

Social Contagion

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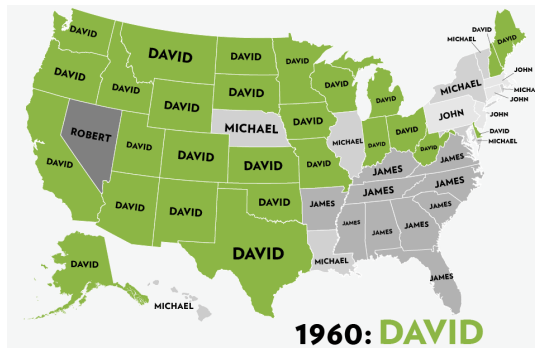
Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

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From the Atlantic

LOL + cute + fail + wtf:

Oopsie!



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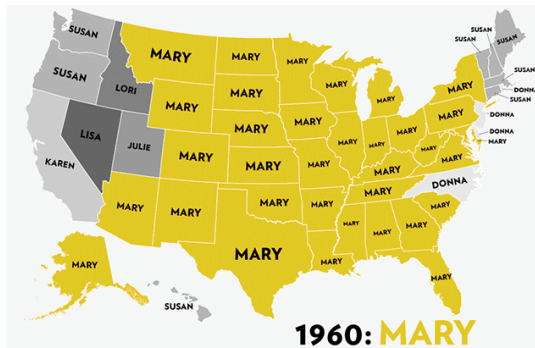
Outline

Social Contagion Models

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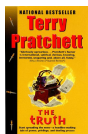
The whole lolcats thing:



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'The rumor spread through the city like wildfire which had quite often spread through Ankh-Morpork since its citizens had learned the words "fire insurance").'



"The Truth" by Terry Pratchett (2000). [22]

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Things that spread well:

buzzfeed.com



Dangerously self aware: [11 Elements that make a perfect viral video](#).

+ News ...

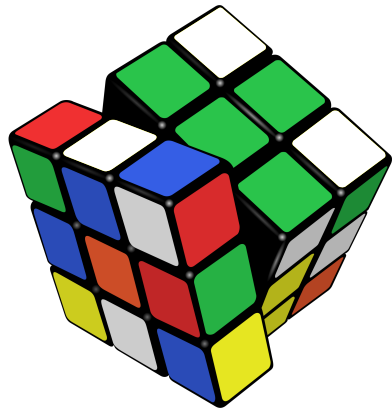
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Some things really stick:

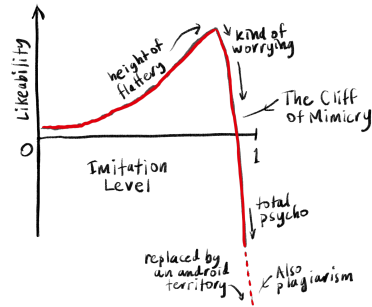


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wtf + geeky + omg:



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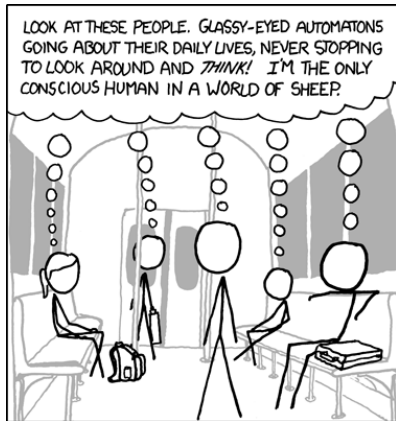
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Market much?

Advertisement enjoyed during "Herstory of Dance" Community S4E08, April 2013.

Why social contagion works so well:



<http://xkcd.com/610/>

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Examples are claimed to abound:

- Fashion
- Striking
- smoking [7]
- Residential segregation [23]
- iPhones and iThings
- obesity [6]
- Stupidity
- Harry Potter
- voting
- gossip
- Rubik's cube
- religious beliefs
- school shootings
- yawning
- leaving lectures

SIR and SIR_S type contagion possible

Classes of behavior versus specific behavior : **dieting, horror movies, getting married, invading countries, ...**

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Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking [7]
- The spread of spreading [6]
- Also: happiness [11], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives

Controversy:

- Are your friends making you fat? (Clive Thompsom, NY Times, September 10, 2009).
- Everything is contagious—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

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- Ugg Boots
- Blundstones

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Mixed messages: Please copy, but also, don't copy ...

- Cindy Harrell appeared in the (terrifying) music video for Ray Parker Jr.'s Ghostbusters.
- In Stranger Things 2, Steve Harrington reveals his Fabergé secret.

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Two focuses for us

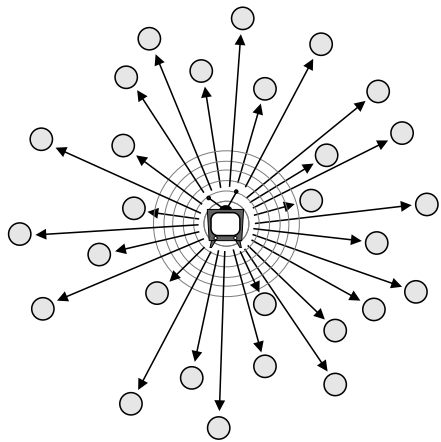
- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers? Highly popularized by Gladwell [12] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [19]

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The hypodermic model of influence



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Why do things spread socially?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are storytellers: *bomo narrativus*
- We like to think things happened for reasons ...
- Reasons for success are usually ascribed to intrinsic properties (examples next).
- Teleological stories of fame are often easy to generate and believe.
- System/group dynamics harder to understand because most of our stories are built around individuals.
- Always good to examine what is said before and after the fact ...

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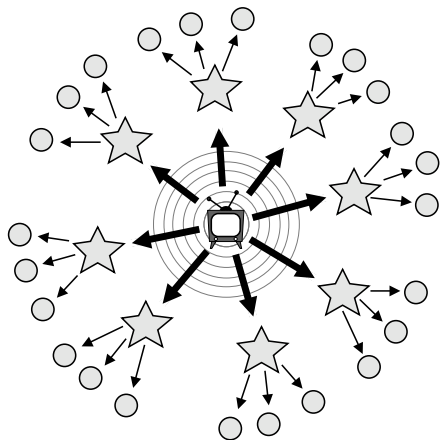
The completely unpredicted fall of Eastern Europe:



Timunr Kuran: [20, 21] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"

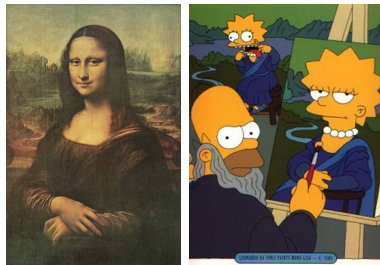
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The two step model of influence [19]



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The Mona Lisa



- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism, parody, ...

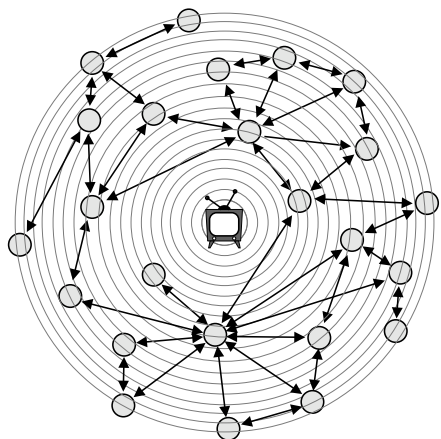
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The dismal predictive powers of editors...



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The general model of influence: the Social Wild



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'Tattooed Guy' Was Pivotal in Armstrong Case [nytimes]



- "... Leogrande's doping sparked a series of events ..."

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From a 2013 Believer Magazine interview with Maurice Sendak

BLVR: Did the success of *Where the Wild Things Are* ever feel like an albatross?

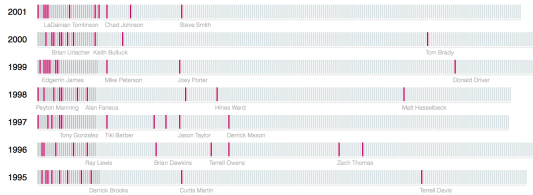
MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention *Moby-Dick*. They're all going to talk about my first book, about ****** maidens in Tahiti*." He was right. No mention of *Moby-Dick* then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came *Moby-Dick* and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. *Moby-Dick* didn't get famous until 1930.

- Sendak named his dog Herman.
- The essential Colbert interview: [Pt. 1](#) and [Pt. 2](#).

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Drafting success in the NFL:

Top Players by Round, 1995-2012



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Social contagion

- Cialdini's modes are heuristics that help us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

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Some possible origins of thresholds:

- Inherent, evolution-derived inclination to coordinate, to conform, to imitate. ^[1]
- Lack of information:** impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities**
 - Externalities = Effects on others not directly involved in a transaction
 - Examples: telephones, fax machine, TikTok, operating systems
 - An individual's utility increases with the adoption level among peers and the population in general

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Messing with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent** 
 - Harnessing of BzzAgents to directly market through social ties.
 - Generally: BzzAgents did not reveal their BzzAgent status and did not want to be paid.
 - NYT, 2004-12-05: "The Hidden (in Plain Sight) Persuaders" 
- One of Facebook's early advertising attempts: **Beacon** 
- All of Facebook's advertising attempts.
- Seriously, Facebook. What could go wrong?

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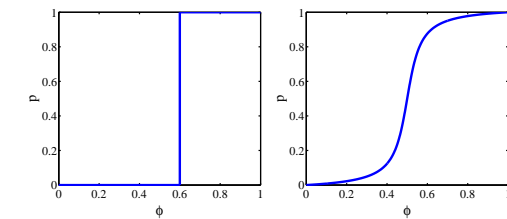
Social Contagion

Some important models:

- Tipping models—Schelling (1971)** ^[23, 24, 25]
 - Simulation on checker boards
 - Idea of thresholds
 - Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.) 
- Threshold models—Granovetter (1978)** ^[15]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992)** ^[2, 3]
 - Social learning theory, Informational cascades,...

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Threshold models—response functions






- Example threshold influence response functions:
 - deterministic and stochastic
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.

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Getting others to do things for you

An influential book: 'Influence' ^[8] by Robert Cialdini 

Six modes of influence:

- Reciprocation:** *The Old Give and Take... and Take;* e.g., Free samples, Hare Krishnas.
- Commitment and Consistency:** *Hobgoblins of the Mind;* e.g., Hazing.
- Social Proof:** *Truths Are Us;* e.g., Jonestown , Kitty Genovese  (contested).
- Liking:** *The Friendly Thief;* e.g., Separation into groups is enough to cause problems.
- Authority:** *Directed Deference;* e.g., Milgram's obedience to authority experiment. 
- Scarcity:** *The Rule of the Few;* e.g., Prohibition.

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Social contagion models

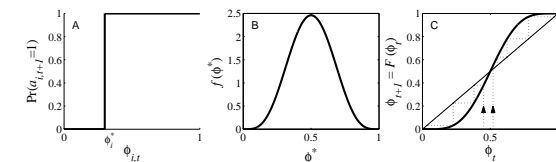
Thresholds

- Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (**unrealistic**).
- Assumption: level of influence per person is uniform (**unrealistic**).

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Threshold models

Action based on perceived behavior of others:

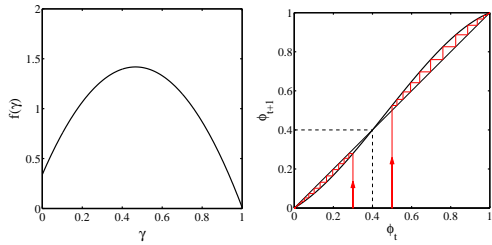


- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a **Critical mass model**

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Threshold models

Another example of critical mass model:



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Threshold models—Nutshell

Implications for collective action theory:

1. Collective uniformity \Rightarrow individual uniformity
2. Small individual changes \Rightarrow large global changes
3. The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
4. System stories live in left null space of our stories—we can't even see them.
5. But we happily impose simplistic, individual-centric stories—we can't help ourselves \square .

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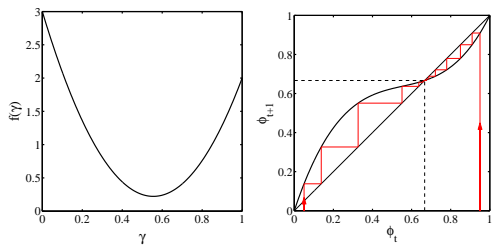
Threshold model on a network

- Interactions between individuals now represented by a network.
- Network is **sparse**.
- Individual i has k_i contacts.
- Influence on each link is **reciprocal** and of **unit weight**.
- Each individual i has a fixed threshold ϕ_i .
- Individuals repeatedly poll contacts on network.
- Synchronous, discrete time updating.
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$.
- Individuals remain active when switched (no recovery = SI model).

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Threshold models

Example of single stable state model:



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Many years after Granovetter and Soong's work:

- “A simple model of global cascades on random networks”
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [27]
- Mean field model \rightarrow network model
- Individuals now have a limited view of the world

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Snowballing

First study random networks:

- Start with N nodes with a degree distribution P_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a **cascade condition**

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We'll also explore:

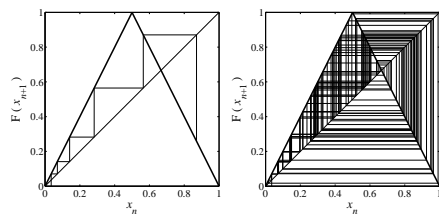
- “Seed size strongly affects cascades on random networks” [14]
Gleeson and Cahalane, Phys. Rev. E, 2007.
- “Direct, physically motivated derivation of the contagion condition for spreading processes on generalized random networks” [10]
Dodds, Harris, and Payne, Phys. Rev. E, 2011
- “Influentials, Networks, and Public Opinion Formation” [28]
Watts and Dodds, J. Cons. Res., 2007.
- “Threshold models of Social Influence” [29]
Watts and Dodds, The Oxford Handbook of Analytical Sociology, 2009.

The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?

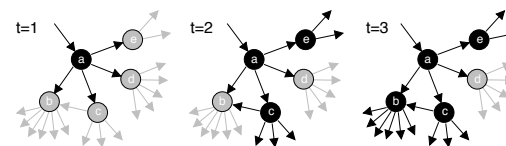
Threshold models

Chaotic behavior possible [17, 16, 9, 18]



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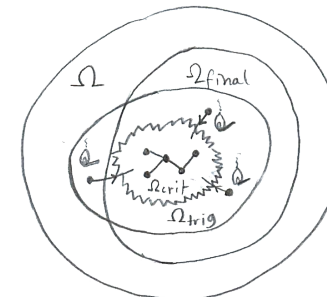
Threshold model on a network



- All nodes have threshold $\phi = 0.2$.

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Example random network structure:



- $\Omega_{crit} = \Omega_{vuln} =$ critical mass = global vulnerable component
- $\Omega_{trig} =$ triggering component
- $\Omega_{final} =$ potential extent of spread
- $\Omega =$ entire network

$$\Omega_{crit} \subset \Omega_{trig}; \Omega_{crit} \subset \Omega_{final}; \text{ and } \Omega_{trig}, \Omega_{final} \subset \Omega.$$

- Period doubling arises as map amplitude r is increased.
- Synchronous update assumption is crucial

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Snowballing

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- We need to understand which nodes can be activated when only one of their neighbors becomes active.

The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active **vulnerables**
- The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a **global cluster of vulnerables** [27]
- Cluster of vulnerables = critical mass**
- Network story: 1 node \rightarrow critical mass \rightarrow everyone.

Cascade condition

Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k .
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

Cascade condition

Next: Vulnerability of linked node

Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) d\phi'_*$$

- If linked node is vulnerable, it produces $k - 1$ new outgoing active links
- If linked node is **not vulnerable**, it produces **no** active links.

Cascade condition

Two special cases:

(1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

(2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

Cascade condition

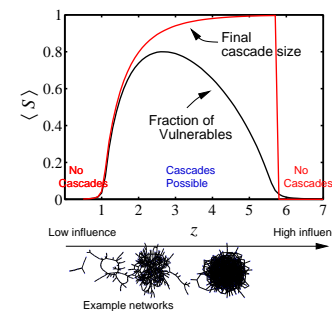
Putting things together:

Expected number of active edges produced by an active edge:

$$R = \left[\sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1-\beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}} \right]$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

Cascades on random networks



- Cascades occur only if size of max vulnerable cluster > 0 .
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

Cascade condition

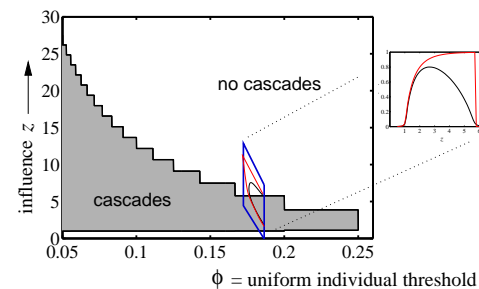
Cascade condition

So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

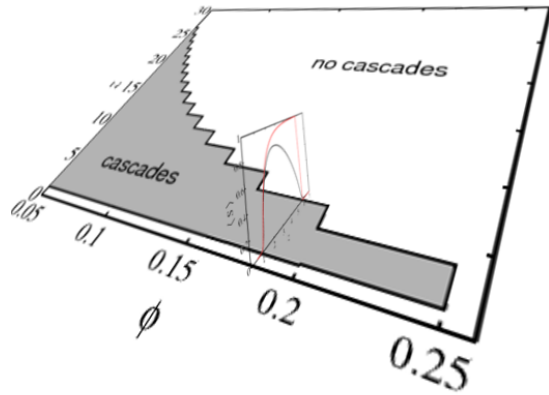
- β_k = probability a degree k node is vulnerable.
- P_k = probability a node has degree k .

Cascade window for random networks



- 'Cascade window' widens as threshold ϕ decreases.
- Lower thresholds enable spreading.

Cascade window for random networks



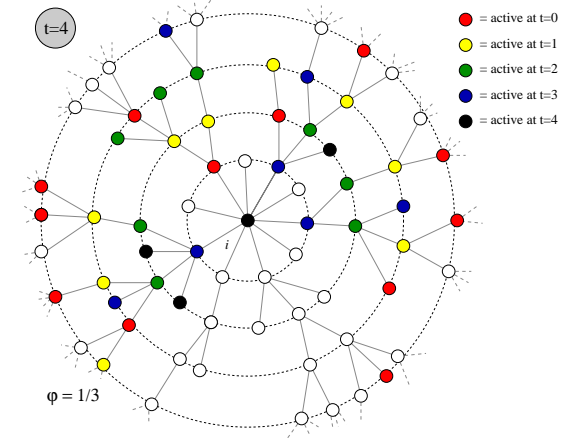
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Threshold contagion on random networks

- Next: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- Problem beautifully solved for infinite seed case by Gleeson and Cahalane: "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007. [14]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [13]

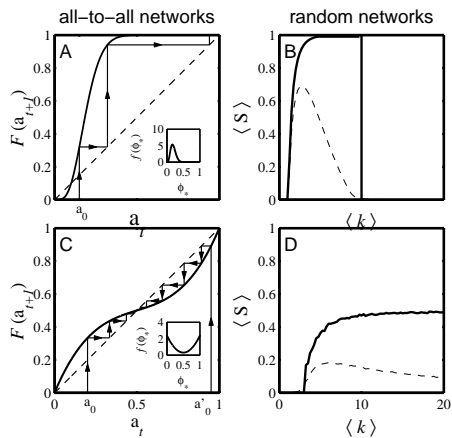
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Expected size of spread



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All-to-all versus random networks



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Determining expected size of spread:

- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node i to become active at time t :
 - $t = 0$: i is one of the seeds (prob = ϕ_0)
 - $t = 1$: i was not a seed but enough of i 's friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = 2$: enough of i 's friends and friends-of-friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i .

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Expected size of spread

Notes:

- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine $\Pr(\text{node of degree } k \text{ switching on at time } t)$.
- Asynchronous updating can be handled too.

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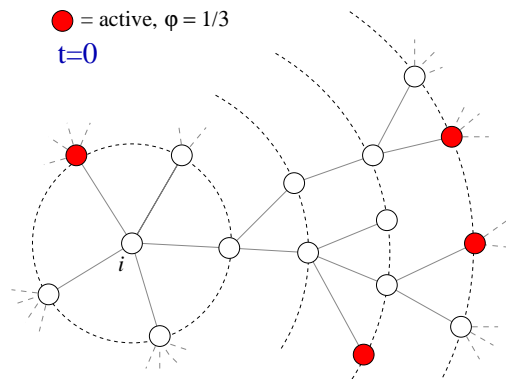
Cascade window—summary

For our simple model of a uniform threshold:

- Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
- High $\langle k \rangle$: Giant component exists but not enough vulnerables.
- Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

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Expected size of spread

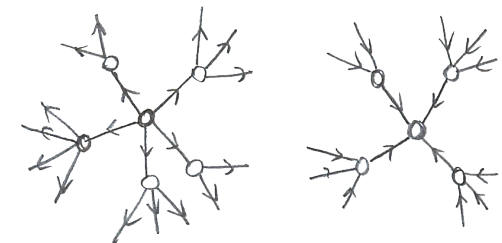


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Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.



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Expected size of spread

- Notation: $\phi_{k,t} = \Pr(\text{a degree } k \text{ node is active at time } t)$.
- Notation: $B_{kj} = \Pr(\text{a degree } k \text{ node becomes active if } j \text{ neighbors are active})$.
- Our starting point: $\phi_{k,0} = \phi_0$.
- $\binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} = \Pr(j \text{ of a degree } k \text{ node's neighbors were seeded at time } t = 0)$.
- Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).
- Probability a degree k node was not a seed at $t = 0$ is $(1 - \phi_0)$.
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k \binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}$$

Expected size of spread

Two pieces: edges first, and then nodes

$$1. \theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

with $\theta_0 = \phi_0$.

$$2. \phi_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}}_{\text{social effects}}$$

- For general t , we need to know the probability an edge coming into a degree k node at time t is active.

Notation: call this probability θ_t .

We already know $\theta_0 = \phi_0$.

Story analogous to $t = 1$ case. For node i :

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i-j} B_{k_i,j}$$

Average over all nodes to obtain expression for ϕ_{t+1} :

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}$$

So we need to compute θ_t ... massive excitement...

Expected size of spread

Iterative map for θ_t is key:

$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}} = G(\theta_t; \phi_0)$$

Expected size of spread:

In words:

- If $G(0; \phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If G has an **unstable fixed point** at $\theta = 0$, then cascades are also always possible.

Non-vanishing seed case:

- Cascade condition is more complicated for $\phi_0 > 0$.
- If G has a **stable fixed point** at $\theta = 0$, and an **unstable fixed point** for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- Tricky point: G depends on ϕ_0 , so as we change ϕ_0 , we also change G .
- A version of a critical mass model again.

Expected size of spread

First connect θ_0 to θ_1 :

$\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^j (1 - \theta_0)^{k-1-j} B_{kj}$$

- $\frac{kP_k}{\langle k \rangle} = R_k = \Pr(\text{edge connects to a degree } k \text{ node})$.
- $\sum_{j=0}^{k-1}$ piece gives $\Pr(\text{degree node } k \text{ activates } j \text{ of its neighbors } k-1 \text{ incoming neighbors are active})$.
- ϕ_0 and $(1 - \phi_0)$ terms account for state of node at time $t = 0$.
- See this all generalizes to give θ_{t+1} in terms of θ_t ...

Expected size of spread:

- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \rightarrow 0$.
- Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

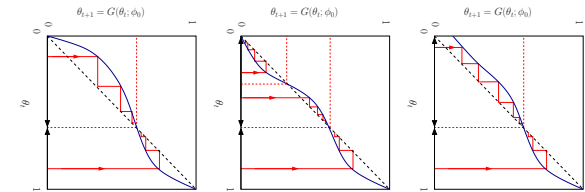
$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.

- If $\theta = 0$ is a fixed point of G (i.e., $G(0; \phi_0) = 0$) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

General fixed point story:

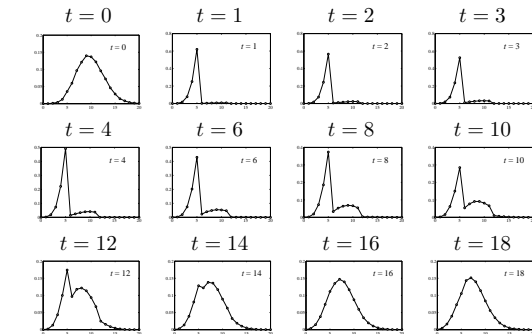


- Given $\theta_0 (= \phi_0)$, θ_{∞} will be the nearest stable fixed point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.
- Important:** Actual form of G depends on ϕ_0 .
- So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G .

Expected size of spread

Expected size of spread:

Early adopters—degree distributions



$P_{k,t}$ versus k

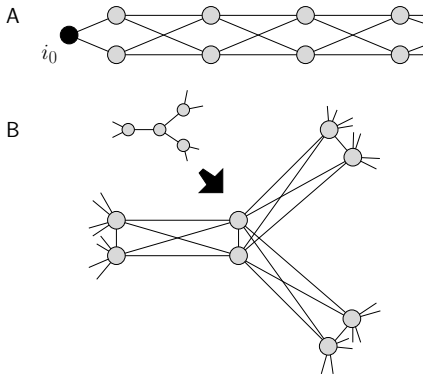


“Influentials, Networks, and Public Opinion Formation”
 Watts and Dodds,
 J. Consum. Res., 34, 441–458, 2007. [28]

- Exploration of threshold model of social contagion on various networks.
- “Influentials” are limited in power.
- Connected groups of weakly influential-vulnerable” individuals are key.
- Average individuals can have more power than well connected ones.

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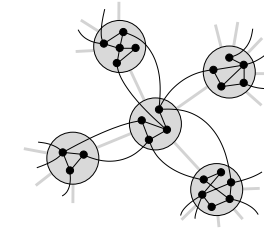
Special subnetworks can act as triggers



$\phi = 1/3$ for all nodes

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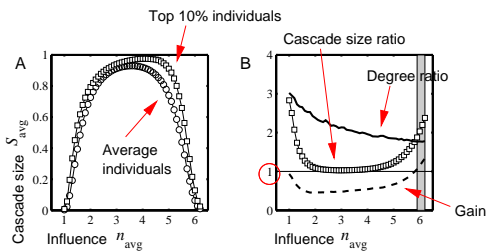
Group structure—Ramified random networks



p = intergroup connection probability
 q = intragroup connection probability.

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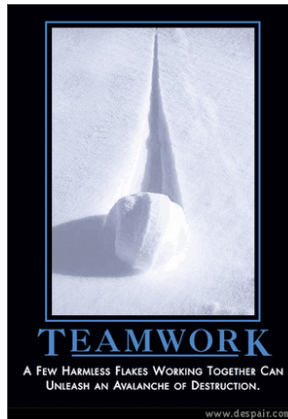
The multiplier effect:



- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

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The power of groups...

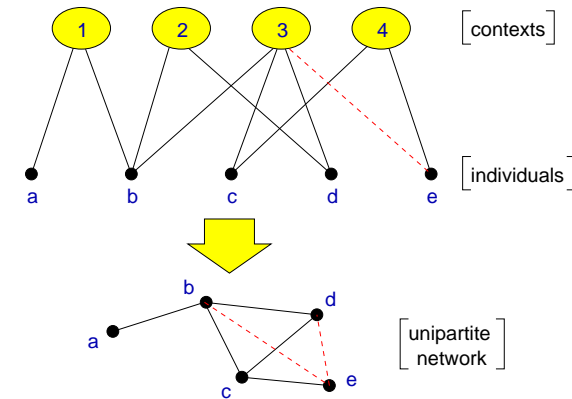


“A few harmless flakes working together can unleash an avalanche of destruction.”

despair.com

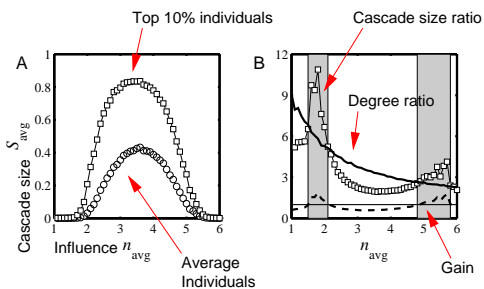
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Bipartite networks



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The multiplier effect:



- Skewed influence distribution example.

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Extensions

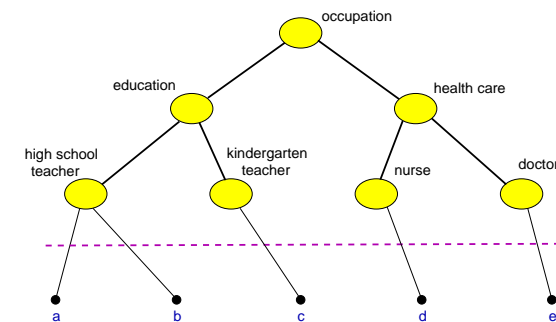


“Threshold Models of Social Influence”
 Watts and Dodds,
 The Oxford Handbook of Analytical Sociology, 34, 475–497, 2009. [29]

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network’s function
- Still, random networks don’t represent all networks
- Major element missing: **group structure**

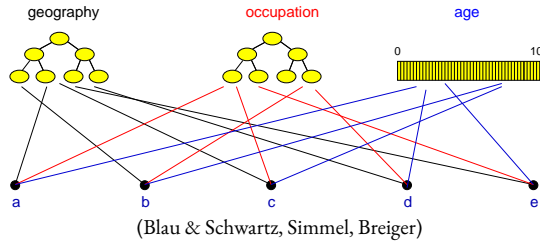
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Context distance



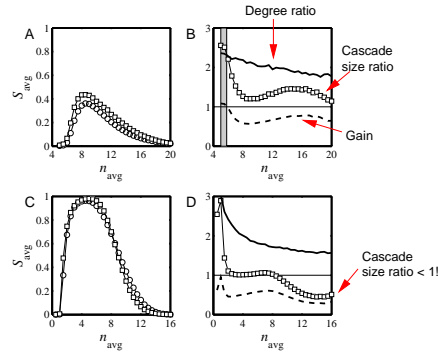
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Generalized affiliation model



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Multiplier effect for group-based networks:



Multiplier almost always below 1.

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Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (fashion), or active = harder to achieve (political messages; even so: buttons and hats).
- Entities can be novel or designed to combine with others, e.g. block another one.

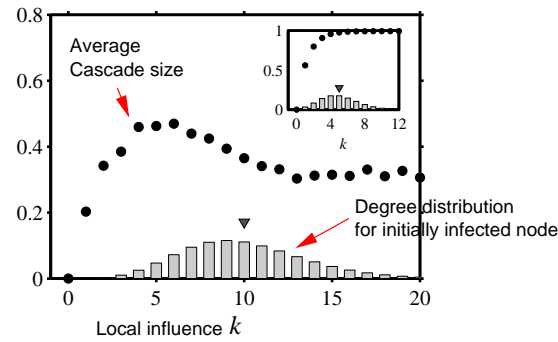
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Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto e^{-\alpha d}$ where α = homophily parameter and d = distance between nodes (height of lowest common ancestor)
- τ_1 = intergroup probability of friend-of-friend connection
- τ_2 = intragroup probability of friend-of-friend connection

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Assortativity in group-based networks



- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

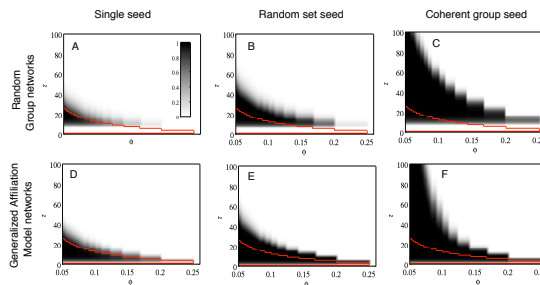
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Cascade windows for group-based networks



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Social contagion

"Without followers, evil cannot spread." -Leonard Nimoy

Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

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