Mechanisms for Generating Power-Law Size Distributions, Part 3

Last updated: 2024/10/03, 19:37:10 EDT

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont

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Principles of Complex Systems
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What's the Story?

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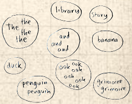

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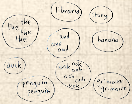
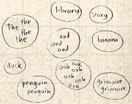


Outline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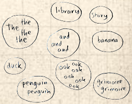

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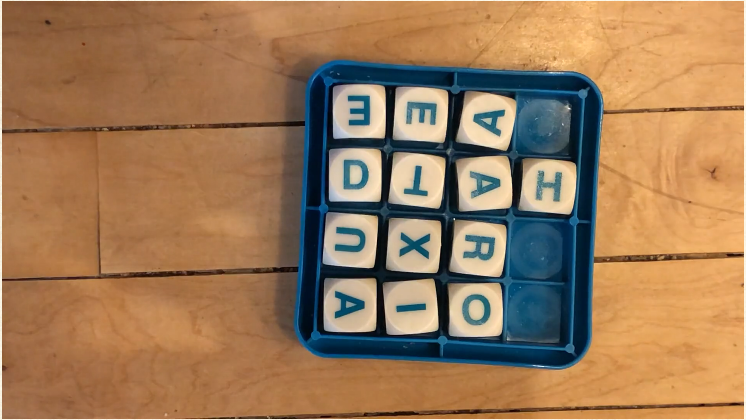
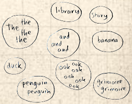
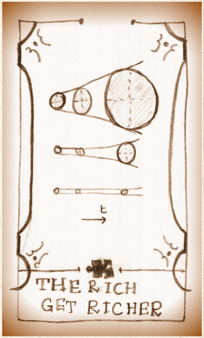





THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TAILS OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE BOOKS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS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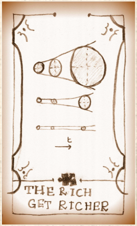
THE RICH GET RICHER

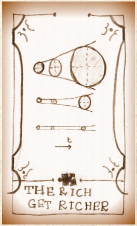
Aggregation:

- Random walks represent additive aggregation



Aggregation:

- Random walks represent additive aggregation
- Mechanism: Random addition and subtraction



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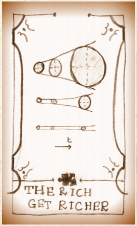
- Compare across realizations, no competition.



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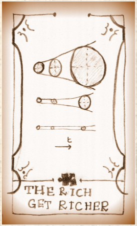

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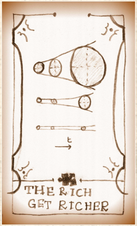
- Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)



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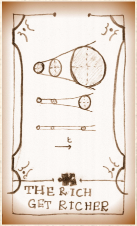
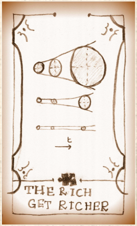
- Competing mechanisms (trickiness)



Pre-Zipf's law observations of Zipf's law

- 1910s: Word frequency examined re Stenography (or shorthand or brachygraphy or tachygraphy), Jean-Baptiste Estoup [6].



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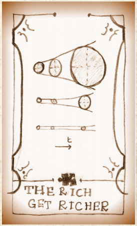

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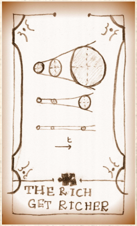

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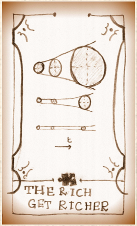
- 1926: Lotka [9]:
  # Scientific papers per author (Lotka's law)



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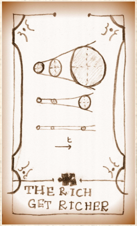

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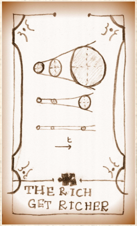

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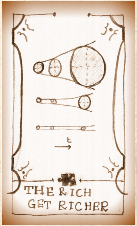

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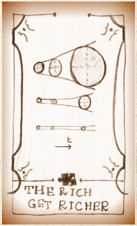

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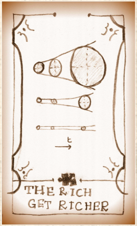
- 1999: Barabasi and Albert [2]:
  The World Wide Web, networks-at-large.



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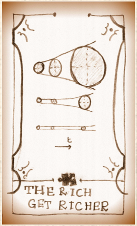

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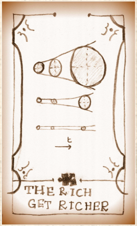

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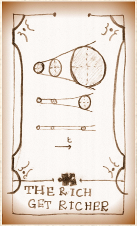

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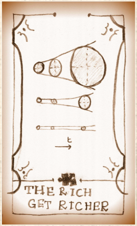

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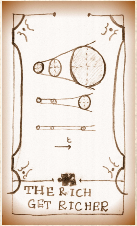

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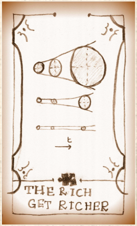
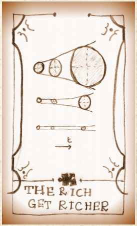
- 1978 Nobel Laureate in Economics
(his Nobel bio is here).



Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at $t = 1$



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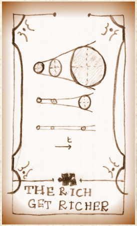

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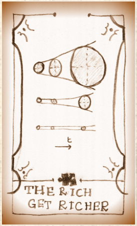

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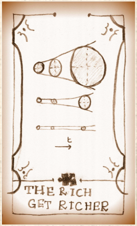

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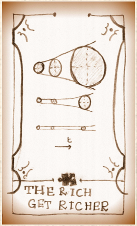

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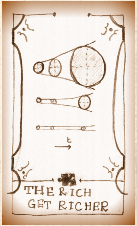
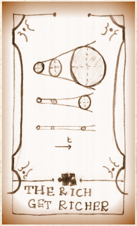


Random Competitive Replication:

Example: Words appearing in a language



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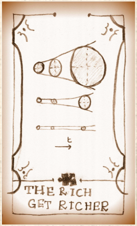

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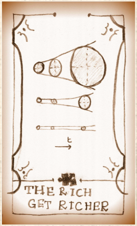
- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared



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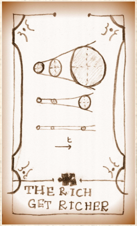

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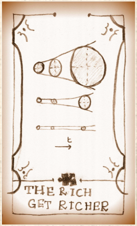

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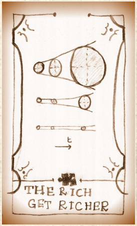

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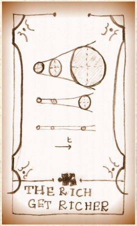
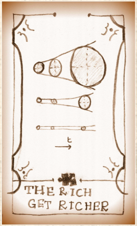
Note: This is a terrible way to write a novel.



For example:

- 21 words used
- next word is new with prob ρ
- next word is a copy with prob $1-\rho$

prob:	next word:
6/21	ook
4/21	the
3/21	and
2/21	penguin
⋮	
1/21	library



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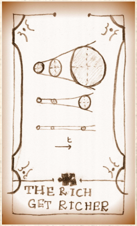

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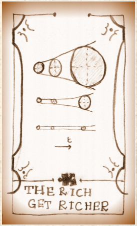

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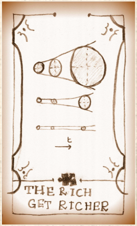

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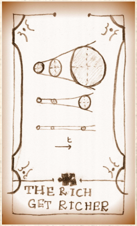

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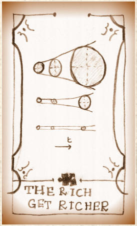

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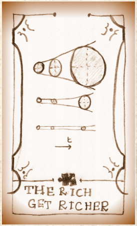

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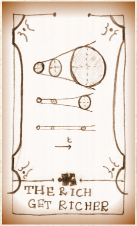

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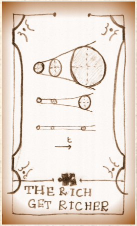

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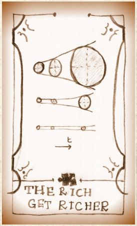
- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.



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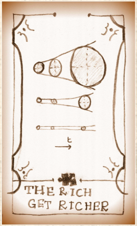

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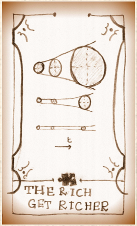

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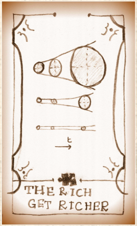

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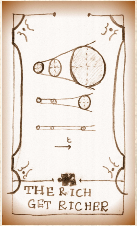

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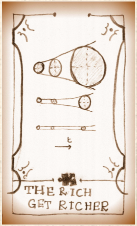

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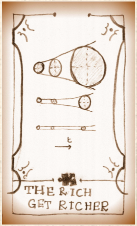

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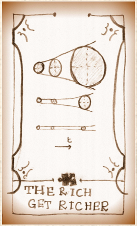

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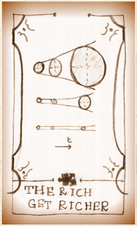

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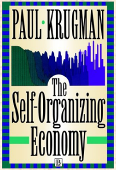
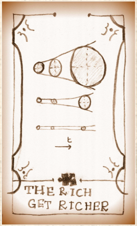
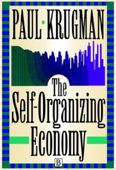
     (But mechanism for selection is not as simple...)



"The Self-Organizing Economy" by Paul Krugman (1996). [8]





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Ch. 3: An Urban Mystery, p. 46

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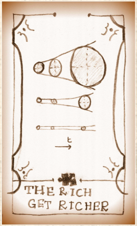

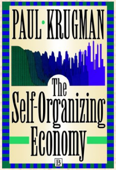

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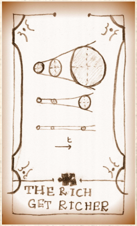

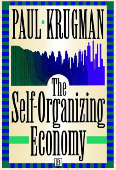

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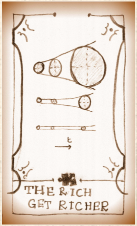

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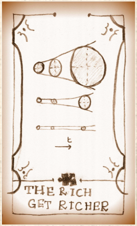
[2] Let's use π for probability because π's not special, right guys?

Outline



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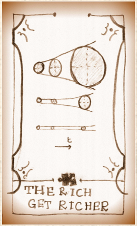
Definitions:

- k_i = size of a group i



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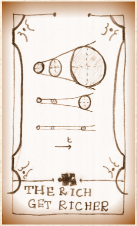

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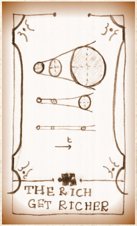
Basic question: How does $N_{k,t}$ evolve with time?



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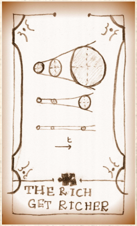
$$\text{First: } \sum_k kN_{k,t} = t = \text{number of elephants at time } t$$



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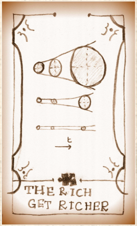

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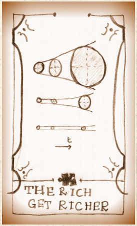

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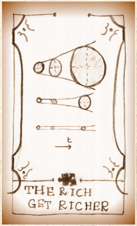

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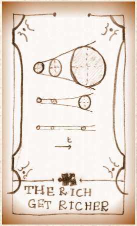

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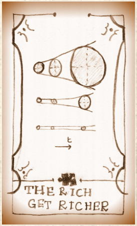
$$P_k(t) = \frac{kN_{k,t}}{t}.$$



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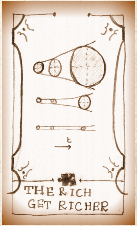
$N_{k,t}$, the number of groups with k elephants, changes at time t if



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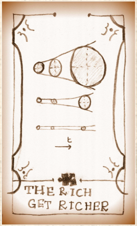

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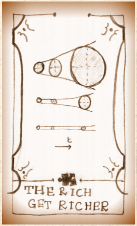

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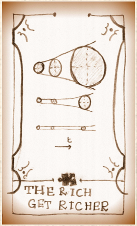

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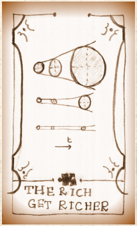

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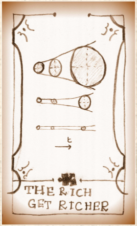

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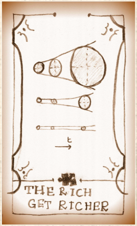

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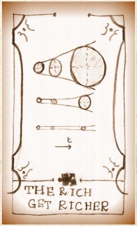
Special case for $N_{1,t}$:



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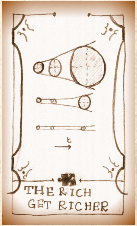

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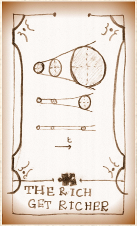

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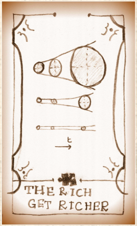

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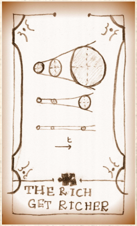

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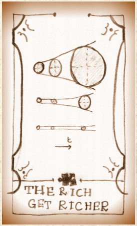

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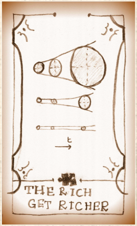

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Putting everything together:

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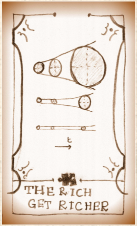

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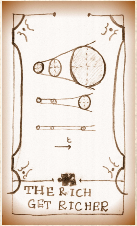

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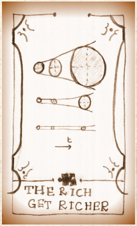

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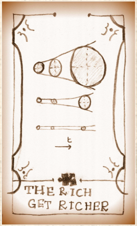

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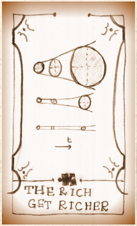

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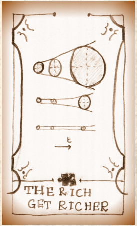

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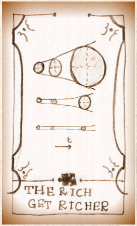
- For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k \cancel{t}}{\rho \cancel{t}} = \frac{n_k}{\rho}.$$



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Stochastic difference equation:

$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t} \right)$$

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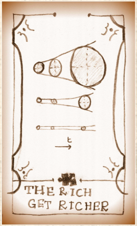

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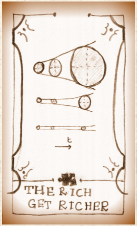

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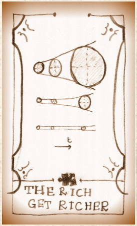

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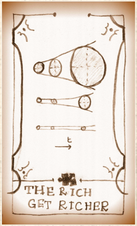

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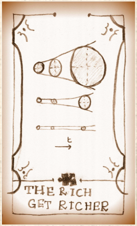

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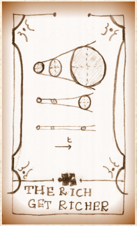

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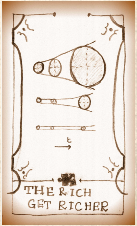

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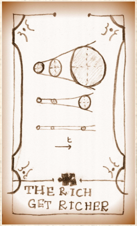

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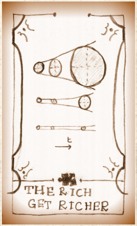

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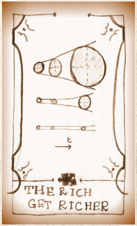

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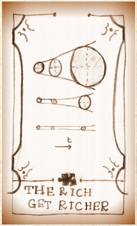
  We (okay, you) find

$$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$



Micro-to-Macro story with ρ and γ measurable.

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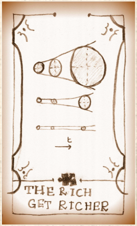

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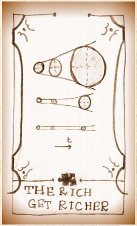

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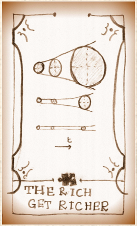

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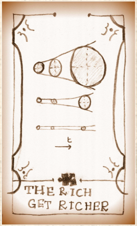

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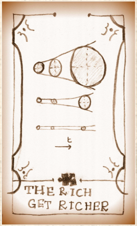

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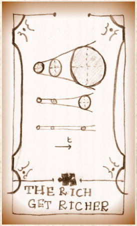

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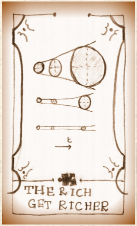
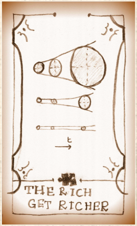
- Upshot: Tunable mechanism producing a family of universality classes.





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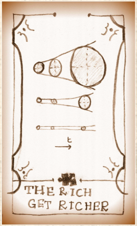

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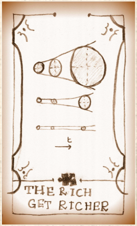

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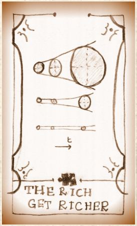

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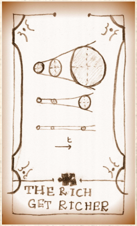

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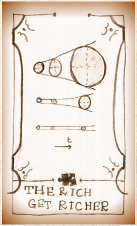

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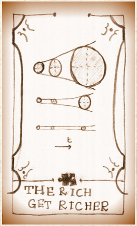

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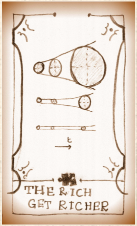

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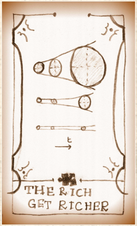

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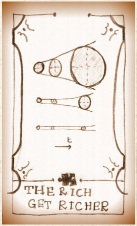

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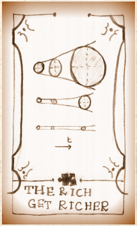
- Rearrange:
$$n_1 + (1-\rho)n_1 = \rho$$



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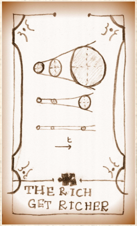
- $$n_1(t+1) - n_1 t = \rho - (1-\rho)1 \cdot \frac{n_1 t}{t}$$
- $$n_1 = \rho - (1-\rho)n_1$$
- Rearrange:
  $$n_1 + (1-\rho)n_1 = \rho$$
- $$\boxed{n_1 = \frac{\rho}{2-\rho}}$$



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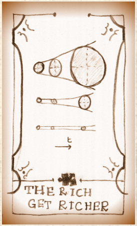

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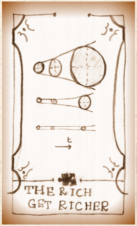

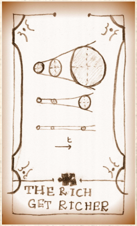

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- Recall number of distinct elephants = ρt.
- Fraction of distinct elephants that are unique (belong to groups of size 1):

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(also = fraction of groups of size 1)

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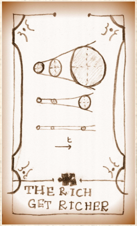

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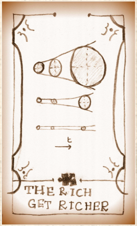

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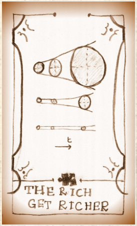

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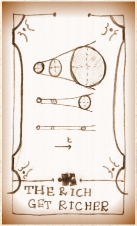
- For ρ small, fraction of unique elephants $\sim 1/2$
- Roughly observed for real distributions
- ρ increases, fraction increases
- Can show fraction of groups with two elephants $\sim 1/6$



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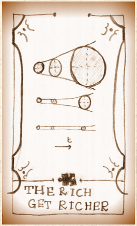

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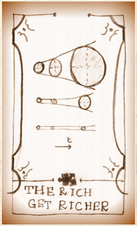

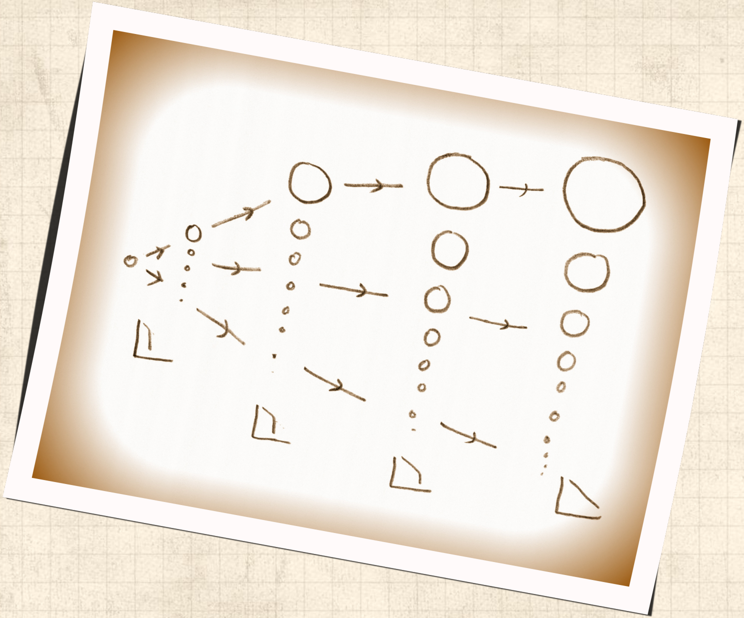

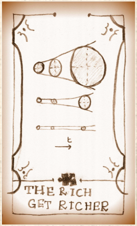
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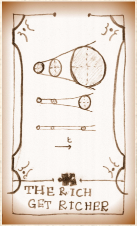
THE RICH
GET RICHER

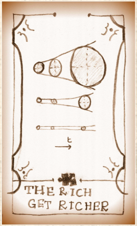
Outline



Words:

From Simon [14]:

Estimate $\rho_{\text{est}} = \#$ unique words/# all words



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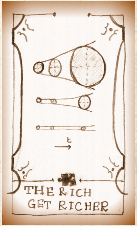

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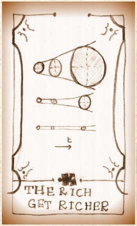
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870



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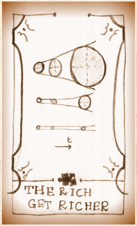

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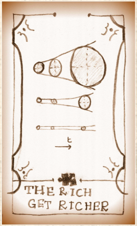

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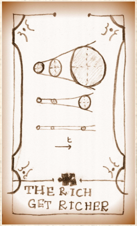

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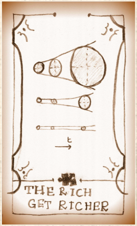

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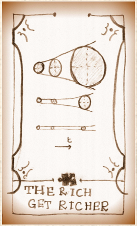

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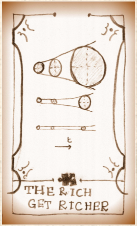

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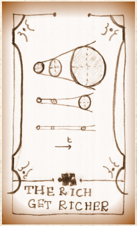

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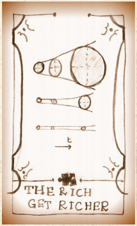

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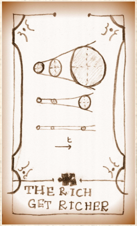
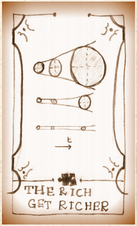


Evolution of catch phrases:

Derek de Solla Price:

- First to study network evolution with these kinds of models.



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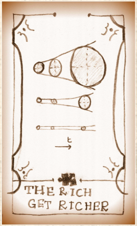

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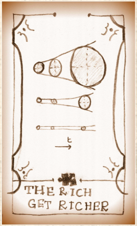

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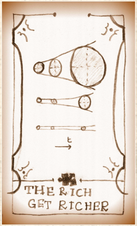

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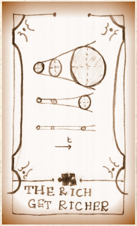

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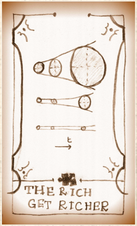
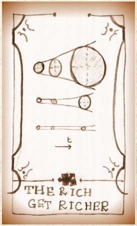
- Directed network
- Two (surmountable) problems:
  1. New papers have no citations
  2. Selection mechanism is more complicated



Evolution of catch phrases:

Robert K. Merton: the Matthew Effect

- Studied careers of scientists and found credit flowed disproportionately to the already famous



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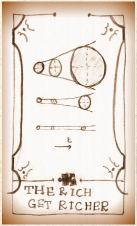

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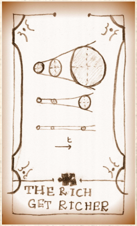

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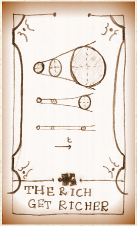

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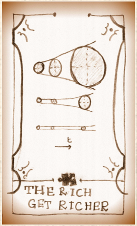

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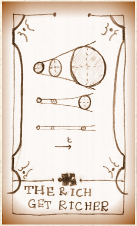

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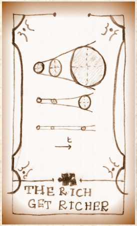
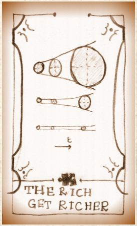
- (Hath = suggested unit of purchasing power.)
- Matilda effect: women's scientific achievements are often overlooked



Evolution of catch phrases:

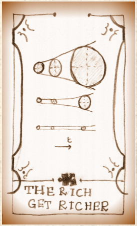
Merton was a catchphrase machine:



Evolution of catch phrases:

Merton was a catchphrase machine:

1. Self-fulfilling prophecy



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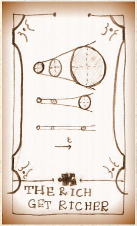
2. Role model



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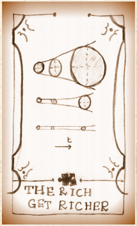

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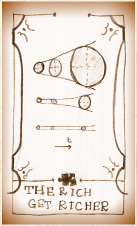

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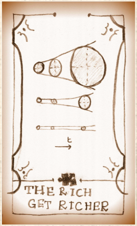
5. Obliteration by incorporation (includes above examples from Merton himself)



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Merton was a catchphrase machine:

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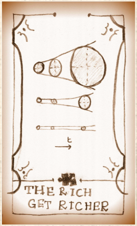

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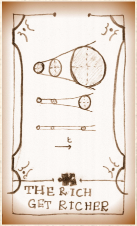
Merton's son, Robert C. Merton, won the Nobel Prize for Economics in 1997.



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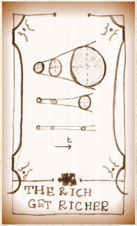

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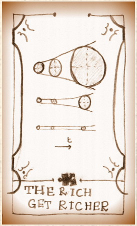

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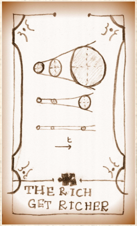

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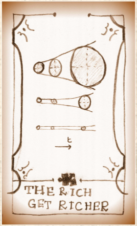

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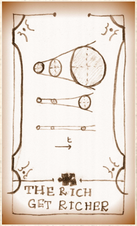

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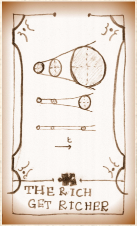

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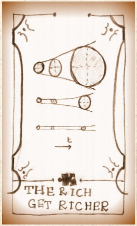

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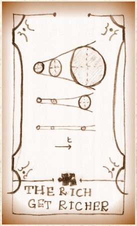
- "Scale-free networks" = food on the table for physicists



Outline

Rich-Get-Richer Mechanism

Simon's Model

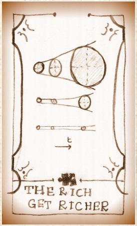
Analysis

Words

Catchphrases

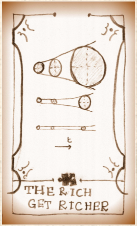
First Mover Advantage

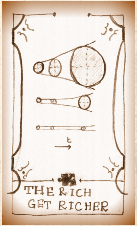
References



Another analytic approach: [5]

- Focus on how the nth arriving group typically grows.





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$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)} \left[\frac{1}{t}\right]^{-(1-\rho)} = \Gamma(2-\rho) \left[\frac{t}{1}\right]^{+(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho} \left[\frac{n-1}{t}\right]^{-(1-\rho)} = \left[\frac{t}{n-1}\right]^{+(1-\rho)} & \text{for } n \geq 2. \end{cases}$$

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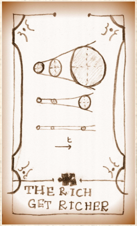
- First mover is a factor $1/\rho$ greater than expected.



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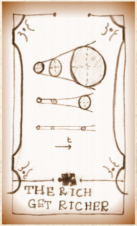
- Because ρ is usually close to 0, the first element is truly an elephant in the room.



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- First mover is a factor $1/\rho$ greater than expected.
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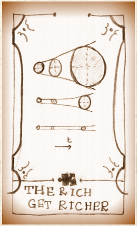
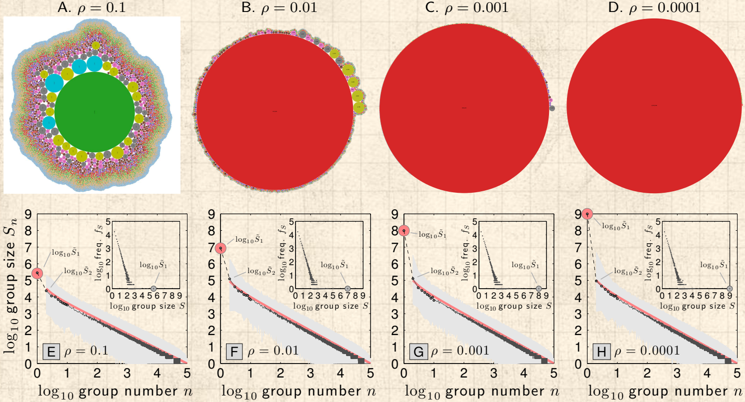
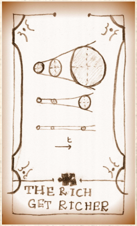
- Appears that this has been missed for 60 years ...



"Simon's fundamental rich-get-richer model entails a dominant first-mover advantage"

Dodds et al.,
Physical Review E, **95**, 052301, 2017. [5]



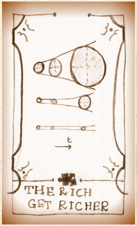


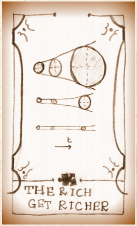
See visualization at paper's online app-endices

Alternate analysis:

- Evolution of the nth arriving group's size:

$$\left\langle S_{n,t+1} - S_{n,t} \right\rangle = (1-\rho_t) \cdot \frac{S_{n,t}}{t} \cdot (+1).$$





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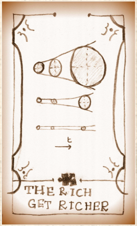
- For $t \geq t_n^{\rm init}$, fix $\rho_t = \rho$ and shift t to $t-1$:

$$S_{n,t} = \left[1 + \frac{(1-\rho)}{t-1} \right] S_{n,t-1}.$$

where $S_{n,t_n^{\rm init}} = 1$.

Betafication ensues:

$$
\begin{aligned}
S_{n,t} &= \left[1 + \frac{(1-\rho)}{t-1}\right]\left[1 + \frac{(1-\rho)}{t-2}\right]\cdots\left[1 + \frac{(1-\rho)}{t_n^{\text{init}}}\right]\cdot 1 \\
&= \left[\frac{t+1-\rho}{t-1}\right]\left[\frac{t-\rho}{t-2}\right]\cdots\left[\frac{t_n^{\text{init}}+1-\rho}{t_n^{\text{init}}}\right] \\
&= \frac{\Gamma(t+1-\rho)\Gamma(t_n^{\text{init}})}{\Gamma(t_n^{\text{init}}+1-\rho)\Gamma(t)} \\
&= \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}.
\end{aligned}
$$

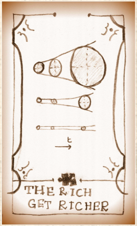


The first mover is really different:

- The issue is t_n^{init} in

$$S_{n,t} = \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}$$

- For $n \geq 2$ and $\rho \ll 1$, the nth group typically arrives at $t_n^{\text{init}} \simeq \left[\frac{n-1}{\rho}\right]$



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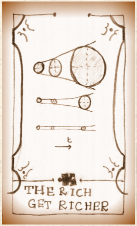

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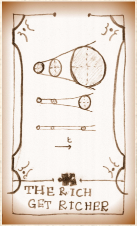
- Simon missed the first mover by working on the size distribution.



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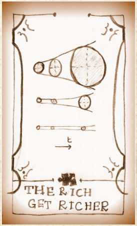
- Contribution to $P_{k,t}$ of the first element vanishes as $t \to \infty$.



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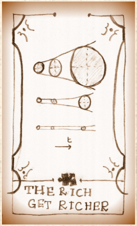
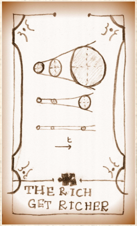
- Note: Does not apply to Barabási-Albert model.



Variability:

- The probability that the nth arriving group, if of size $S_{n,t} = k$ at time t, first replicates at time $t + \tau$:



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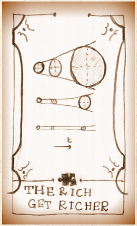
$$\mathbf{Pr}\left(S_{n,t+\tau} = k+1 \,|\, S_{n,t+i} = k \ \text{ for } i = 0, \ldots, \tau - 1\right)$$

$$= \prod_{i=0}^{\tau-1}\left[1 - (1-\rho)\frac{k}{t+i}\right] \cdot (1-\rho)\frac{k}{t+\tau}$$

$$= k\frac{B(\tau, t)}{B\left(\tau, t-(1-\rho)\right)}\frac{1-\rho}{t+\tau} \propto \frac{\tau^{-(1-\rho)k}}{t+\tau} \sim \tau^{-(2-\rho)k}.$$



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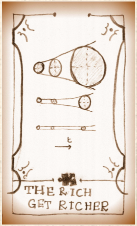
- Upshot: nth arriving group starting at size 1 will on average wait for an infinite time to replicate.



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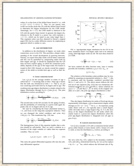
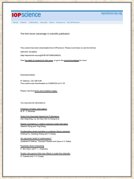
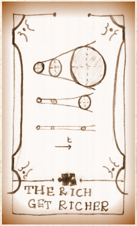


"Organization of Growing Random Networks"
Krapivsky and Redner,
Phys. Rev. E, **63**, 066123, 2001. [7]



"The first-mover advantage in scientific publication"
M. E. J. Newman,
Europhysics Letters, **86**, 68001, 2009. [11]



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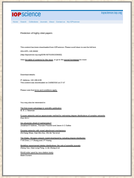
IOPscience

Prediction of highly cited papers

"Prediction of highly cited papers"
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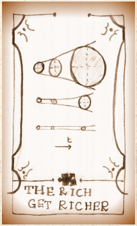
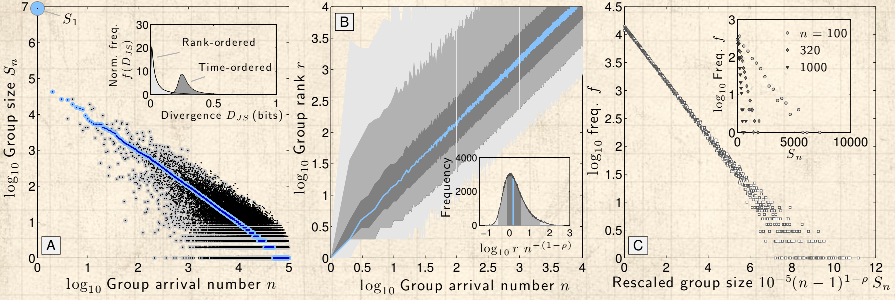
Europhysics Letters, **105**, 28002, 2014. [12]


The effect of the initial network configuration on preferential attachment

"The effect of the initial network configuration on preferential attachment"
Berset and Medo,
The European Physical Journal B, **86**, 1–7, 2013. [3]

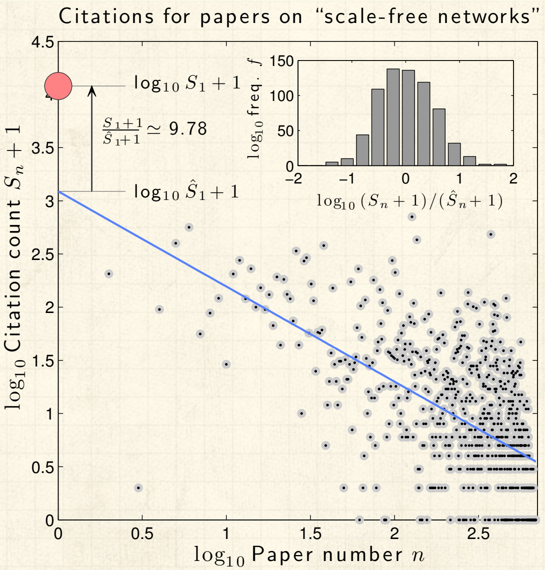


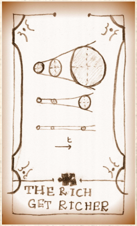
Arrival variability:



- Any one simulation shows a high amount of disorder.
- Two orders of magnitude variation in possible rank.
- Rank ordering creates a smooth Zipf distribution.
- Size distribution for the nth arriving group show exponential decay.

Self-referential citation data:

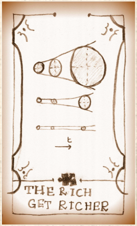




More mattering:

Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.



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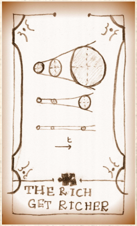
- People, wealth, sports, music, movies, books, schools, cities, countries, dogs (13/10), ...



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Rich-get-richerness in social contagion:

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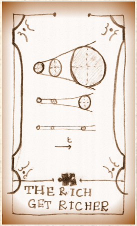
- Gameable: payola, astroturfing, sockpuppetry, John Barron (the sockpuppet hype man), ...



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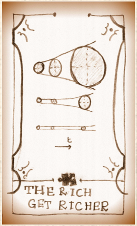
Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.
- People, wealth, sports, music, movies, books, schools, cities, countries, dogs (13/10), ...
- Gameable: payola, astroturfing, sockpuppetry, John Barron (the sockpuppet hype man), ...
- Black-box ranking algorithms make ranking opaque.



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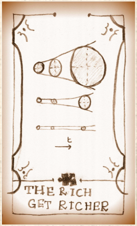

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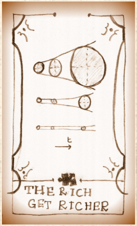

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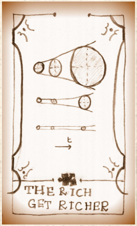

[1] "With great power comes great responsibility." –S. Man.

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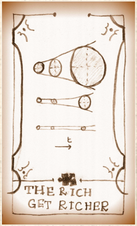

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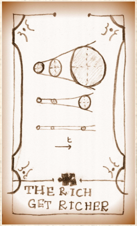
- What if a healthier Facebook is just ... Instagram? (hahahhaaha)



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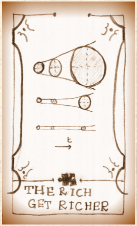

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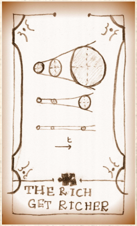

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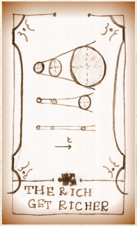

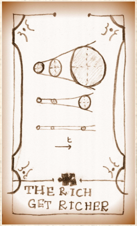

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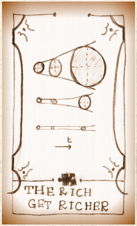

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