

Overview of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

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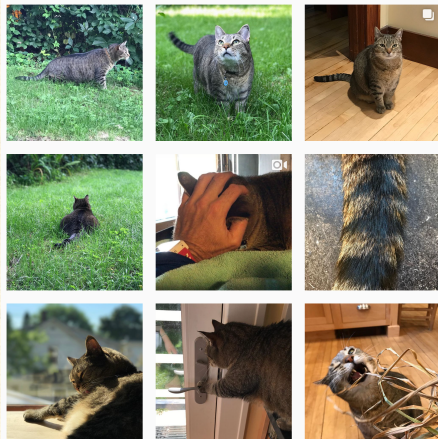
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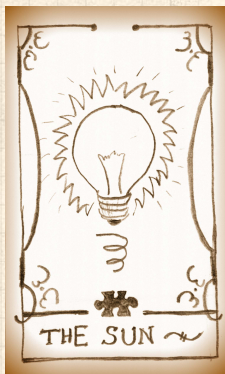
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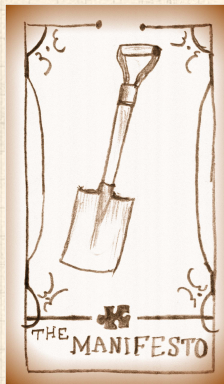
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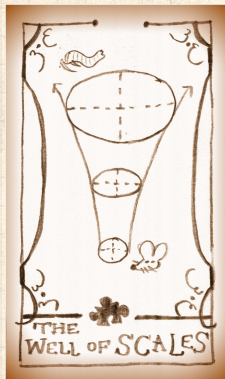
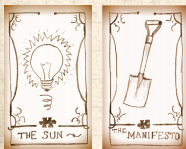
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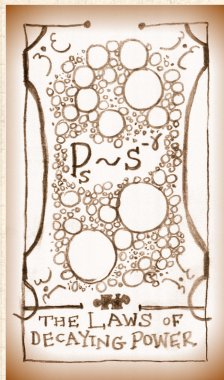
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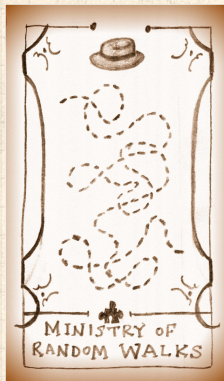
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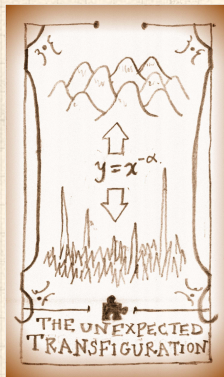
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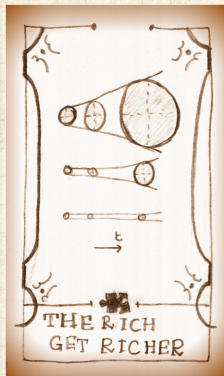
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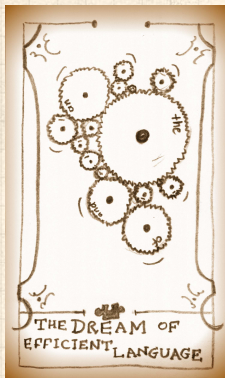
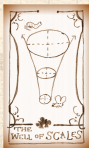
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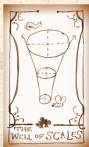
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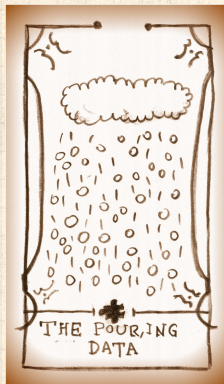
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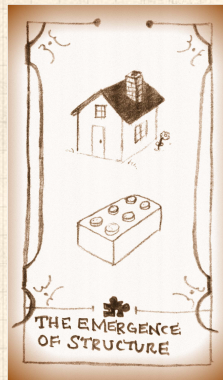
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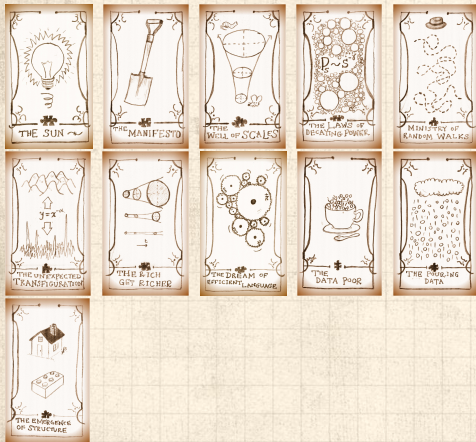
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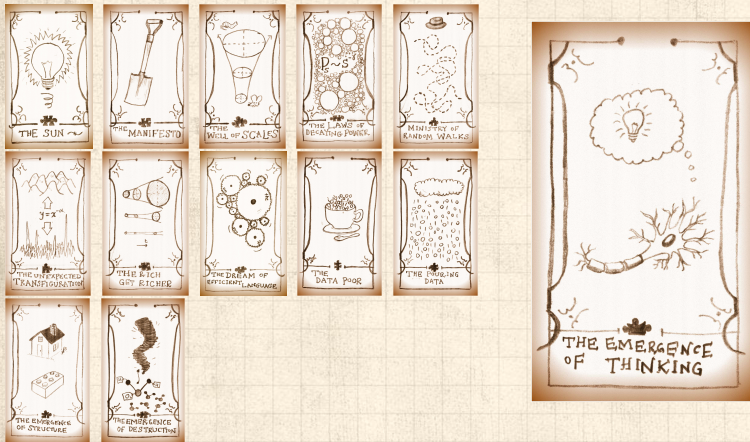
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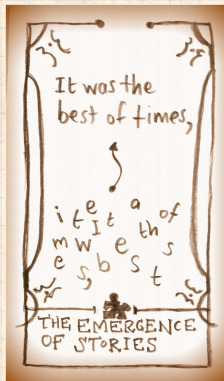
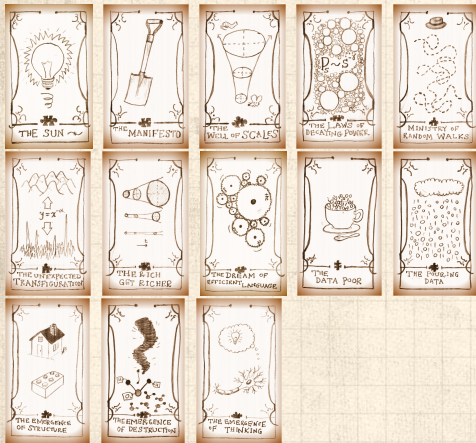
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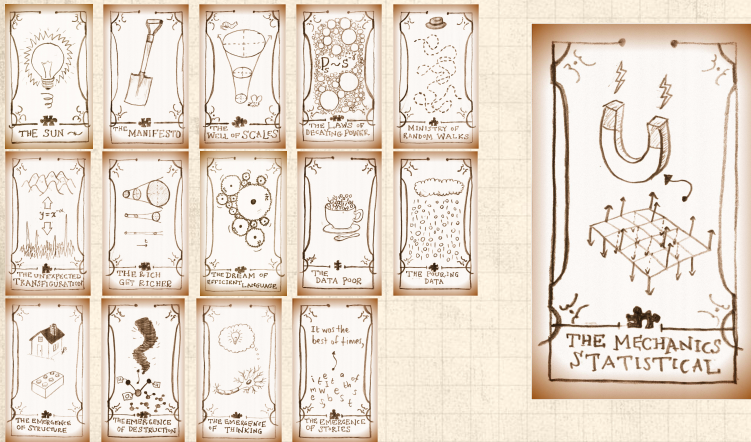
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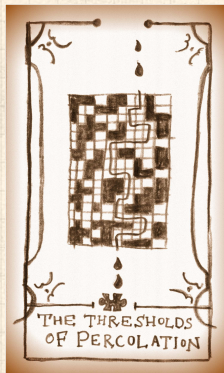
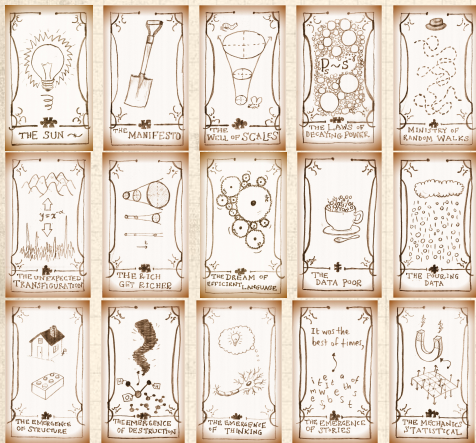
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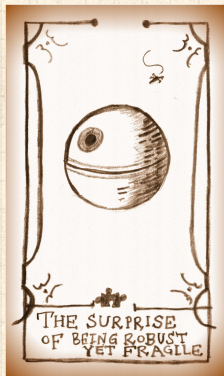
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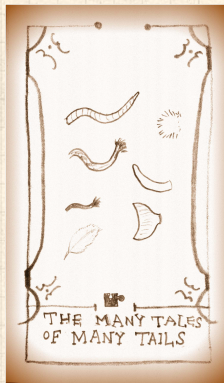
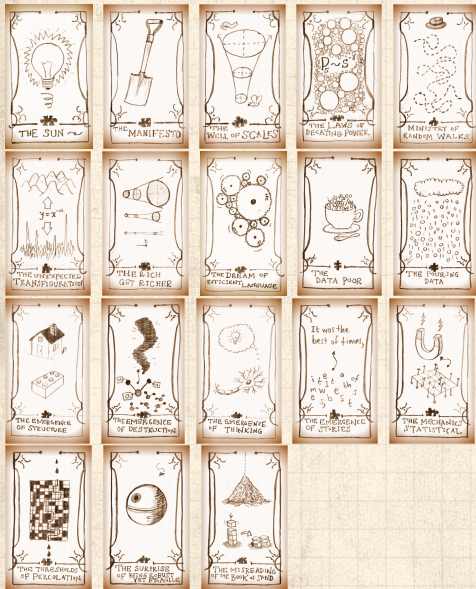
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net•work |'net,wɜrk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*


Thesaurus deliciousness:

network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's etymological investigation: 



Opus reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

Ancestry:

First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

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-  1883–: distribution network of electrical cables

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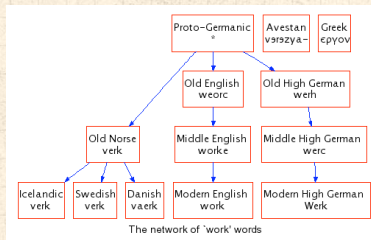
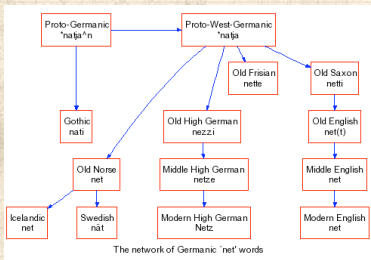
-  1658–: reticulate structures in animals
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-  1869–: railways
-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

Ancestry:

Net and Work are venerable old words:

 'Net' first used to mean spider web (King Ælfréd, 888).

 'Work' appear to have long meant purposeful action.



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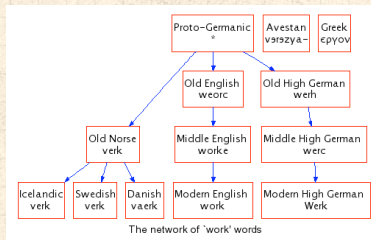
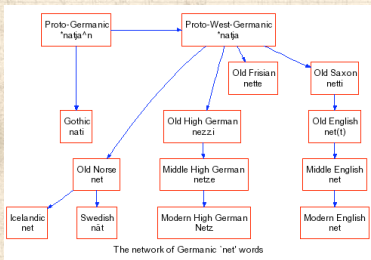
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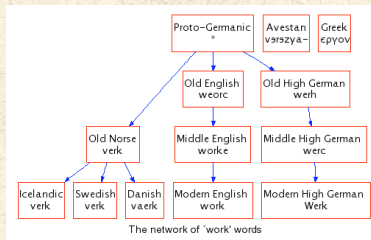
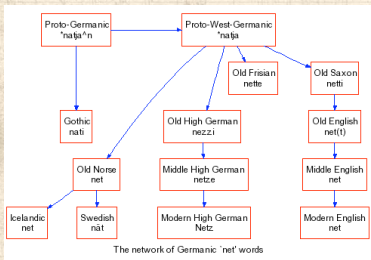
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 c.f., ironwork, stonework, fretwork.

Key Observation:



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-  *Piranha physicus*
-  Hunt in packs.
-  Feast on new and interesting ideas (see chaos, cellular automata, ...)

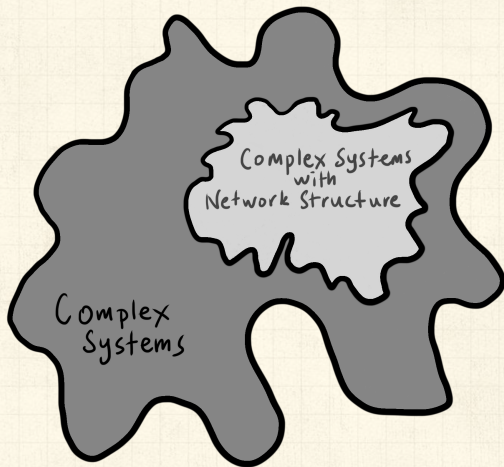
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-  *Piranha physicus*
-  Hunt in packs.
-  Feast on new and interesting ideas (see chaos, cellular automata, ...)
-  See also: <https://xkcd.com/793/> 

Complex Systems is the Big Story:



Only a bit networky: Fluids-at-large (the atmosphere, oceans, ...), organism cells, ...

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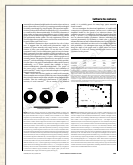
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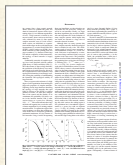



Popularity (according to Google Scholar)



“Collective dynamics of ‘small-world’ networks” 
Watts and Strogatz,
Nature, **393**, 440–442, 1998. ^[20]

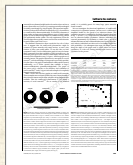
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“Emergence of scaling in random networks” 
Barabási and Albert,
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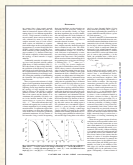
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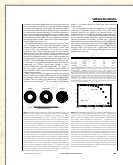
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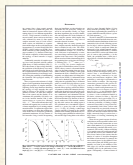
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“The structure and function of complex networks” 

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Popularity according to textbooks:

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Popularity according to textbooks:

Textbooks:




Mark Newman (Physics, Michigan)

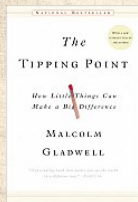
“Networks: An Introduction” 



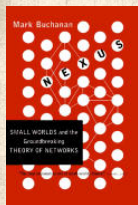
David Easley and Jon Kleinberg (Economics and Computer
Science, Cornell)

“Networks, Crowds, and Markets: Reasoning About a Highly
Connected World” 

Popularity according to popular books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [9]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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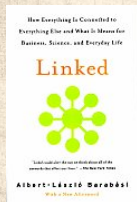
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Popularity according to popular books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts ^[19]

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Numerous others ...

-  Complex Social Networks—F. Vega-Redondo ^[18]
-  Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo ^[16]
-  Random Graph Dynamics—R. Durrett
-  Scale-Free Networks—Guido Caldarelli
-  Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
-  Complex Graphs and Networks—Fan Chung
-  Social Network Analysis—Stanley Wasserman and Kathleen Faust
-  Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
-  Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes ^[8]

More observations



But surely **networks aren't new** ...

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But surely **networks aren't new** ...



Graph theory was well established ...

More observations



But surely **networks aren't new** ...



Graph theory was well established ...



Study of social networks started in the 1930's ...

More observations



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So why all this 'new' research on networks?








More observations

-  But surely **networks aren't new** ...
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-  So why all this 'new' research on networks?
-  **Answer:** Oodles of Easily Accessible Data.









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-  We can now inform (alas) our theories with a much more measurable reality.*









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-  **Answer:** Oodles of Easily Accessible Data.
-  We can now inform (alas) our theories with a much more measurable reality.*
-  Graph theory missed “becoming”: Stories = Characters + Time

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-  **Answer:** Oodles of Easily Accessible Data.
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-  Graph theory missed "becoming": Stories = Characters + Time
-  A worthy goal: establish **mechanistic explanations**.

More observations

-  But surely **networks aren't new** ...
-  Graph theory was well established ...
-  Study of social networks started in the 1930's ...
-  So why all this 'new' research on networks?
-  **Answer:** Oodles of Easily Accessible Data.
-  We can now inform (alas) our theories with a much more measurable reality.*
-  Graph theory missed “becoming”: Stories = Characters + Time
-  A worthy goal: establish **mechanistic explanations**.

**If this is upsetting, maybe string theory is for you ...*

More observations



Internet-scale data sets can be overly **exciting**.

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Witness:

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 For scientists, description is only part of the battle.


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But:

 For scientists, description is only part of the battle.

 We still need to **understand**.

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

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 e.g., people, forks in rivers, proteins, webpages, organisms, ...

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Other spiffing words: vertices and edges.

Super Basic definitions

Node degree = Number of links per node

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 Notation: Node i 's degree = k_i .

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 Defn: N_i = the set of i 's k_i neighbors


Super Basic definitions


Adjacency matrix:

 We can represent a network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

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$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$


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 For numerical work, we always use sparse matrices.

 For many real networks, A is a function of time.

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So what passes for a complex network?

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So what passes for a complex network?

 Complex networks are **large** (in node number)





So what passes for a complex network?

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So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**
-  Complex networks can be social, economic, natural, informational, abstract, ...

Examples


Physical networks


River networks



Examples

Physical networks




 River networks

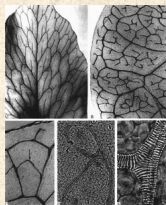
 Neural networks



Examples





Physical networks

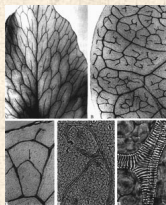
-  River networks
-  Neural networks
-  Trees and leaves



Examples





Physical networks


-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

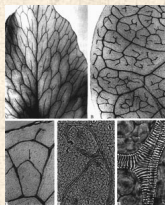


Examples

Physical networks





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-  Neural networks
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-  Blood networks


 The Internet (pipes)




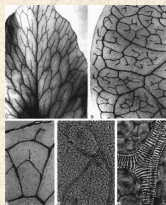
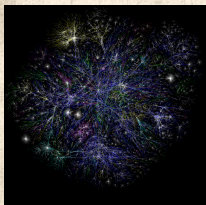
Examples

Physical networks

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-  Neural networks
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-  Blood networks





 The Internet (pipes)




 Road networks

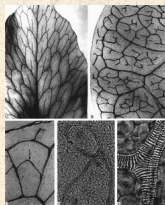


Examples

Physical networks





-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks




-  The Internet (pipes)
-  Road networks
-  Power grids

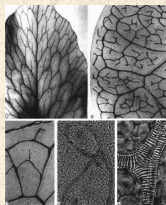



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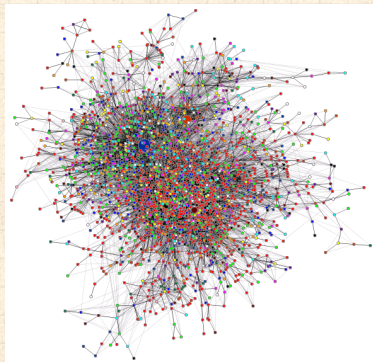



 **Distribution** (branching) versus **redistribution** (cyclical)

Examples

Interaction networks

The Blogosphere (RIP)



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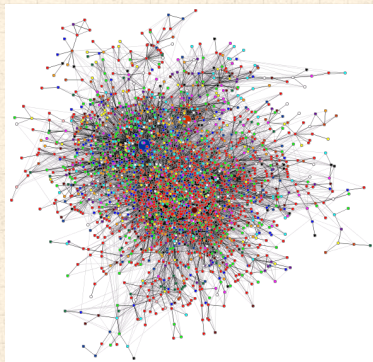
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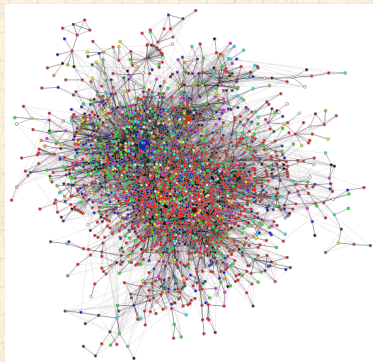
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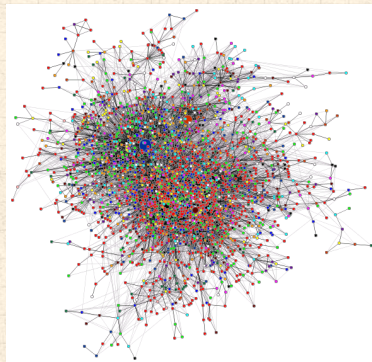
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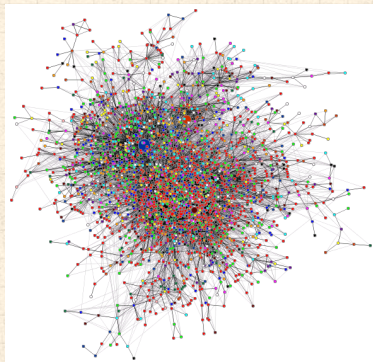
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
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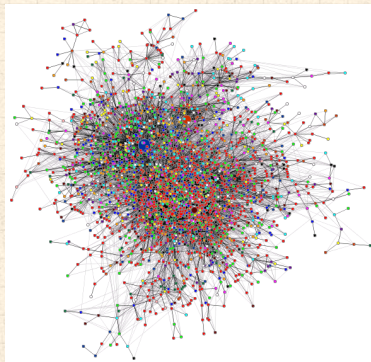



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Examples

Interaction networks








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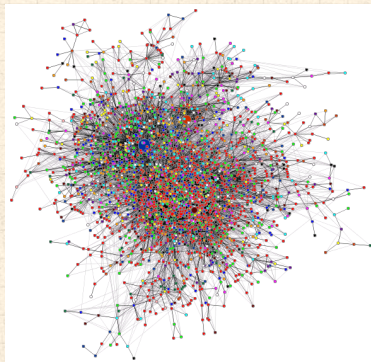



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Examples

Interaction networks









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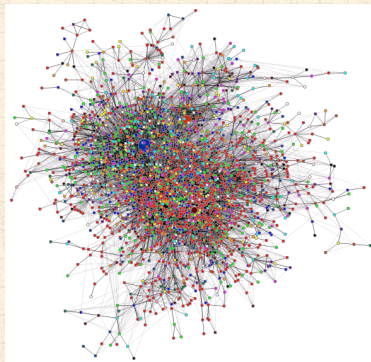



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Examples

Interaction networks

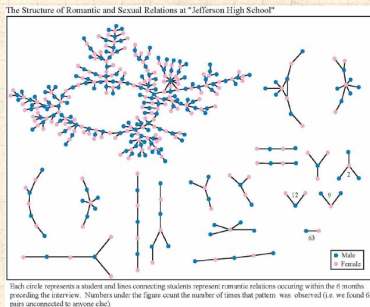
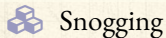
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-  The internet (World Wide Web)



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Examples

Interaction networks: social networks



(Bearman *et al.*, 2004)

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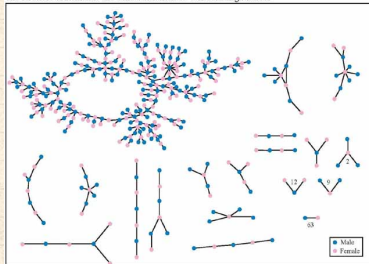
Examples

Interaction networks: social networks

 Snogging

 Friendships

The Structure of Romantic and Sexual Relations at "Jefferson High School"






Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

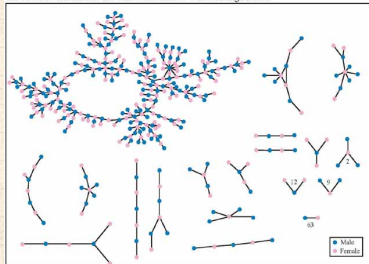
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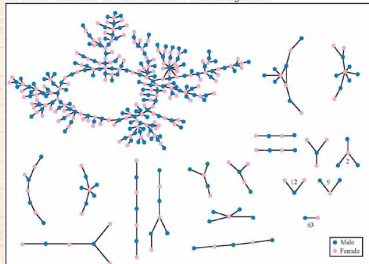
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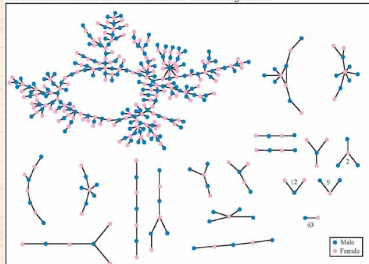
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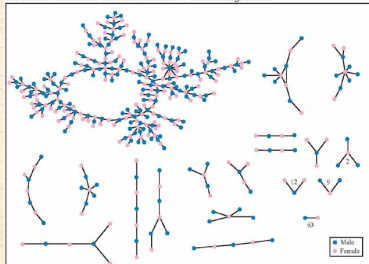
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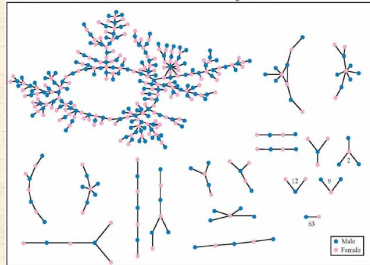
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‘Remotely sensed’ by: email activity, instant messaging, phone logs

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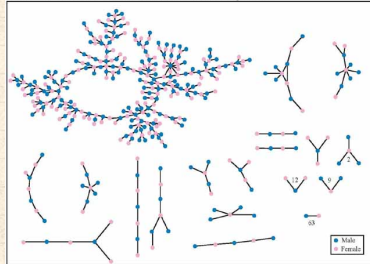
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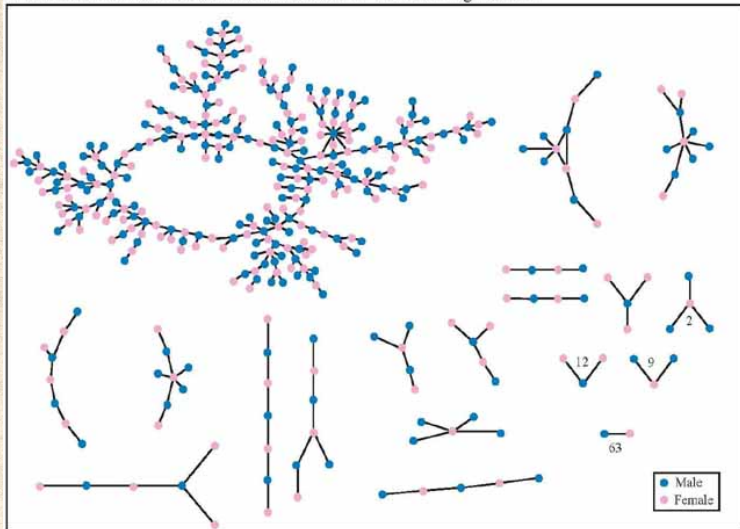


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Examples

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Relational networks



Consumer purchases

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Consumer purchases
(Walmart, Target, Amazon, ...)

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-  Thesauri: Networks of words generated by meanings

Examples

Relational networks

-  Consumer purchases
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-  Thesauri: Networks of words generated by meanings
-  Knowledge/Databases/Ideas








Relational networks

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-  Metadata—Tagging, Keywords bit.ly  [flickr](https://www.flickr.com/photos/complex-networks/) 

common tags cloud | [list](#)

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

Relational networks

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-  Large Language Models

common tags cloud | [list](#)

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

Neural reboot (NR):

Dog has fun.

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<https://www.you>



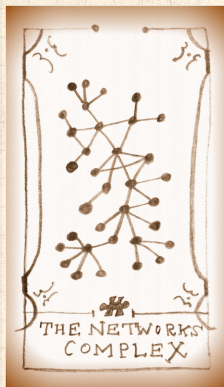
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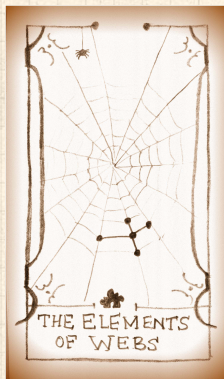
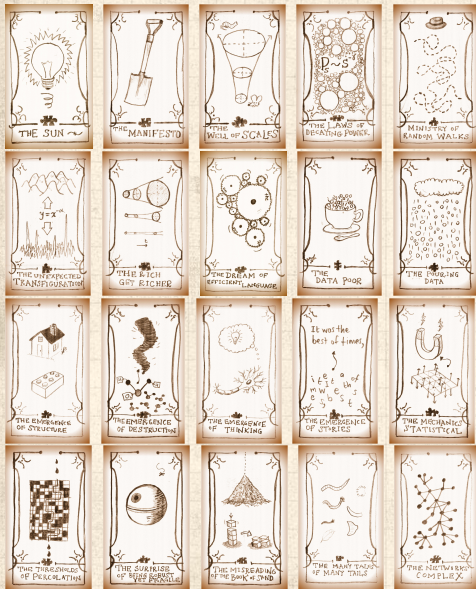
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A notable feature of large-scale networks:

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A notable feature of large-scale networks:

 Graphical renderings are often just a big mess.

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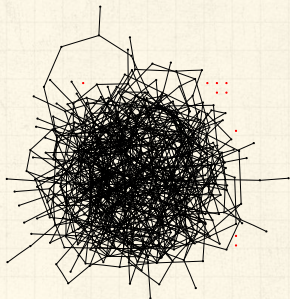
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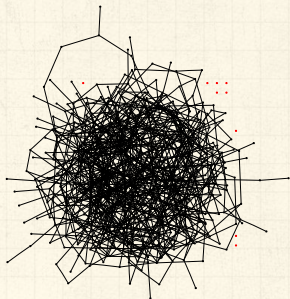


⇐ Typical hairball




-  number of nodes $N = 500$
-  number of edges $m = 1000$
-  average degree $\langle k \rangle = 4$


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 Graphical renderings are often just a big mess.




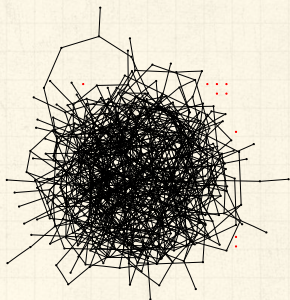
⇐ Typical hairball

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 And even when renderings somehow look good:


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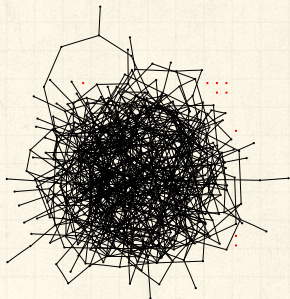
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-  average degree $\langle k \rangle = 4$

 And even when renderings somehow look good:
“That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way”




said Ponder [Stibbons] —*Making Money*, T. Pratchett.


A notable feature of large-scale networks:

 Graphical renderings are often just a big mess.



⇐ Typical hairball


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 And even when renderings somehow look good:
“That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way”


said Ponder [Stibbons] — *Making Money*, T. Pratchett.


 We need to extract **digestible, meaningful aspects.**


Some key aspects of real complex networks:


 degree distribution*


 assortativity


 homophily


 clustering


 motifs


 modularity


 concurrency


 hierarchical scaling


 network distances

 centrality

 efficiency

 interconnectedness

 robustness

 Plus coevolution of network structure
and processes on networks.

* Degree distribution is the elephant in the room that we are
now all very aware of ...

Properties

1. degree distribution P_k

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 **ex 1:** Pure (Erdős-Rényi) random networks have Poisson degree distributions:


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 link cost controls skew.

 hubs may facilitate or impede contagion.

Properties

Note:



Pure (Erdős-Rényi) random networks are a *mathematical construct*.

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Note:


 Pure (Erdős-Rényi) random networks are a *mathematical construct*.


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 “Becoming”: Stories = Characters + Time

2. Assortativity/3. Homophily:





 Social networks: Homophily  = birds of a feather

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Often social: company directors, coauthors, actors.

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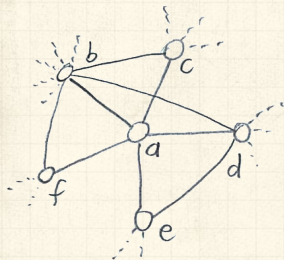
Often social: company directors, coauthors, actors.

 **Disassortative** network: high degree nodes connecting to low
degree nodes.

*Often technological or biological: Internet, WWW, protein
interactions, neural networks, food webs.*

Local socialness:

4. Clustering:

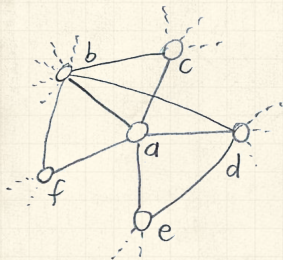


Local socialness:

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Your friends tend to know each other.



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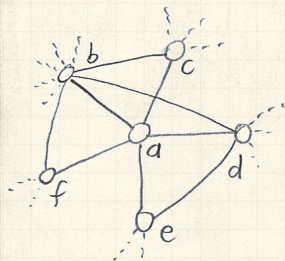
Two measures (explained on following slides):

1. Watts & Strogatz [20]

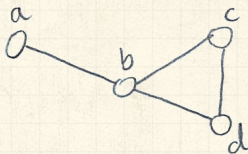
$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

2. Newman [15]

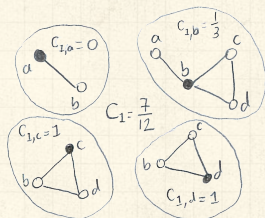
$$C_2 = \frac{3 \times \#triangles}{\#triples}$$



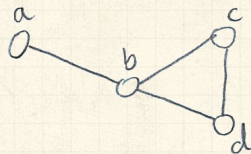
Example network:




Calculation of C_1 :

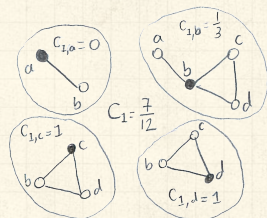


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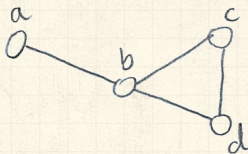



 C_1 is the **average fraction of pairs of neighbors who are connected.**


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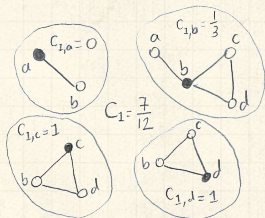


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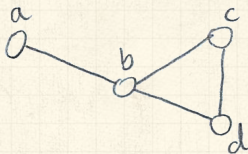
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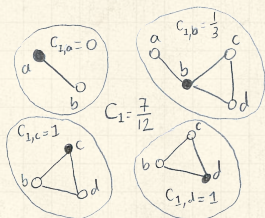


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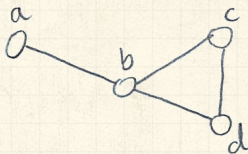



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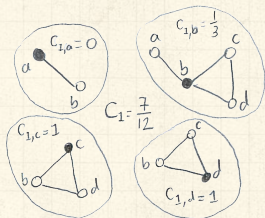


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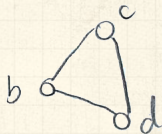
Triples and triangles

Example network:

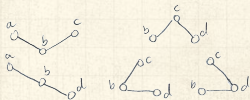


Nodes i_1 , i_2 , and i_3 form a **triple** around i_1 if i_1 is connected to i_2 and i_3 .

Triangles:



Triples:



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Triples and triangles

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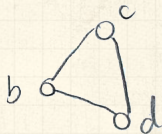


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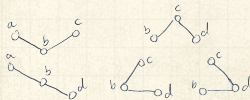


Nodes i_1 , i_2 , and i_3 form a **triangle** if each pair of nodes is connected

Triangles:



Triples:



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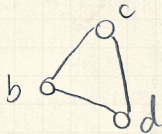


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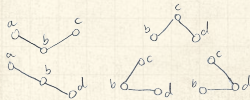


The definition $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ measures the fraction of **closed triples**

Triangles:



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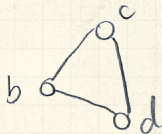
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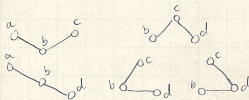
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The **'3'** appears because for each triangle, we have 3 closed triples.

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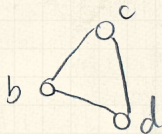
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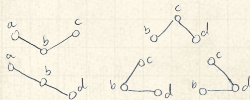
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Social Network Analysis (SNA):
fraction of **transitive triples**.

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Clustering:

Sneaky counting for undirected, unweighted networks:

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 If the path $i-j-\ell$ exists then $a_{ij}a_{j\ell} = 1$.

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For sparse networks, C_1 tends to discount highly connected nodes.


Properties


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 C_1 is a global average of a local ratio.

 C_2 is a ratio of two global quantities.

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5. motifs:

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5. motifs:

 small, recurring functional subnetworks

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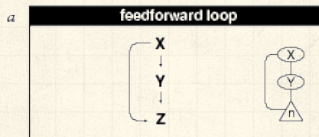
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5. motifs:

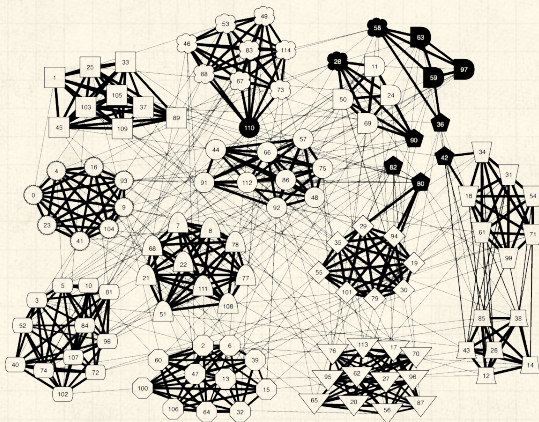
 small, recurring functional subnetworks

 e.g., Feed Forward Loop:



Shen-Orr, Uri Alon, *et al.* [17]

6. modularity and structure/community detection:




Clauset *et al.*, 2006^[7]: NCAA football


7. concurrency:




transmission of a contagious element only occurs during contact


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-  beware cumulated network data
-  Kretzschmar and Morris, 1996 ^[13]
-  “Temporal networks” become a concrete area of study for Piranha Physicus in 2013.

8. Horton-Strahler ratios:

 Metrics for branching networks:



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Metrics for branching networks:



Method for ordering streams hierarchically



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Number: $R_n = N_\omega / N_{\omega+1}$



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Metrics for branching networks:

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- Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_\omega \rangle$



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9. network distances:

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9. network distances:

(a) shortest path length d_{ij} :

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9. network distances:

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Fewest number of steps between nodes i and j .

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(b) average path length $\langle d_{ij} \rangle$:





Average shortest path length in whole network.

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(a) shortest path length d_{ij} :

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-  (Also called the chemical distance between i and j .)

(b) average path length $\langle d_{ij} \rangle$:

-  Average shortest path length in whole network.
-  Good algorithms exist for calculation.
-  Weighted links can be accommodated.

9. network distances:



network diameter d_{\max} :

Maximum shortest path length between any two nodes.

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closeness $d_{cl} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:

Average 'distance' between any two nodes.

9. network distances:



network diameter d_{\max} :

Maximum shortest path length between any two nodes.



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9. network distances:



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$d_{\text{cl}} = \infty$ only when all nodes are isolated.



Closeness perhaps compresses too much into one number

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10. centrality:

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ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg^[12])

Interconnected networks and robustness (two for one deal):

“Catastrophic cascade of failures in interdependent networks” [6].

Buldyrev et al., Nature 2010.

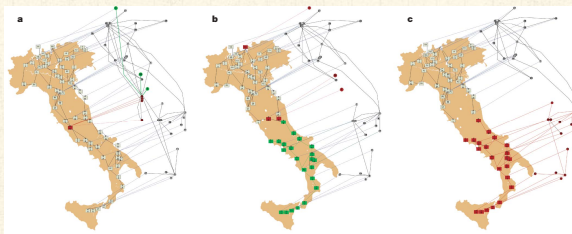




Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003³⁶. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a**, One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network)


at the next step are marked in green. **b**, Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c**, Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).


Overview Key Points:


 The field of complex networks came into existence in the late 1990s.

 To solve network problems: “Follow the edges.”




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-  Three main (blurred) categories:
 1. **Physical** (e.g., river networks),
 2. **Interactional** (e.g., social networks),
 3. **Abstract** (e.g., thesauri).
-  To solve network problems: “Follow the edges.”

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