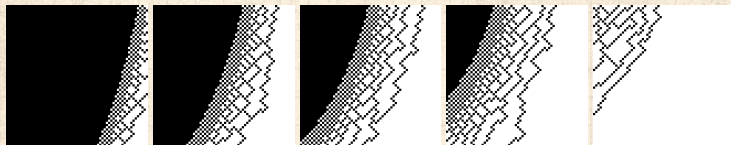


FIG. 3. Cumulative distributions of events $F(c)$: (a) at peak yield for $D = 1, 2, N$, and N^2 with $N = 64$, and (b) for $D = N^2$, and $N = 64$ at equal density increments of 0.1, ranging at $\rho = 0.1$ (bottom curve) to $\rho = 0.9$ (top curve).



Variable density story does not hold up:



HOT model simulations for:²

 $N = 64, D = N^2 = 4,096$  

 $N = 128, D = N^2 = 16,384$  

 $N = 256, D = N^2 = 65,536$ (symmetric)  

 $N = 256, D = N^2 = 65,536$ (skewed)  

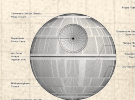
 Density measure should be for forested part only.³

 Distribution is missing spike for size zero forests.

 Distribution tail grows with tree addition.

²Simulations and videos by David Matthews, PoCS 2020

³And it would be high, far above p_c



Outline

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Robustness

HOT theory

Random forests

Self-Organized Criticality

COLD theory

Network robustness

Extras

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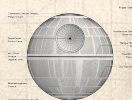
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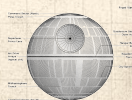
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$D = 1$: Random forests = Percolation ^[11]

 Randomly add trees.



Random Forests

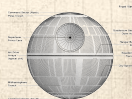
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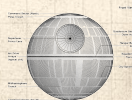
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Random Forests

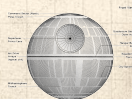
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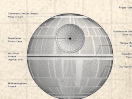
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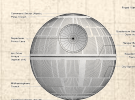
HOT forests nutshell:



Highly structured.



Claim power law distribution of tree cluster sizes for a broad range of ρ , including below ρ_c (but model's dynamic growth path is odd).



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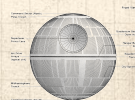
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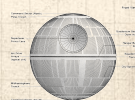


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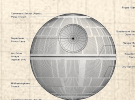
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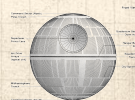
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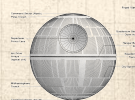
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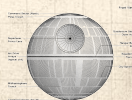
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- HOT theory is more general than just this toy model.



HOT theory:

The abstract story, using figurative forest fires:

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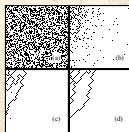
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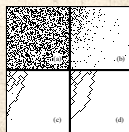


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Given some measure of failure size y_i and correlated resource size x_i with relationship $y_i = x_i^{-\alpha}$, $i = 1, \dots, N_{\text{sites}}$.



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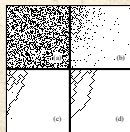
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 Design system to minimize $\langle y \rangle$ subject to a constraint on the x_i .

 Minimize cost:

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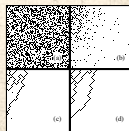
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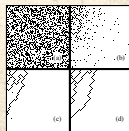
Subject to $\sum_{i=1}^{N_{\text{sites}}} x_i = \text{constant}$.



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$$C_{\text{fire}} \propto \sum_{i=1}^{N_{\text{sites}}} p_i a_i.$$

a_i = area of i th site's region, and p_i = avg. prob. of fire at i th site over some time frame.



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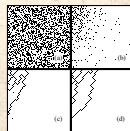
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Per unit area, and over same time frame:

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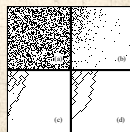
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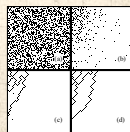
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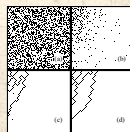
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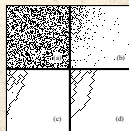
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3. Insert assignment question to find:

$$\Pr(a_i) \propto a_i^{-\gamma}.$$

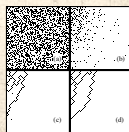


Continuum version:

1. Cost function:

$$\langle C \rangle = \int C(\vec{x}) p(\vec{x}) d\vec{x}$$

where C is some cost to be evaluated at each point in space $\vec{x}$
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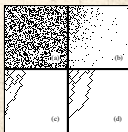


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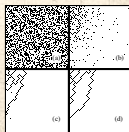
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$$\int R(\vec{x}) d\vec{x} = c$$

where c is a constant.



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where C is some cost to be evaluated at each point in space $\vec{x}$ (e.g., $V(\vec{x})^\alpha$), and $p(\vec{x})$ is the probability an Ewok jabs position $\vec{x}$ with a sharpened stick (or equivalent).

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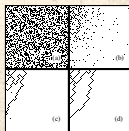
$$\int R(\vec{x}) d\vec{x} = c$$

where c is a constant.



Claim/observation is that typically^[4]

$$V(\vec{x}) \sim R^{-\beta}(\vec{x})$$



Continuum version:

1. Cost function:

$$\langle C \rangle = \int C(\vec{x}) p(\vec{x}) d\vec{x}$$

where C is some cost to be evaluated at each point in space $\vec{x}$ (e.g., $V(\vec{x})^\alpha$), and $p(\vec{x})$ is the probability an Ewok jabs position $\vec{x}$ with a sharpened stick (or equivalent).

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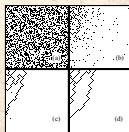


Claim/observation is that typically^[4]

$$V(\vec{x}) \sim R^{-\beta}(\vec{x})$$



For spatial systems with barriers: $\beta = d$.



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Self-Organized Criticality

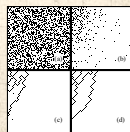
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The HOT model in the wild



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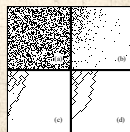
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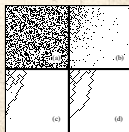
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SOC = Self-Organized Criticality

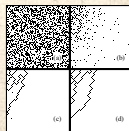


Idea: natural dissipative systems exist at ‘critical states’;



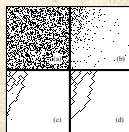
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- 🧱 Idea: natural dissipative systems exist at ‘critical states’;
- 🧱 Analogy: Ising model with temperature somehow self-tuning;



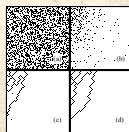
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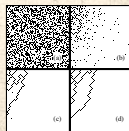
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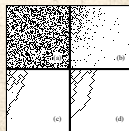
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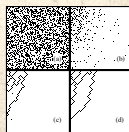
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- ✉ Idea: natural dissipative systems exist at ‘critical states’;
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- ✉ Self-tuning not always possible;
- ✉ Much criticism and arguing...



Robustness

HOT theory

Random forests

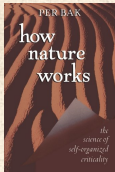
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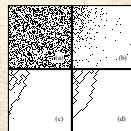
References



“How Nature Works: the Science of Self-Organized Criticality”

by Per Bak (1997). [2]

Avalanches of Sand and Rice ...



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“Complexity and robustness” 

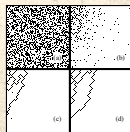
Carlson and Doyle,

Proc. Natl. Acad. Sci., **99**, 2538–2545, 2002. [6]

HOT versus SOC



Both produce power laws



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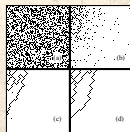
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Optimization versus self-tuning



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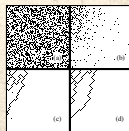
Both produce power laws



Optimization versus self-tuning



Claim: HOT systems viable over a wide range of high densities (false)



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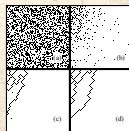
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HOT versus SOC

-  Both produce power laws
-  Optimization versus self-tuning
-  Claim: HOT systems viable over a wide range of high densities (false)
-  True: SOC systems have one special density



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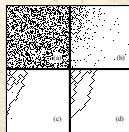
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-  True: SOC systems have one special density
-  HOT systems produce specialized structures



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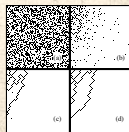
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HOT versus SOC

-  Both produce power laws
-  Optimization versus self-tuning
-  Claim: HOT systems viable over a wide range of high densities (false)
-  True: SOC systems have one special density
-  HOT systems produce specialized structures
-  SOC systems produce generic structures



HOT theory—Summary of designed tolerance [6]

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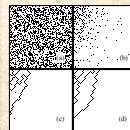
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Table 1. Characteristics of SOC, HOT, and data

	Property	SOC	HOT and Data
1	Internal configuration	Generic, homogeneous, self-similar	Structured, heterogeneous, self-dissimilar
2	Robustness	Generic	Robust, yet fragile
3	Density and yield	Low	High
4	Max event size	Infinitesimal	Large
5	Large event shape	Fractal	Compact
6	Mechanism for power laws	Critical internal fluctuations	Robust performance
7	Exponent α	Small	Large
8	α vs. dimension d	$\alpha \approx (d - 1)/10$	$\alpha \approx 1/d$
9	DDOFs	Small (1)	Large (∞)
10	Increase model resolution	No change	New structures, new sensitivities
11	Response to forcing	Homogeneous	Variable



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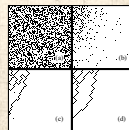
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Robustness and narrative causality:



Robust-yet-fragile, enstoried.⁴



⁴See also: Achilles 

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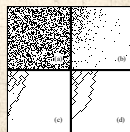
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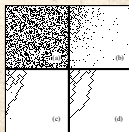
References



Avoidance of large-scale failures



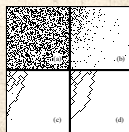
Constrained Optimization with Limited Deviations^[9]



Avoidance of large-scale failures

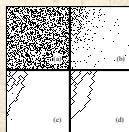
 Constrained Optimization with Limited Deviations^[9]

 Weight cost of large losses more strongly



Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations^[9]
- Weight cost of large losses more strongly
- Increases average cluster size of burned trees...



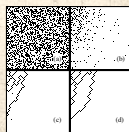
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 Weight cost of large losses more strongly

 Increases average cluster size of burned trees...

 ... but reduces chances of catastrophe



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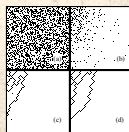
 Constrained Optimization with Limited Deviations ^[9]

 Weight cost of large losses more strongly

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 Power law distribution of fire sizes is truncated



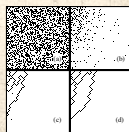
Observed:



Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.



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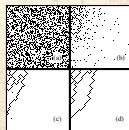
 Power law distributions often have an exponential cutoff

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 May be Weibull distributions:

$$P(x) \sim x^{-\gamma} e^{-ax^{-\gamma+1}}$$



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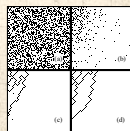
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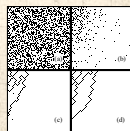
We'll return to this later on (maybe):

 Network robustness.

 Albert et al., Nature, 2000:
“Error and attack tolerance of complex networks” [1]

 General contagion processes acting on complex networks. [13, 12]

 Similar robust-yet-fragile stories ...



HOT forests—Real data:

“Complexity and Robustness,” Carlson & Dolye ^[6]

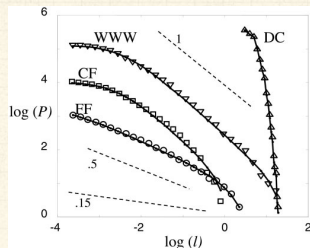


Fig. 1. Log-log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0, 0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$, respectively) and the SOC FF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5, 1$ (dashed) are included. The cumulative distributions of frequencies $\mathcal{P}(l \geq l_i)$ vs. l_i describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km² (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.



These are CCDFs ...

EEK: $P, \mathcal{P}(l \geq l_i)$,



PLR =
probability-loss-resource.



Minimize cost subject to
resource (barrier) constraints:

$$C = \sum_i p_i l_i$$

given

$$l_i = f(r_i) \text{ and } \sum r_i \leq R.$$



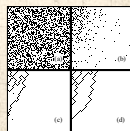
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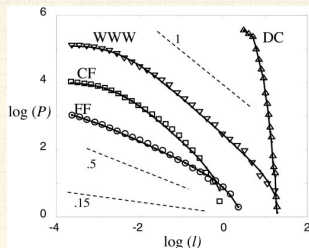


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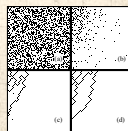
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Horror: log. Screaming: “The
base! What is the base!? You
monsters!”

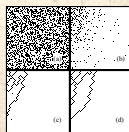


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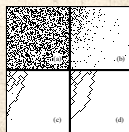
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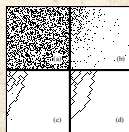
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