

# Overview of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D  
CSYS/MATH 6701, 6713, & a pretend number, 2025–2026

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Computational Story Lab | Vermont Complex Systems Center  
Santa Fe Institute | University of Vermont



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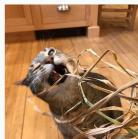
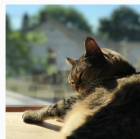
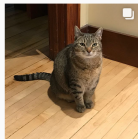
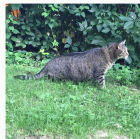
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 On Instagram at [pratchett\\_the\\_cat](https://www.instagram.com/pratchett_the_cat) 

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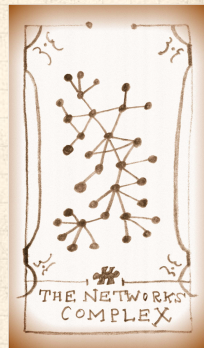
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# net•work |'netwɜrk|

noun

- 1 an arrangement of intersecting horizontal and vertical lines.
  - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- 2 a group or system of interconnected people or things : *a trade network.*
  - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
  - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj. ] *network television.*
  - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
  - a system of connected electrical conductors.

verb [ trans. ]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj. ] ( **networked**) *networked workstations.*
- [ intrans. ] [often as n. ] ( **networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*



## Thesaurus deliciousness:

# network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.



From Keith Briggs's excellent etymological investigation: 



Opus reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]



# Ancestry:

First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

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1883–: distribution network of electrical cables

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-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

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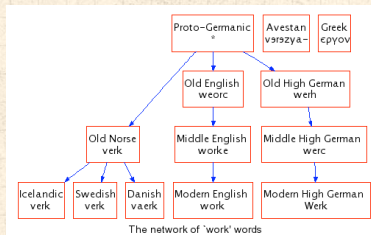
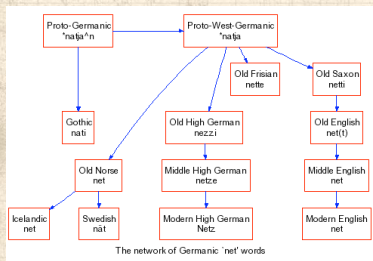
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'Work' appear to have long meant purposeful action.



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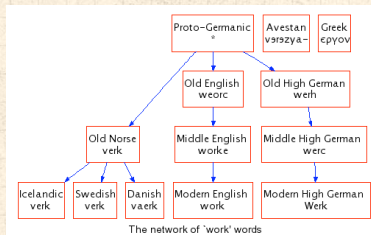
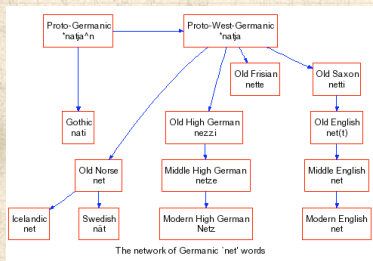
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'Network' = something built based on the idea of natural, flexible lattice or web.

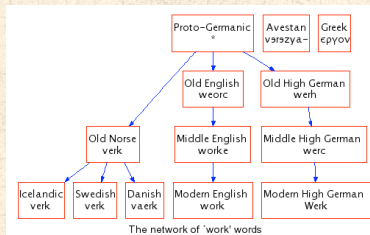
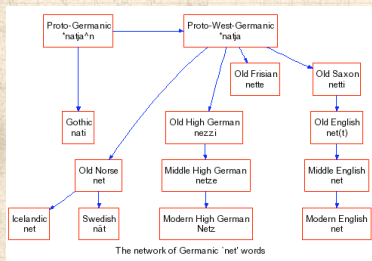


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 **'Work'** appear to have long meant purposeful action.



 **'Network'** = something built based on the idea of natural, flexible lattice or web.

 c.f., ironwork, stonework, fretwork.





## Key Observation:



Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.

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## Key Observation:



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Opens door to mathematical and numerical analysis.



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Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.



## Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...

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 Hunt in packs.



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- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)





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- See also: <https://xkcd.com/793/>





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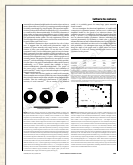
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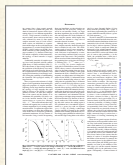
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“Collective dynamics of ‘small-world’ networks”   
Watts and Strogatz,  
Nature, **393**, 440–442, 1998. <sup>[16]</sup>

Times cited: 

(as of October 24, 2018)



“Emergence of scaling in random networks”   
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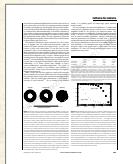
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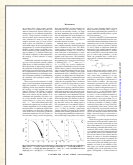
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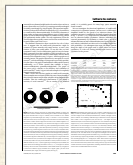
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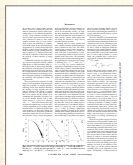
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### “Complex Networks: Structure and Dynamics”

Boccaletti et al.,

Physics Reports, **424**, 175–308, 2006. <sup>[3]</sup>

Times cited:  ~ **8,533** (as of October 24, 2018)



### “The structure and function of complex networks”

M. E. J. Newman,

SIAM Rev., **45**, 167–256, 2003. <sup>[12]</sup>

Times cited:  ~ **17,782** (as of October 24, 2018)



### “Statistical mechanics of complex networks”

Albert and Barabási,

Rev. Mod. Phys., **74**, 47–97, 2002. <sup>[1]</sup>

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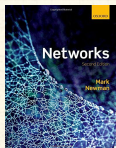
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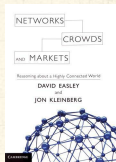
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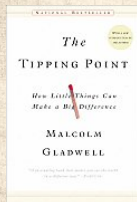


“Networks, crowds, and markets: Reasoning about a  
highly connected world”    
by Easley and Kleinberg (2010). <sup>[7]</sup>

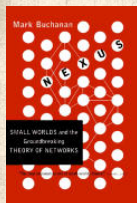
<http://cs.cornell.edu/home/kleinber/networks-book/> 



# Popularity according to books:



## The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



## Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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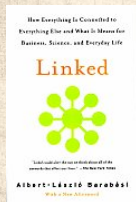
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# Popularity according to books:



## Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



## Six Degrees: The Science of a Connected Age—Duncan Watts<sup>[15]</sup>

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## Numerous others ...

- Complex Social Networks—F. Vega-Redondo <sup>[14]</sup>
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo <sup>[13]</sup>
- Random Graph Dynamics—R. Durrett
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster <sup>[5]</sup>
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes <sup>[6]</sup>



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But surely **networks** aren't new ...

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Graph theory is well established ...

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Study of social networks started in the 1930's ...

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But surely **networks** aren't new ...



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So why all this 'new' research on networks?

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But surely **networks** aren't new ...



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**Answer:** Oodles of Easily Accessible Data.

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**Answer:** Oodles of Easily Accessible Data.



We can now inform (alas) our theories  
with a much more measurable reality.\*



# More observations

- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.\*
- A worthy goal: establish **mechanistic explanations**.



# More observations



But surely **networks** aren't new ...



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**Answer:** Oodles of Easily Accessible Data.



We can now inform (alas) our theories  
with a much more measurable reality.\*



A worthy goal: establish **mechanistic** explanations.

*\*If this is upsetting, maybe string theory is for you ...*



# More observations



Web-scale data sets can be overly **exciting**.

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 Web-scale data sets can be overly **exciting**.

Witness:

 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

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# More observations

 Web-scale data sets can be overly **exciting**.

Witness:

 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

 “The Unreasonable Effectiveness of Data,”  
Halevy et al. <sup>[9]</sup>.

 c.f. Wigner’s “The Unreasonable Effectiveness of Mathematics in the Natural Sciences” <sup>[17]</sup>

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But:

 For scientists, description is only part of the battle.

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But:

 For scientists, description is only part of the battle.

 We still need to **understand**.

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# Super Basic definitions

**Nodes** = A collection of entities which have properties that are somehow related to each other

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# Super Basic definitions

**Nodes** = A collection of entities which have properties that are somehow related to each other



e.g., people, forks in rivers, proteins, webpages, organisms, ...

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 **Links** may be directed or undirected.



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# Super Basic definitions

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Other spiffing words: vertices and edges.



# Super Basic definitions

Node degree = Number of links per node

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# Super Basic definitions

**Node degree** = Number of links per node

 Notation: Node  $i$ 's degree =  $k_i$ .

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 Notation: the average degree of a network =  $\langle k \rangle$





# Super Basic definitions

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(and sometimes  $z$ )



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 Connection between number of edges  $m$  and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$



# Super Basic definitions

Node degree = Number of links per node

 Notation: Node  $i$ 's degree =  $k_i$ .

  $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network =  $\langle k \rangle$   
(and sometimes  $z$ )

 Connection between number of edges  $m$  and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

 Defn:  $N_i$  = the set of  $i$ 's  $k_i$  neighbors



# Super Basic definitions

## Adjacency matrix:



We represent a directed network by a matrix  $A$  with link weight  $a_{ij}$  for nodes  $i$  and  $j$  in entry  $(i, j)$ .

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## Adjacency matrix:



We represent a directed network by a matrix  $A$  with link weight  $a_{ij}$  for nodes  $i$  and  $j$  in entry  $(i, j)$ .



e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$





# Super Basic definitions

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 We represent a directed network by a matrix  $A$  with link weight  $a_{ij}$  for nodes  $i$  and  $j$  in entry  $(i, j)$ .

 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 (n.b., for numerical work, we always use sparse matrices.)



# Examples

So what passes for a complex network?

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So what passes for a complex network?



Complex networks are **large** (in node number)

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So what passes for a complex network?



Complex networks are **large** (in node number)



Complex networks are **sparse** (low edge to node ratio)



So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**





## So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**
-  Complex networks can be social, economic, natural, informational, abstract, ...



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## Physical networks



### River networks



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# Examples

## Physical networks

 River networks

 Neural networks



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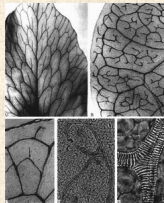
# Examples

## Physical networks

 River networks

 Neural networks

 Trees and leaves

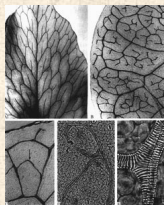




# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

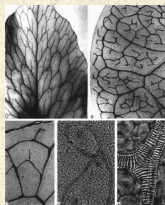


# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

## The Internet



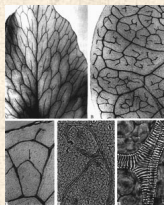
# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

 The Internet

 Road networks

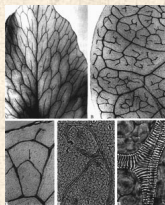


# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids



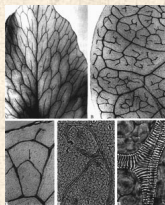


# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids



 **Distribution** (branching) versus **redistribution** (cyclical)

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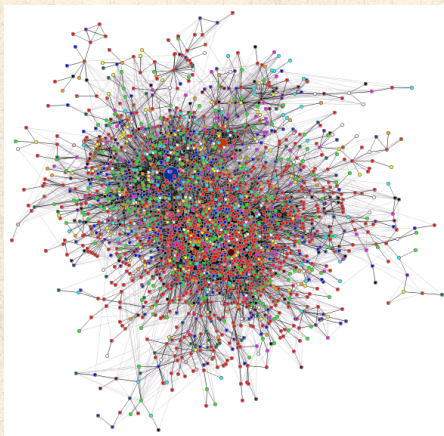
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# Examples

## Interaction networks

-  The Blogosphere (RIP)
-  Biochemical networks
-  Gene-protein networks
-  Food webs: who eats whom
-  The internet<sup>1</sup>
-  Airline networks
-  Call networks (AT&T)
-  The Media
-  Social Media



[datamining.typepad.com](http://datamining.typepad.com) 



<sup>1</sup>What was the World Wide Web, then the Internet, then the internet

topics:

 Hidalgo et al.'s “The Product Space Conditions the Development of Nations” [10]



## References



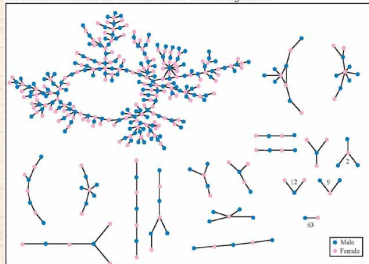
# Examples

## Interaction networks: social networks



Snogging

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else)

(Bearman *et al.*, 2004)

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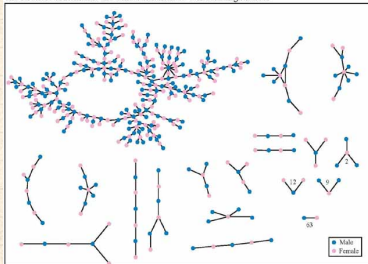


Snogging



Friendships

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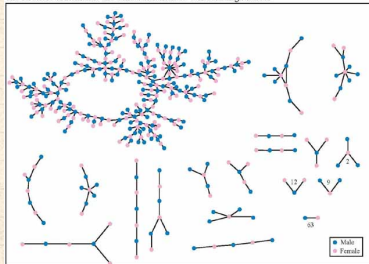


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-  Snogging
-  Friendships
-  Acquaintances

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## Interaction networks: social networks



Snogging



Friendships

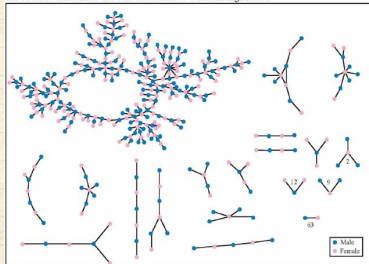


Acquaintances



Boards and directors

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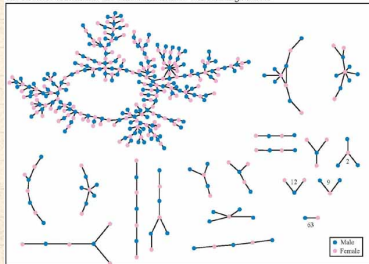


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## Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations

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## Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook ↗, ↗  
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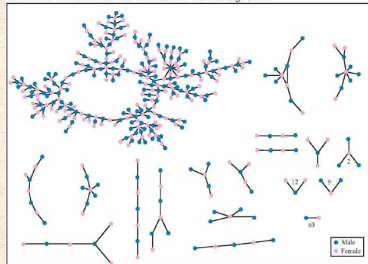
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## Interaction networks: social networks

 Snogging Friendships

## Acquaintances



## Boards and directors

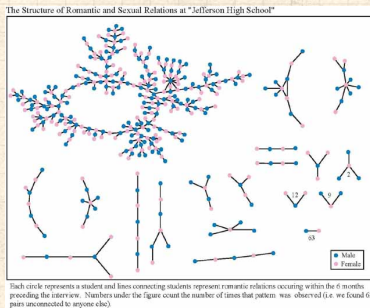


## Organizations



 facebook  twitter ,

'Remotely sensed' by: email activity, instant messaging, phone logs



(Bearman *et al.*, 2004)

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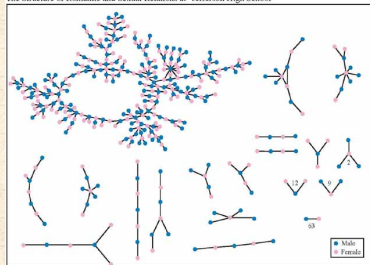
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- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook ↗, ↗  
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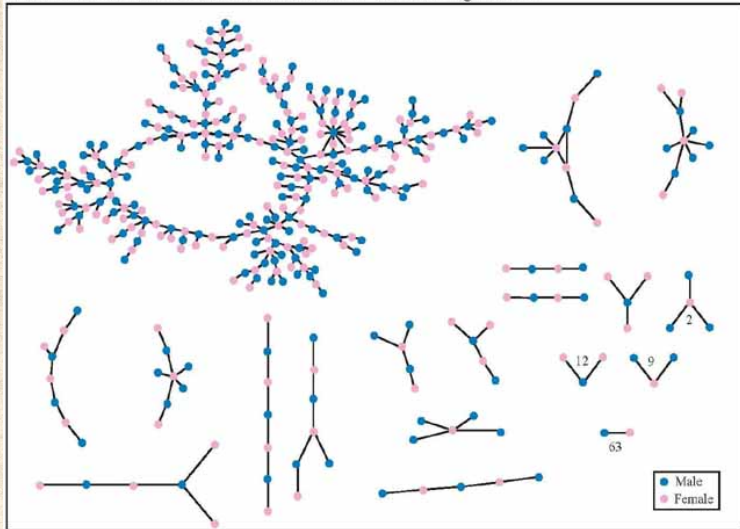
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## Relational networks



Consumer purchases

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## Relational networks



Consumer purchases  
(Walmart, Target, Amazon, ...)

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## Relational networks



Consumer purchases  
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Thesauri: Networks of words generated by meanings

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## Relational networks



Consumer purchases  
(Walmart, Target, Amazon, ...)



Thesauri: Networks of words generated by meanings



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## Relational networks



Consumer purchases  
(Walmart, Target, Amazon, ...)



Thesauri: Networks of words generated by meanings



Knowledge/Databases/Ideas



Metadata—Tagging: [\\_bit.ly](#)  [flickr](#) 

**common tags**    cloud | [list](#)

community   daily   dictionary   education   **encyclopedia**  
english   free   imported   info   information   internet   knowledge  
learning   news   **reference**   research   resource  
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