

System Robustness

Last updated: 2025/10/28, 19:39:10 EDT

Principles of Complex Systems,
Vols. 1, 2, 3D, 4 Fouever, V for Vendetta

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Robustness

Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

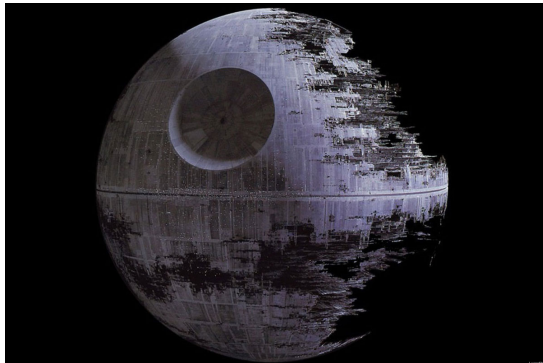
- Blackouts
- Disease outbreaks
- Wildfires
- Earthquakes
- Organisms, individuals and societies
- Ecosystems
- Cities
- Myths: Achilles.

But complex systems also show persistent **robustness** (not as exciting but important...)

Robustness and Failure may be a power-law story...

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Our emblem of Robust-Yet-Fragile:



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System robustness may result from

- Evolutionary processes
- Engineering/Design

Idea: Explore systems optimized to perform under **uncertain conditions**.

The handle:
'Highly Optimized Tolerance' (HOT) ^[4, 5, 6, 10]

The catchphrase: **Robust-Yet-Fragile**

The people: Jean Carlson and John Doyle 

Great abstracts of the world #73: "There aren't any." ^[7]

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Features of HOT systems: ^[5, 6]

- High performance and robustness
- Designed/evolved to handle known stochastic environmental variability
- Fragile** in the face of unpredicted environmental signals
- Highly specialized, low entropy configurations
- Power-law distributions appear (of course...)

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HOT combines things we've seen:

- Variable transformation
- Constrained optimization

Need power law transformation between variables:
($Y = X^{-\alpha}$)

Recall PLIPLO is bad...

MIWO is good: Mild In, Wild Out

X has a characteristic size but Y does not

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Forest fire example: ^[5]

- Square $N \times N$ grid¹
- Sites contain a tree with probability ρ = density
- Sites are empty with probability $1 - \rho$
- Fires start at location (i, j) according to some distribution P_{ij}
- Fires spread from tree to tree (nearest neighbor only)
- Connected clusters of trees burn completely
- Empty sites block fire
- Best case scenario:**
Build firebreaks to maximize average # trees left intact given one spark

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Forest fire example: ^[5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree
- D = **design parameter**
- Average over P_{ij} = spark probability
- $D = 1$: random addition
- $D = N^2$: test all possibilities

Measure average area of forest left untouched

- $f(c)$ = distribution of fire sizes c (= cost)
- Yield = $Y = \rho - \langle c \rangle$

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¹This is bad notation. Would be better to have $N = L \times L$

Specifics:



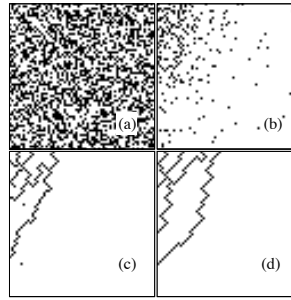
$$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$

where

$$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$

- In the original work, $b_y > b_x$
- Distribution has more width in y direction.

HOT Forests [5]



$N = 64$

- (a) $D = 1$
- (b) $D = 2$
- (c) $D = N$
- (d) $D = N^2$

- P_{ij} has an asymmetric, offset normal decay
- White square = tree
- Black square = no tree

- Optimized forests do well on average (robustness)
- But rare, extreme events occur (fragility)

HOT Forests

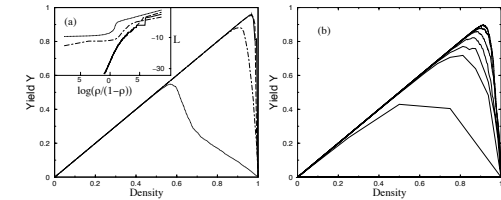


FIG. 2. Yield vs density $Y(\rho)$: (a) for design parameters $D = 1$ (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with $N = 64$, and (b) for $D = 2$ and $N = 2, 2^2, \dots, 2^7$ running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions $L = \log[\langle f \rangle / (1 - \langle f \rangle)]$, on a scale which more clearly differentiates between the curves.

[5]

HOT Forests:

- $Y =$ ‘the average density of trees left unburned in a configuration after a single spark hits.’ [5]

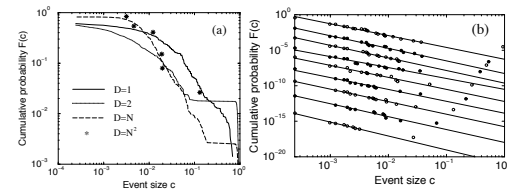
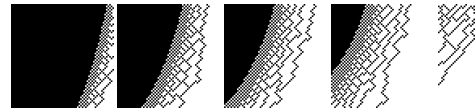


FIG. 3. Cumulative distributions of events $F(c)$: (a) at peak yield for $D = 1, 2, N$, and N^2 with $N = 64$, and (b) for $D = N^2$, and $N = 64$ at equal density increments of 0.1 , ranging at $\rho = 0.1$ (bottom curve) to $\rho = 0.9$ (top curve).

Variable density story does not hold up:



HOT model simulations for:²

- $N = 64, D = N^2 = 4,096$
- $N = 128, D = N^2 = 16,384$
- $N = 256, D = N^2 = 65,536$ (symmetric)
- $N = 256, D = N^2 = 65,536$ (skewed)

- Density measure should be for forested part only.³
- Distribution is missing spike for size zero forests.
- Distribution tail grows with tree addition.

²Simulations and videos by David Matthews, PoCS 2020

³And it would be high, far above p_c

HOT theory:

The abstract story, using figurative forest fires:

- Given some measure of failure size y_i and correlated resource size x_i with relationship $y_i = x_i^{-\alpha}$, $i = 1, \dots, N_{\text{sites}}$.
- Design system to minimize $\langle y \rangle$ subject to a constraint on the x_i .
- Minimize cost:

$$C = \sum_{i=1}^{N_{\text{sites}}} \Pr(y_i) y_i$$

Subject to $\sum_{i=1}^{N_{\text{sites}}} x_i = \text{constant}$.

Random Forests

$D = 1$: Random forests = Percolation [11]

- Randomly add trees.
- Below critical density ρ_c , no fires take off.
- Above critical density ρ_c , percolating cluster of trees burns.
- Only at ρ_c , the critical density, is there a power-law distribution of tree cluster sizes.
- Forest is random and featureless.

1. Cost: Expected size of fire:

$$C_{\text{fire}} \propto \sum_{i=1}^{N_{\text{sites}}} p_i a_i.$$

a_i = area of i th site’s region, and p_i = avg. prob. of fire at i th site over some time frame.

2. Constraint: building and maintaining firewalls.
Per unit area, and over same time frame:

$$C_{\text{firewalls}} \propto \sum_{i=1}^{N_{\text{sites}}} a_i^{1/2} a_i^{-1}.$$

- We are assuming isometry.
- In d dimensions, $1/2$ is replaced by $(d - 1)/d$

3. Insert assignment question to find:

$$\Pr(a_i) \propto a_i^{-\gamma}.$$

HOT forests nutshell:

- Highly structured.
- Claim power law distribution of tree cluster sizes for a broad range of ρ , including below ρ_c (but model’s dynamic growth path is odd).
- Claim: No specialness of ρ_c (oops).
- Forest states are **tolerant**.
- Uncertainty is okay if well characterized.
- If P_{ij} is characterized poorly or changes too fast, failure becomes **highly likely**.
- Growth is key to toy model which is both algorithmic and physical.
- HOT theory is more general than just this toy model.

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