Mechanisms for Generating Power-Law Size Distributions, Part 3

Last updated: 2025/10/08, 21:11:29 EDT

Principles of Complex Systems, Vols. 1, 2, 3D, 4 Fourever, V for Vendetta

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Institute
University of Vermont | Santa Fe Institute

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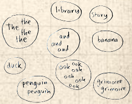
Principles of Complex Systems
@pocsvox
What's the Story?



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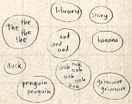

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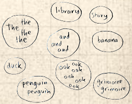

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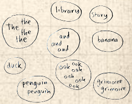
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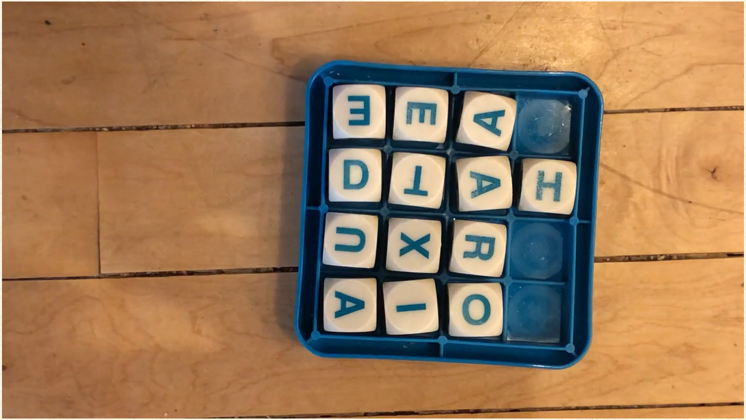
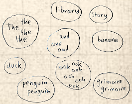
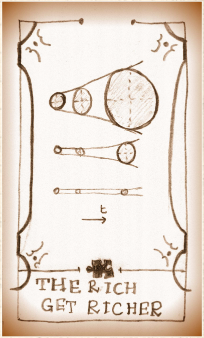





THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TAILS OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE BOOKS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS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THE RICH GET RICHER

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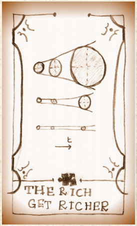
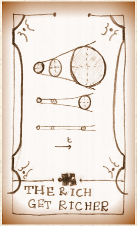


Aggregation:

- Random walks represent additive aggregation



Aggregation:

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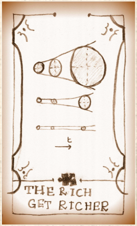
- Mechanism: Random addition and subtraction



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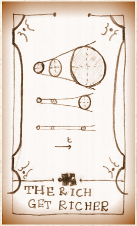

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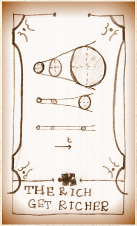

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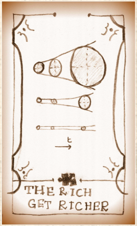

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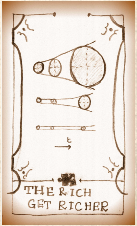
- Competing mechanisms (trickiness)



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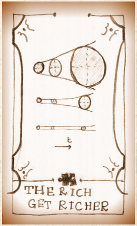

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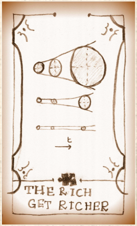

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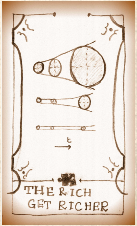

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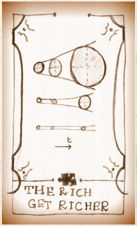

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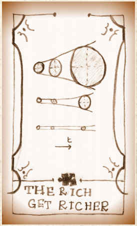
- 1926: Lotka [7]:
  # Scientific papers per author (Lotka's law)



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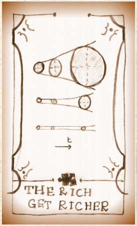

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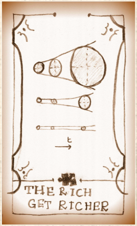

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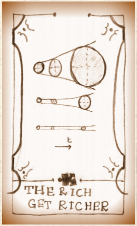

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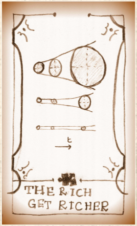

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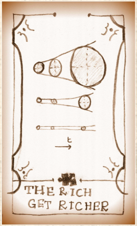
- 1999: Barabasi and Albert [2]:
  The World Wide Web, networks-at-large.



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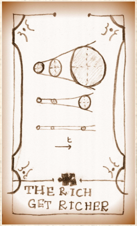

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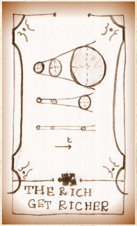

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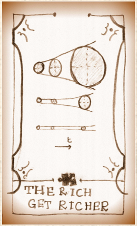

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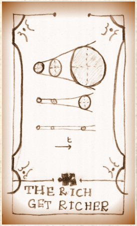

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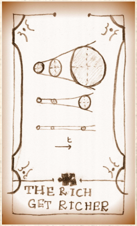

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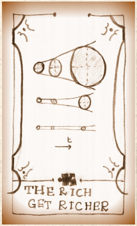
- 1978 Nobel Laureate in Economics (his Nobel bio is here).



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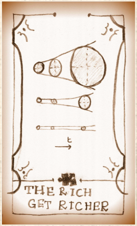
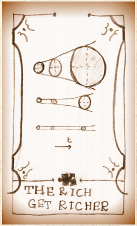
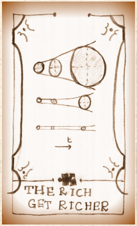


Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 Moo Deng (or element) of a particular flavor at $t = 1$





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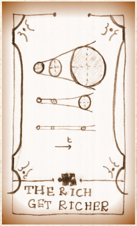

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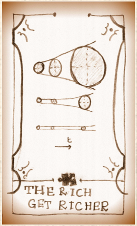

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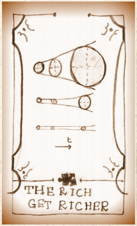

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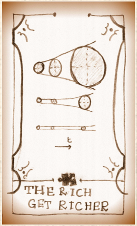
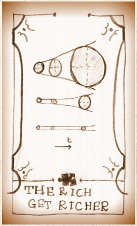


Random Competitive Replication—Simon's rich-get-richer model: [10]

Example: Words appearing in a language



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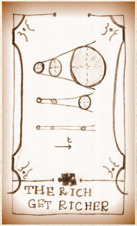

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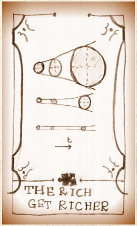

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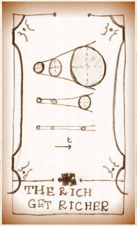

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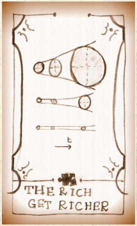

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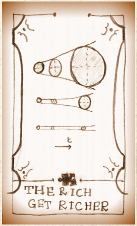
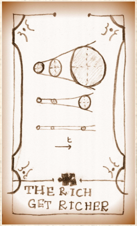


Random Competitive Replication—Simon's rich-get-richer model: [10]



Example: Words appearing in a language

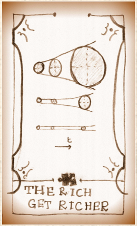
- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
  = Mutation/Innovation
- With probability $1 - \rho$, randomly choose one word from all words that have come before, and reuse this word
  = Replication/Imitation

Note: This is a terrible way to write a novel.

For example:

- 21 words used
- next word is new with prob ρ
- next word is a copy with prob $1-\rho$

prob:	next word:
6/21	ook
4/21	the
3/21	and
2/21	penguin
⋮	
1/21	library



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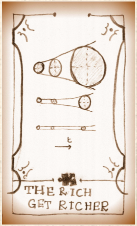

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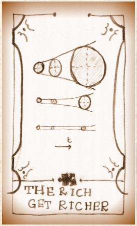

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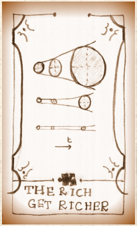

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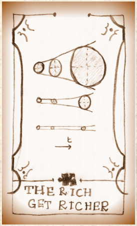

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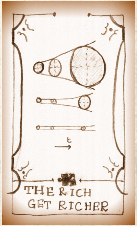

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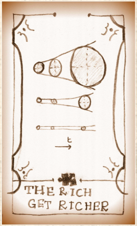

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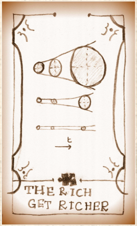
Your free set of tofu knives:



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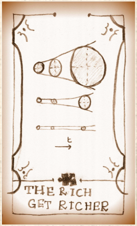
- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.



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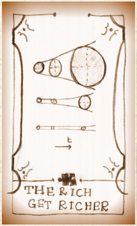
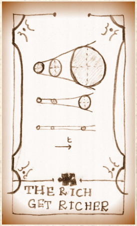
- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.





Language Log Home About Comments policy

The long tail of religious studies?

August 5, 2010 @ 10:33 am · Filed by Mark Liberman under Computational linguistics

« previous post | next post »

Google Books isn't the only outfit that sometimes has trouble with metadata. I happened to notice this morning that Oxford University Press has classified Herbert A. Simon's "On a class of skew distribution functions" (*Biometrika* 43:425-440, 1955) as "Religious Studies..Death":

BIOMETRIKA

ABOUT THIS JOURNAL CONTACT THIS JOURNAL SUBSCRIPTIONS

Oxford Journals > Mathematics & Physical Sciences > Biometrika > Volume 42, Numb

Biometrika 1955 42(3-4):425-440; doi:10.1093/biomet/42.3-4.425
© 1955 by Biometrika Trust

ON A CLASS OF SKEW DISTRIBUTION FUNCTIONS

HERBERT A. SIMON

You have reached the most complete version of this article accessible without further authentication.
More complete versions are available.
Link to article

Article topics:

- Religious Studies..Death

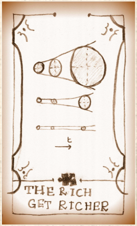
Carl Burke said,
August 5, 2010 @ 12:14 pm

If I had to guess at features that suggest 'Religious Studies — Death', I'd have to go with the word 'urn' and the suffix 'xion', almost never seen except on 'crucifixion'. Granted that Biometrika is published by Oxford University Press, and 'connexion' is a perfectly good British word, the classification algorithm might be more familiar with the American form 'connection'.

[(myl) Looking a bit further into the paper, one finds things like

> it is well known that the negative binomial and the log series distributions can be obtained as the stationary solutions of certain stochastic processes. For example, J.H. Darwin (1953) derives these from birth and death processes, with appropriate assumptions as to the birth- and death-rates and the initial conditions.

]

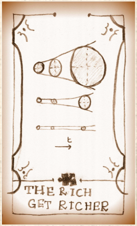


Language Log blog post, 2010/08/05

Random Competitive Replication:

Some observations:

- Steady growth of system: +1 elephant per unit time.



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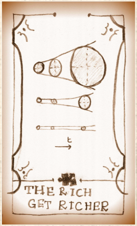
- Steady growth of system: +1 elephant per unit time.
- Steady growth of distinct flavors at rate ρ



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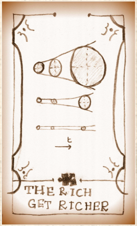

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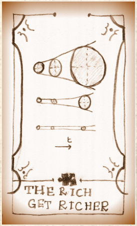

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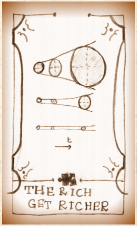

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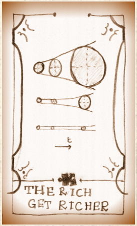

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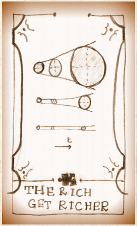

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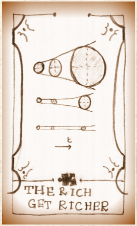
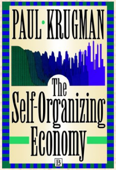
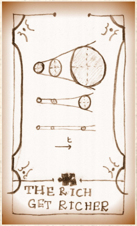
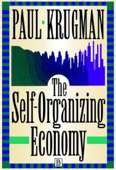
  4. Different selection based on group size
     (But mechanism for selection is not as simple...)





"The Self-Organizing Economy" by Paul Krugman (1996). [6]





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Ch. 3: An Urban Mystery, p. 46

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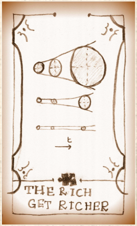

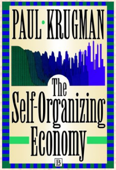

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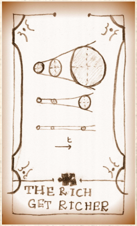
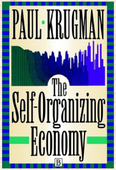


[1] Krugman's book was handed to the Deliverator by a certain Álvaro Cartea many years ago at the Santa Fe Institute Summer School.



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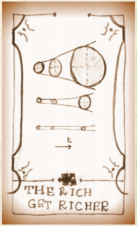
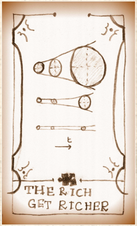
[2] Let's use π for probability because π's not special, right guys?



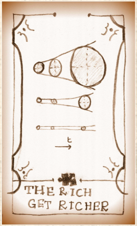
Outline



Random Competitive Replication:

Definitions:

- k_i = size of a group i



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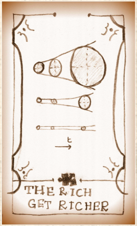

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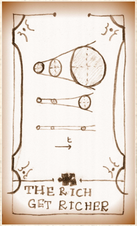

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Basic question: How does $N_{k,t}$ evolve with time?

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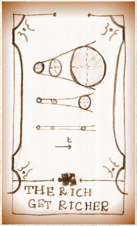
Basic question: How does $N_{k,t}$ evolve with time?

First: $$\sum_k kN_{k,t} = t = \text{number of elephants at time } t$$



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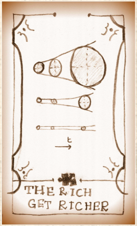
$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:



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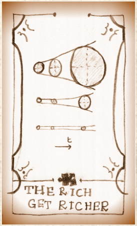

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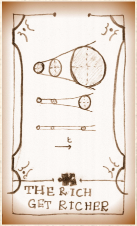

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$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

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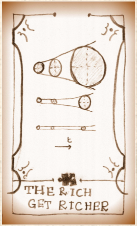
- $\Rightarrow kN_{k,t}$ elephants in size k groups
- t elephants overall



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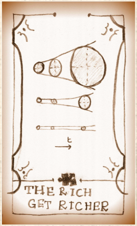
$$P_k(t) = \frac{kN_{k,t}}{t}.$$



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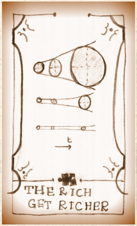
$N_{k,t}$, the number of groups with k elephants, changes at time t if



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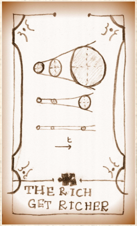

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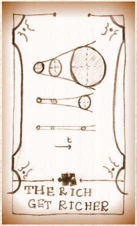

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$N_{k,t}$, the number of groups with k elephants, changes at time t if

1. An elephant belonging to a group with k elephants is replicated:
   $N_{k,t+1} = N_{k,t} - 1$

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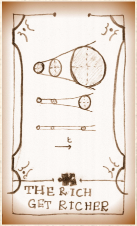

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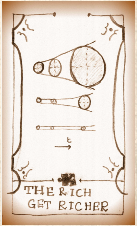

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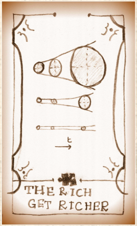

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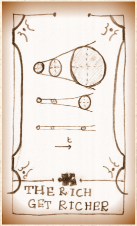
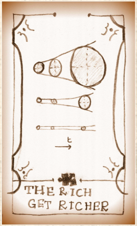


Random Competitive Replication:

Special case for $N_{1,t}$:



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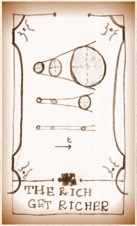

Special case for $N_{1,t}$:

1. The new elephant is a new flavor:

Random Competitive Replication:

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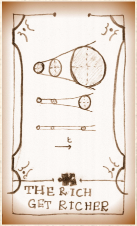
2. A unique elephant is replicated:



Random Competitive Replication:

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   $N_{1,t+1} = N_{1,t} + 1$

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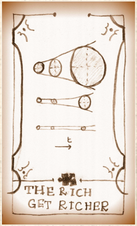

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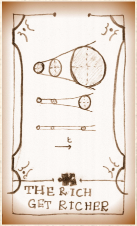

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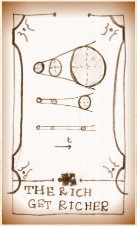

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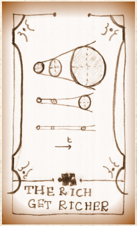
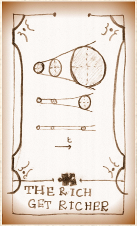


Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t} \right)$$



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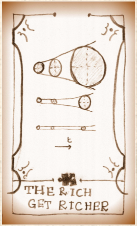
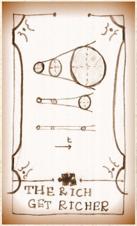


Random Competitive Replication:

Assume distribution stabilizes: $N_{k,t} = n_k t$

(Reasonable for t large)



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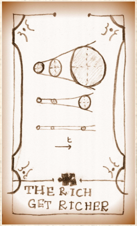

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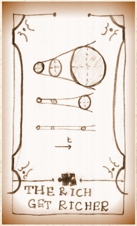

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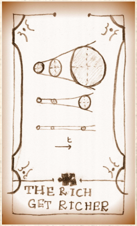

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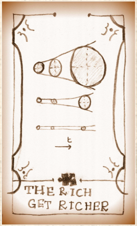
- For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k \cancel{t}}{\rho \cancel{t}} = \frac{n_k}{\rho}.$$



Random Competitive Replication:

Stochastic difference equation:

$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t}\right)$$

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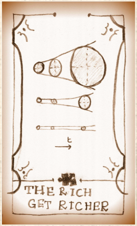
$$n_k(t+1) - n_k t = (1-\rho)\left((k-1)\frac{n_{k-1}t}{t} - k\frac{n_k t}{t}\right)$$



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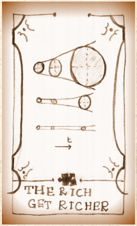

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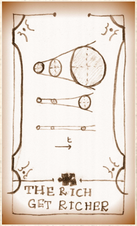

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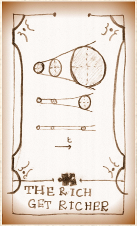
$$\Rightarrow n_k\left(1 + (1-\rho)k\right) = (1-\rho)(k-1)n_{k-1}$$



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We have a simple recursion:

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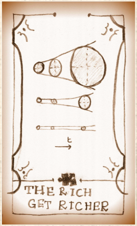

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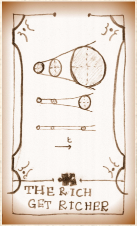

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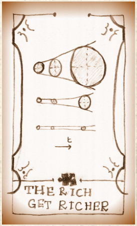

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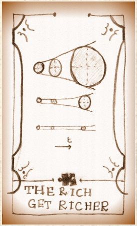

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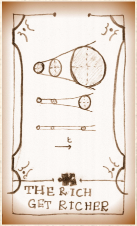

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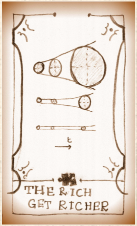

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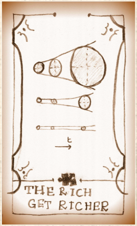
  We (okay, you) find

$$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$



Micro-to-Macro story with ρ and γ measurable.

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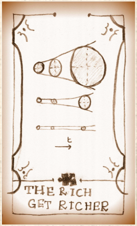

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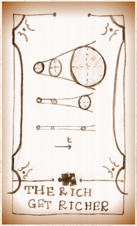

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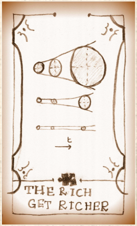

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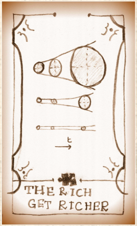

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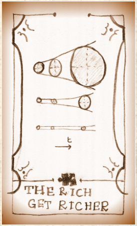

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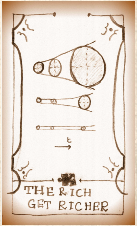

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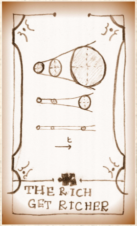
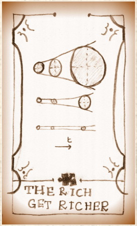
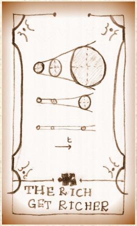
- Upshot: Tunable mechanism producing a family of universality classes.



Recall size-ranking law: $s_r \sim r^{-\alpha}$
(s_r = size of the rth largest group of elephants)





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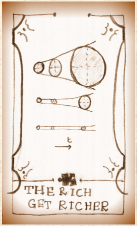

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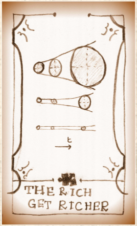

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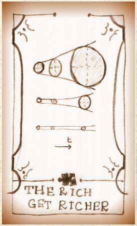

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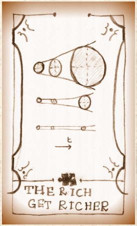

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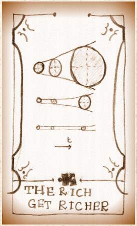
- Still, other quite different mechanisms are possible...
- Must look at the details to see if mechanism makes sense... more later.



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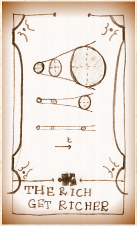

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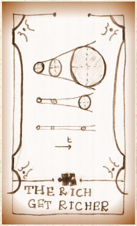

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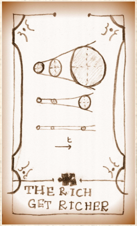
- $$n_1 = \rho - (1-\rho) n_1$$



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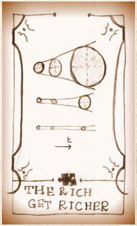

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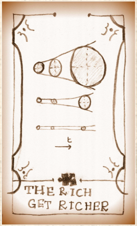
- $$\boxed{n_1 = \frac{\rho}{2-\rho}}$$



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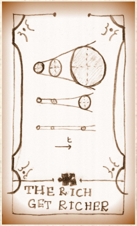

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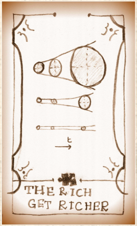

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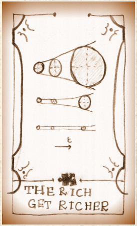
$$\frac{1}{\rho t} N_{1,t} = \frac{1}{\cancel{\rho t}} \frac{\cancel{\rho t}}{2-\rho} = \frac{1}{2-\rho}$$

(also = fraction of groups of size 1)



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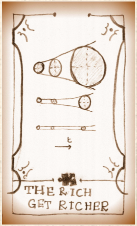

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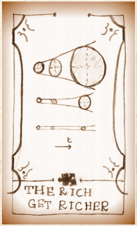

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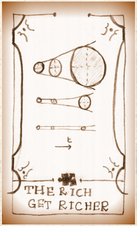

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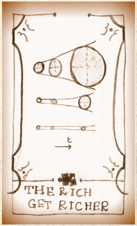
- Can show fraction of groups with two elephants $\sim 1/6$



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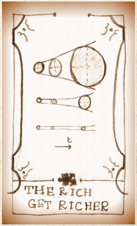

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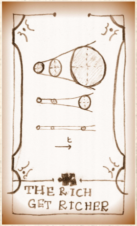

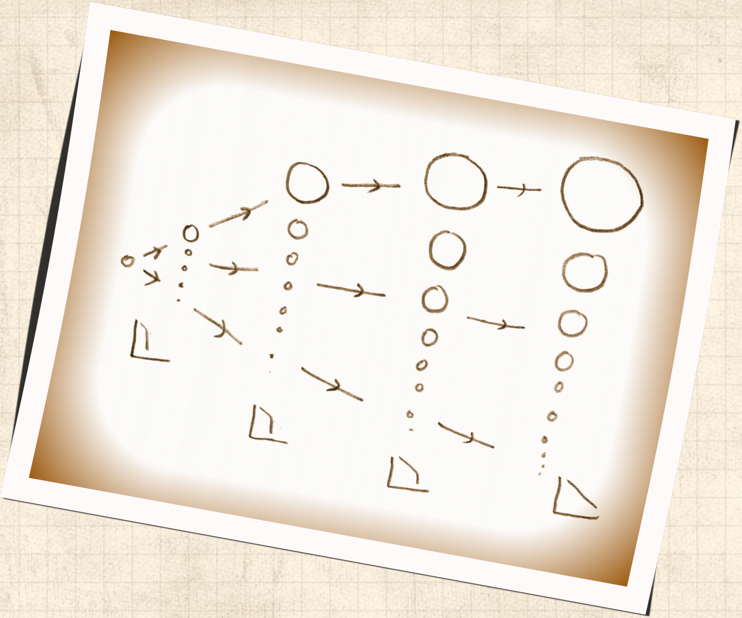

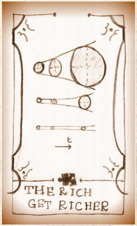
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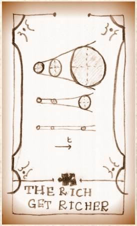
GET RICHER

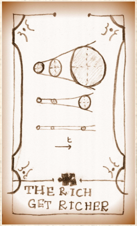
Outline



Words:

From Simon [10]:

Estimate $\rho_{\text{est}} = \#$ unique words/$\#$ all words



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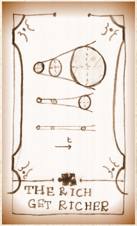

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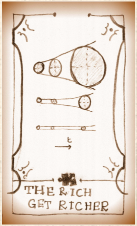

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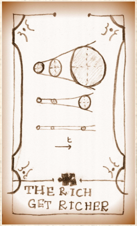
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870



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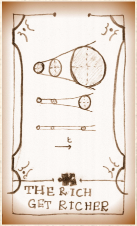

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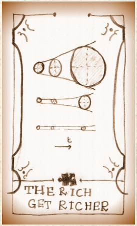

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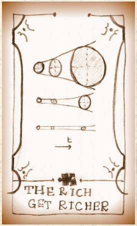

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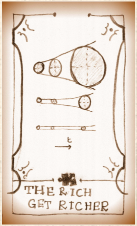

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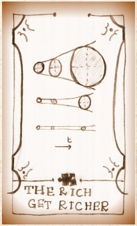

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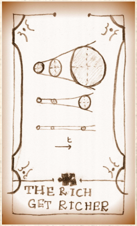

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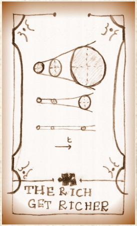

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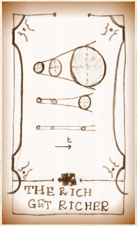

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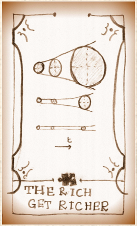

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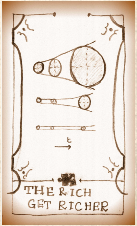

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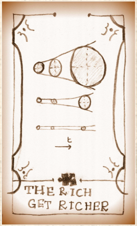

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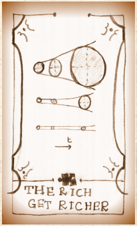

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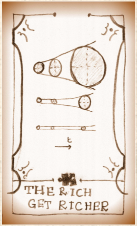

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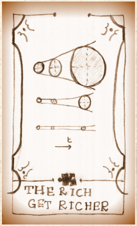
- Directed network
- Two (surmountable) problems:
  1. New papers have no citations
  2. Selection mechanism is more complicated



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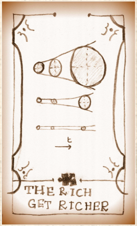

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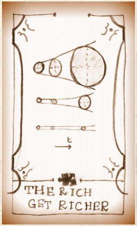

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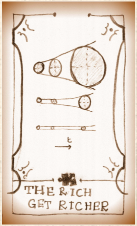

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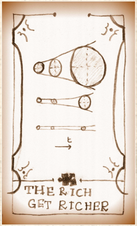

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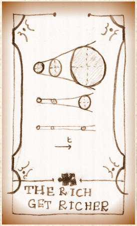

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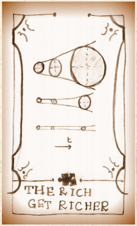
- (Hath = suggested unit of purchasing power.)



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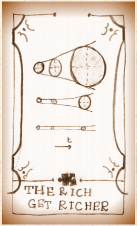
- Studied careers of scientists and found credit flowed disproportionately to the already famous

  From the Gospel of Matthew:
  "For to every one that hath shall be given...
  (Wait! There's more....)
  but from him that hath not, that also which he seemeth to have shall be taken away.
  And cast the worthless servant into the outer darkness; there men will weep and gnash their teeth."
- (Hath = suggested unit of purchasing power.)
- Matilda effect: women's scientific achievements are often overlooked



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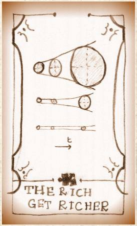
Merton was a catchphrase machine:



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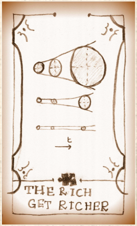

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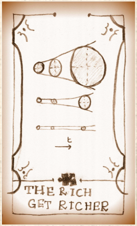

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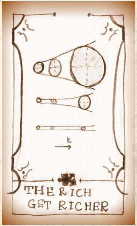

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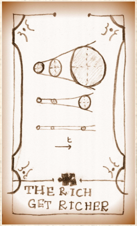

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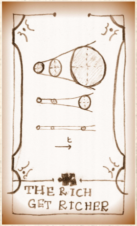

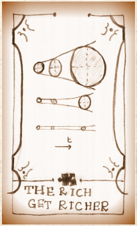

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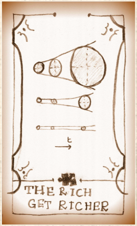
And just to be clear...

Merton's son, Robert C. Merton, won the Nobel Prize for Economics in 1997.



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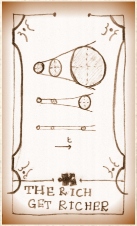
- Barabasi and Albert [2]—thinking about the Web



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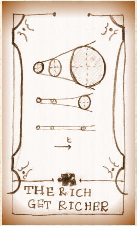

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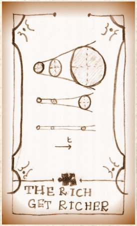

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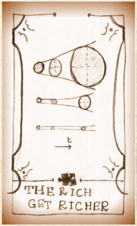

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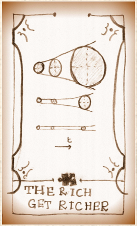

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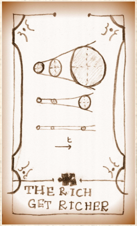

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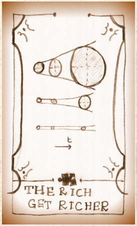

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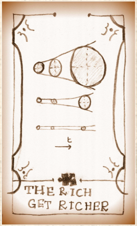
- "Scale-free networks" = food on the table for physicists



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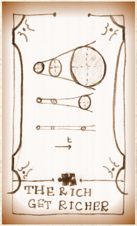
Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.



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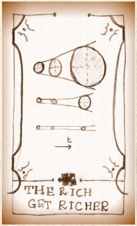

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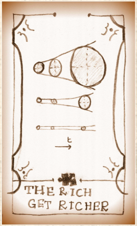

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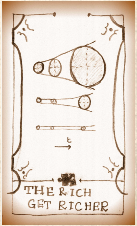

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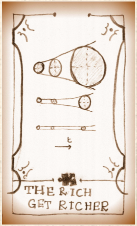

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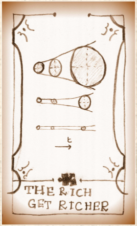

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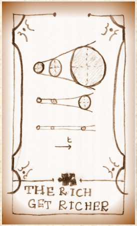

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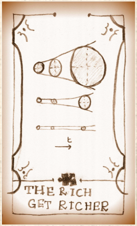

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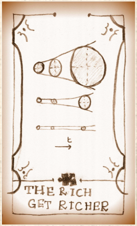
- What if a healthier Facebook is just ... Instagram? (hahahhaaha)



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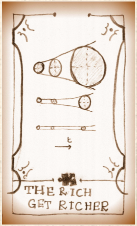

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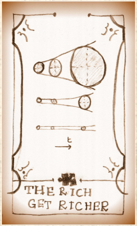

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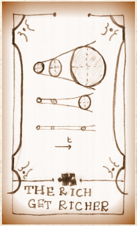

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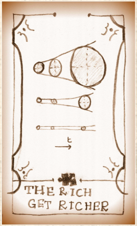
Simon's Model



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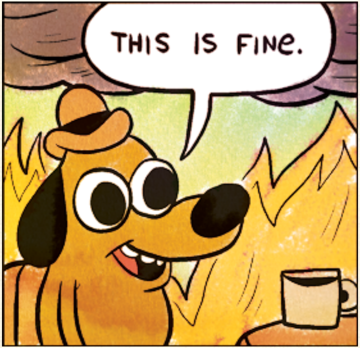

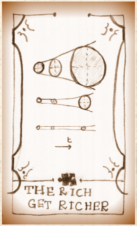

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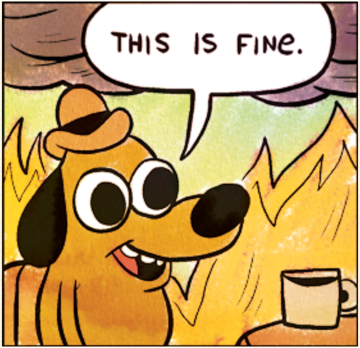

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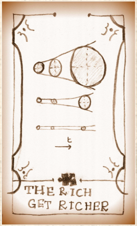

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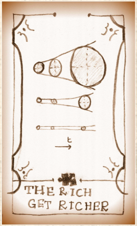

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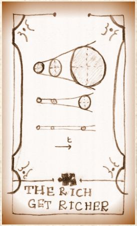

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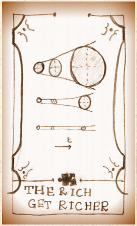

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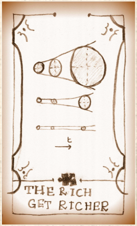

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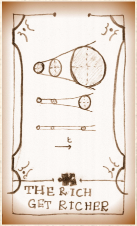
To be continued ...



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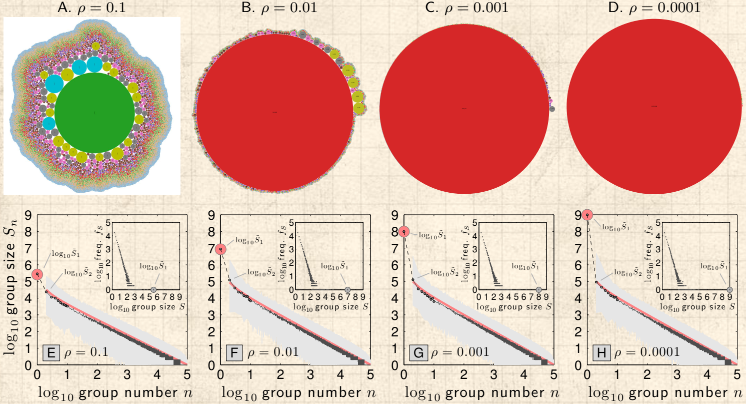
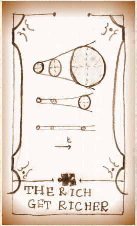
"Simon's fundamental rich-get-richer model entails a dominant first-mover advantage"
Dodds et al.,
Physical Review E, **95**, 052301, 2017. [4]





See Fletcher Hazlehurst's visualization at paper's online app-endices

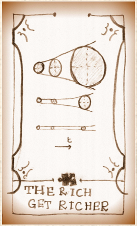
References I

[1] F. Auerbach.
Das gesetz der bevölkerungskonzentration.
Petermanns Geogr. Mitteilungen, 59:73–76, 1913.

[2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. pdf

[3] D. J. de Solla Price.
Networks of scientific papers.
Science, 149:510–515, 1965. pdf

[4] P. S. Dodds, D. R. Dewhurst, F. F. Hazlehurst, C. M. Van Oort, L. Mitchell, A. J. Reagan, J. R. Williams, and C. M. Danforth.
Simon's fundamental rich-get-richer model entails a dominant first-mover advantage.
Physical Review E, 95:052301, 2017. pdf

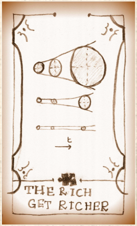


References II

[5] J.-B. Estoup.
Gammes sténographiques: méthode et exercices pour l'acquisition de la vitesse.
Institut Sténographique, 1916.

[6] P. Krugman.
The Self-Organizing Economy.
Blackwell Publishers, Cambridge, Massachusetts, 1996.

[7] A. J. Lotka.
The frequency distribution of scientific productivity.
Journal of the Washington Academy of Science, 16:317–323, 1926.

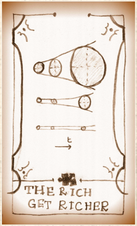


References III

[8] B. B. Mandelbrot.
An informational theory of the statistical structure of languages.
In W. Jackson, editor, Communication Theory, pages 486–502. Butterworth, Woburn, MA, 1953. pdf

[9] D. D. S. Price.
A general theory of bibliometric and other cumulative advantage processes.
Journal of the American Society for Information Science, pages 292–306, 1976. pdf

[10] H. A. Simon.
On a class of skew distribution functions.
Biometrika, 42:425–440, 1955. pdf



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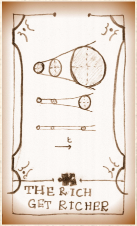

[11] G. U. Yule.
A mathematical theory of evolution, based on the conclusions of Dr J. C. Willis, F.R.S.
Phil. Trans. B, 213:21–87, 1925. pdf

[12] G. K. Zipf.
Human Behaviour and the Principle of Least-Effort.
Addison-Wesley, Cambridge, MA, 1949.