

Properties of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2025–2026

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont



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Degree distributions

Assortativity

Clustering

Motifs

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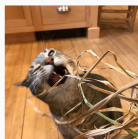
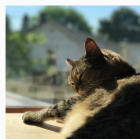
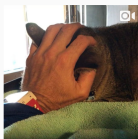
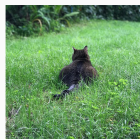
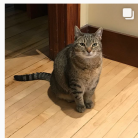
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 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 

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A notable feature of large-scale networks:

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A notable feature of large-scale networks:



Graphical renderings are often just a big mess.

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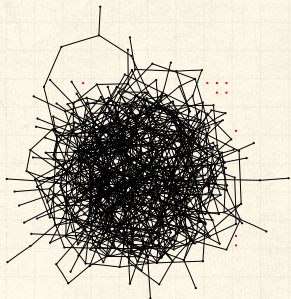
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A notable feature of large-scale networks:



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⇐ Typical hairball



number of nodes $N = 500$



number of edges $m = 1000$



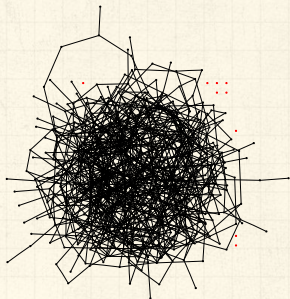
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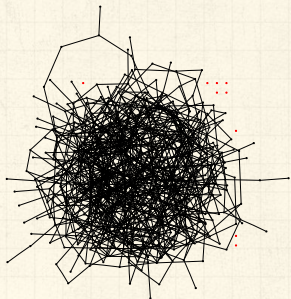
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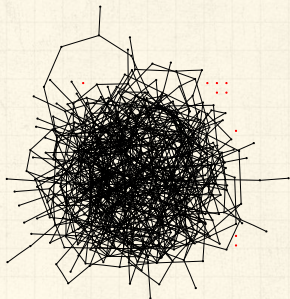
said Ponder [Stibbons] — *Making Money*, T. Pratchett.



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said Ponder [Stibbons] — *Making Money*, T. Pratchett.



We need to extract **digestible, meaningful aspects**.



Some key aspects of real complex networks:

 degree distribution*

 assortativity

 homophily

 clustering

 motifs

 modularity



 hierarchical scaling

 concurrency

 network distances

 centrality

 multilayerness

 efficiency

 robustness

 Plus coevolution of network structure and processes on networks.

- * Degree distribution is the elephant in the room that we are now all very aware of...



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1. degree distribution P_k

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 P_k is the probability that a randomly selected node has degree k .

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 link cost controls skew.

 hubs may facilitate or impede contagion.



Properties

Note:



Erdős-Rényi random networks are a *mathematical construct*.

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 Erdős-Rényi random networks are a *mathematical construct*.

 'Scale-free' networks are **growing networks** that form according to a **plausible mechanism**.

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Note:

 Erdős-Rényi random networks are a *mathematical construct*.

 'Scale-free' networks are **growing networks** that form according to a **plausible mechanism**.

 Randomness is out there, just not to the degree of a completely random network.



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2. Assortativity/3. Homophily:

 Social networks: Homophily  = birds of a feather

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 **Disassortative** network: high degree nodes connecting to low degree nodes.

*Often **techological** or **biological**: Internet, WWW, protein interactions, neural networks, food webs.*



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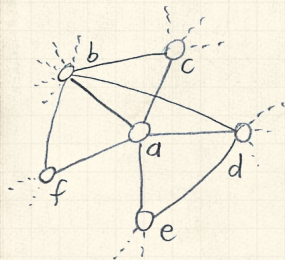
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Local socialness:

4. Clustering:



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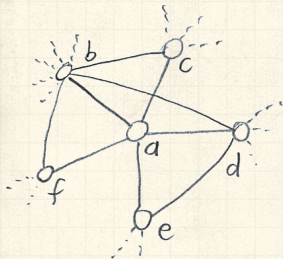


Local socialness:

4. Clustering:



Your friends tend to know each other.



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Local socialness:

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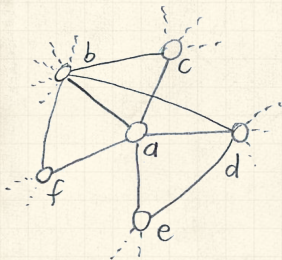
Two measures (explained on following slides):

1. Watts & Strogatz [8]

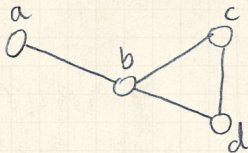
$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

2. Newman [6]

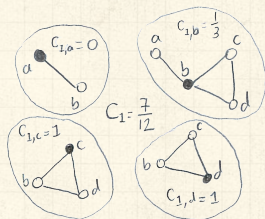
$$C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$$



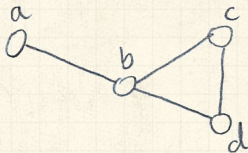
Example network:



Calculation of C_1 :

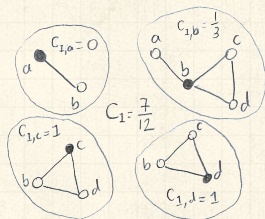


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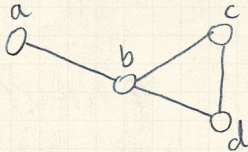


C_1 is the **average fraction of pairs of neighbors who are connected.**

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Example network:



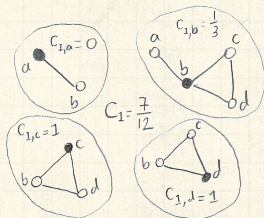
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Fraction of pairs of neighbors who are connected is

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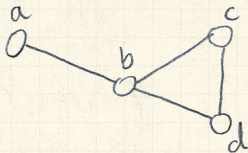
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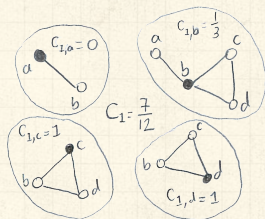
where k_i is node i 's degree, and N_i is the set of i 's neighbors.



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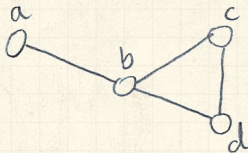


Averaging over all nodes, we have:

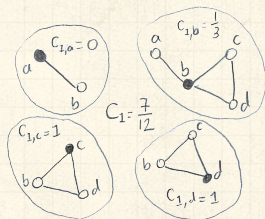
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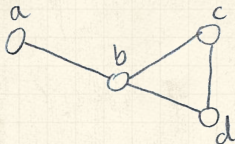
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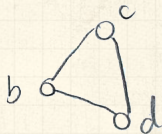
Triples and triangles

Example network:

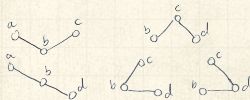


Nodes i_1 , i_2 , and i_3 form a **triple** around i_1 if i_1 is connected to i_2 and i_3 .

Triangles:

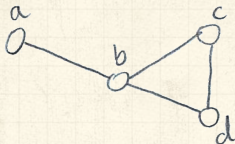


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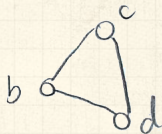


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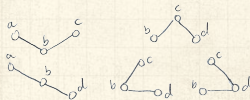
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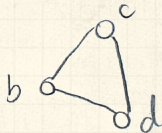


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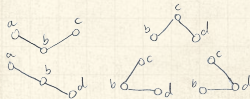
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The definition $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ measures the fraction of **closed triples**

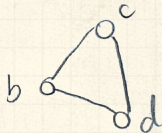


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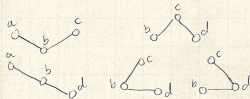
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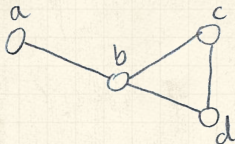


The '**3**' appears because for each triangle, we have 3 closed triples.

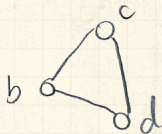


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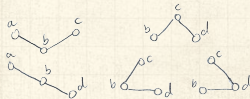
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The '**3**' appears because for each triangle, we have 3 closed triples.



Social Network Analysis (SNA): fraction of **transitive triples**.



Clustering:

Sneaky counting for undirected, unweighted networks:

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Clustering:

Sneaky counting for undirected, unweighted networks:

 If the path $i-j-\ell$ exists then $a_{ij}a_{j\ell} = 1$.

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 Otherwise, $a_{ij}a_{j\ell} = 0$.

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 Otherwise, $a_{ij}a_{j\ell} = 0$.

 We want $i \neq \ell$ for good triples.



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 In general, a path of n edges between nodes i_1 and i_n travelling through nodes $i_2, i_3, \dots, i_{n-1}$ exists $\iff$
$$a_{i_1 i_2} a_{i_2 i_3} a_{i_3 i_4} \cdots a_{i_{n-2} i_{n-1}} a_{i_{n-1} i_n} = 1.$$

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$$\#\text{triangles} = \frac{1}{6} \text{Tr} A^3$$



Properties



For sparse networks, C_1 tends to discount highly connected nodes.

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Properties



For sparse networks, C_1 tends to discount highly connected nodes.



C_2 is a useful and often preferred variant

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In general, $C_1 \neq C_2$.

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C_1 is a global average of a local ratio.



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For sparse networks, C_1 tends to discount highly connected nodes.



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In general, $C_1 \neq C_2$.



C_1 is a global average of a local ratio.



C_2 is a ratio of two global quantities.



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5. motifs:

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5. motifs:



small, recurring functional subnetworks

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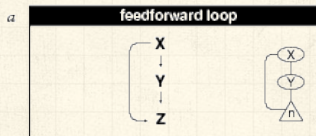
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5. motifs:

 small, recurring functional subnetworks

 e.g., Feed Forward Loop:

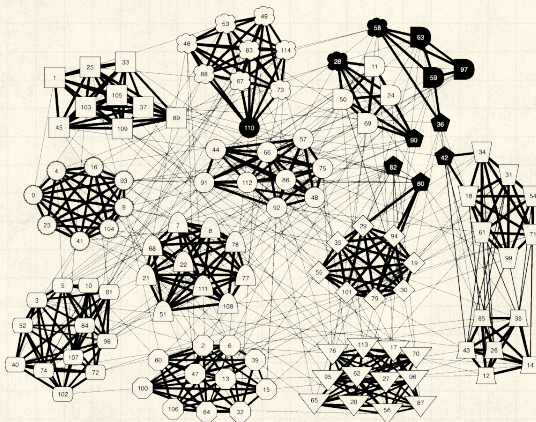


Shen-Orr, Uri Alon, *et al.* [7]



Properties

6. modularity and structure/community detection:



Clauset *et al.*, 2006^[2]: NCAA football

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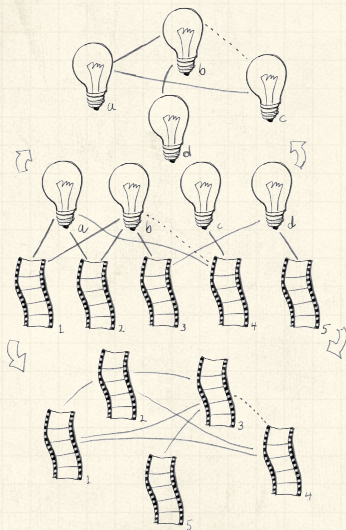
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Bipartite/multipartite affiliation structures:



Many real-world networks have an underlying multi-partite structure.

-  Stories-tropes.
-  Boards and directors.
-  Films-actors-directors.
-  Classes-teachers-students.
-  Upstairs-downstairs.



Unipartite networks may be induced or co-exist.



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7. concurrency:



transmission of a contagious element only occurs during contact



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 Kretzschmar and Morris, 1996 ^[4]



7. concurrency:

 transmission of a contagious element only occurs during contact

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 beware cumulated network data

 Kretzschmar and Morris, 1996 ^[4]

 “Temporal networks” become a concrete area of study for Piranha Physicus in 2013.



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8. Horton-Strahler ratios:



Metrics for branching networks:



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8. Horton-Strahler ratios:



Metrics for branching networks:



Method for ordering streams hierarchically



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Number: $R_n = N_\omega / N_{\omega+1}$



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Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$



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Metrics for branching networks:



Method for ordering streams hierarchically



Number: $R_n = N_\omega / N_{\omega+1}$



Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$



Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_\omega \rangle$



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9. network distances:

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9. network distances:

(a) shortest path length d_{ij} :

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Properties

9. network distances:

(a) shortest path length d_{ij} :



Fewest number of steps between nodes i and j .

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Average shortest path length in whole network.



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Weighted links can be accommodated.



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9. network distances:



network diameter $d_{\max}$:

Maximum shortest path length between any two nodes.



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Average 'distance' between any two nodes.



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Closeness handles disconnected networks ($d_{ij} = \infty$)



$d_{\text{cl}} = \infty$ only when all nodes are isolated.



Closeness perhaps compresses too much into one number



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10. centrality:

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10. centrality:



Many such measures of a node's 'importance.'

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ex 1: Degree centrality: k_i .

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ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg^[3])



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Interconnected networks and robustness (two for one deal):

“Catastrophic cascade of failures in interdependent networks”^[1].

Buldyrev et al., Nature 2010.

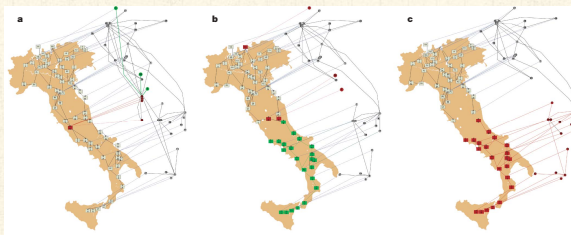


Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003²⁶. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a**, One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network)

at the next step are marked in green. **b**, Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c**, Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).



Overview Key Points:



The field of complex networks came into existence in the late 1990s.

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 Explosion of papers and interest since 1998/99.



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-  The field of complex networks came into existence in the late 1990s.
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-  Three main (blurred) categories:
 1. **Physical** (e.g., river networks),
 2. **Interactional** (e.g., social networks),
 3. **Abstract** (e.g., thesauri).



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