

Overview of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

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Popularity
Graph theory?
Basic definitions

Examples of Complex
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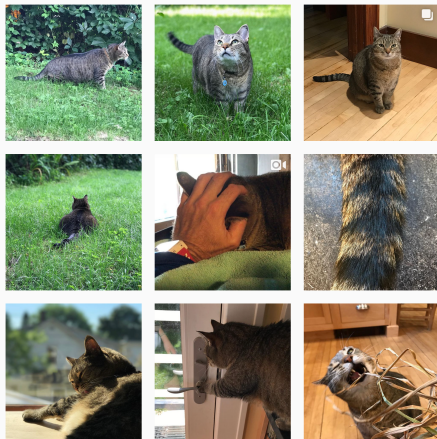
Physical networks
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Outline

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net•work |'net,wɜrk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*



Thesaurus deliciousness:

network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.



From Keith Briggs's excellent etymological investigation: 



Opus reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]








Ancestry:

First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’


From the OED via Briggs:


-  1658–: reticulate structures in animals
-  1839–: rivers and canals
-  1869–: railways
-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

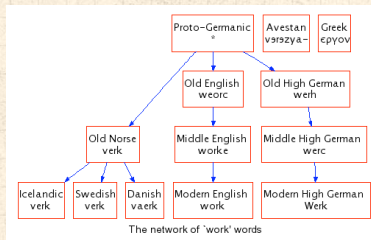
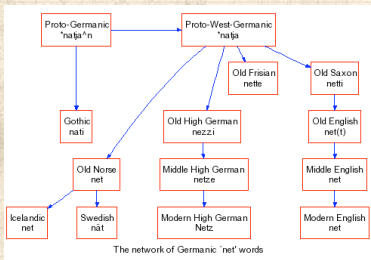



Ancestry:


Net and Work are venerable old words:

 'Net' first used to mean spider web (King Ælfréd, 888).

 'Work' appear to have long meant purposeful action.



 'Network' = something built based on the idea of natural, flexible lattice or web.






 c.f., ironwork, stonework, fretwork.



Key Observation:

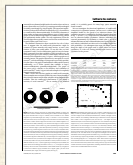
- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:




-  *Piranha physicus*
-  Hunt in packs.
-  Feast on new and interesting ideas (see chaos, cellular automata, ...)
-  See also: <https://xkcd.com/793/> 

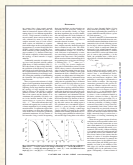



Popularity (according to Google Scholar)



“Collective dynamics of ‘small-world’ networks” 
Watts and Strogatz,
Nature, **393**, 440–442, 1998. ^[16]

Times cited:  ~ **37,460** (as of October 24, 2018)




“Emergence of scaling in random networks” 
Barabási and Albert,
Science, **286**, 509–511, 1999. ^[2]

Times cited:  ~ **32,093** (as of October 24, 2018)




Review articles:




“Complex Networks: Structure and Dynamics” 

Boccaletti et al.,

Physics Reports, **424**, 175–308, 2006. ^[3]


Times cited:  **~ 8,533** (as of October 24, 2018)




“The structure and function of complex networks” 

M. E. J. Newman,

SIAM Rev., **45**, 167–256, 2003. ^[12]


Times cited:  **~ 17,782** (as of October 24, 2018)



“Statistical mechanics of complex networks” 

Albert and Barabási,

Rev. Mod. Phys., **74**, 47–97, 2002. ^[1]

Times cited:  **~ 20,531** (as of October 24, 2018)

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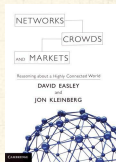
References



Popularity according to textbooks:



“Networks” [a](#) [↗](#)
by Mark Newman (2018). [11]



“Networks, crowds, and markets: Reasoning about a highly connected world” [a](#) [↗](#)
by Easley and Kleinberg (2010). [7]

<http://cs.cornell.edu/home/kleinber/networks-book/> [↗](#)

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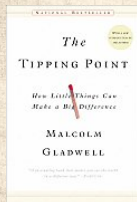
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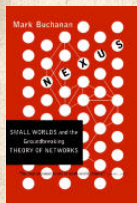
References



Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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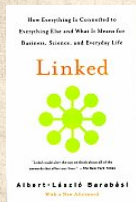
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Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts ^[15]

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Numerous others ...

- Complex Social Networks—F. Vega-Redondo ^[14]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo ^[13]
- Random Graph Dynamics—R. Durrett
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes ^[6]




More observations

- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish **mechanistic explanations**.



**If this is upsetting, maybe string theory is for you ...*





More observations

 Web-scale data sets can be overly **exciting**.


Witness:


 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

 “The Unreasonable Effectiveness of Data,”
Halevy et al. [9].

 c.f. Wigner’s “The Unreasonable Effectiveness of Mathematics in the Natural Sciences” [17]

But:


 For scientists, description is only part of the battle.

 We still need to **understand**.





Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

 **Links** may be directed or undirected.


 **Links** may be binary or weighted.


Other spiffing words: vertices and edges.





Super Basic definitions

Node degree = Number of links per node


 Notation: Node i 's degree = k_i .

 $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)

 Connection between number of edges m and average degree:


$$\langle k \rangle = \frac{2m}{N}.$$


 Defn: N_i = the set of i 's k_i neighbors




Super Basic definitions

Adjacency matrix:

 We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .





 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 (n.b., for numerical work, we always use sparse matrices.)







So what passes for a complex network?




-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**
-  Complex networks can be social, economic, natural, informational, abstract, ...

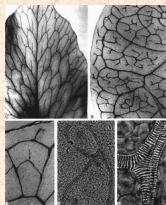
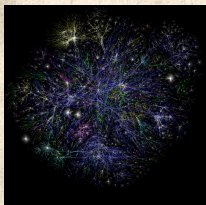



Examples

Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids



 **Distribution** (branching) versus **redistribution** (cyclical)

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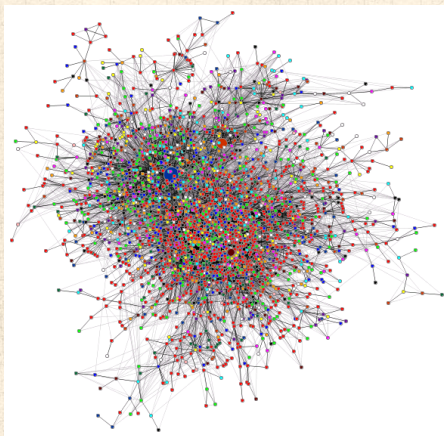
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Examples

Interaction networks

-  The Blogosphere (RIP)
-  Biochemical networks
-  Gene-protein networks
-  Food webs: who eats whom
-  The internet¹
-  Airline networks
-  Call networks (AT&T)
-  The Media
-  Social Media





datamining.typepad.com 




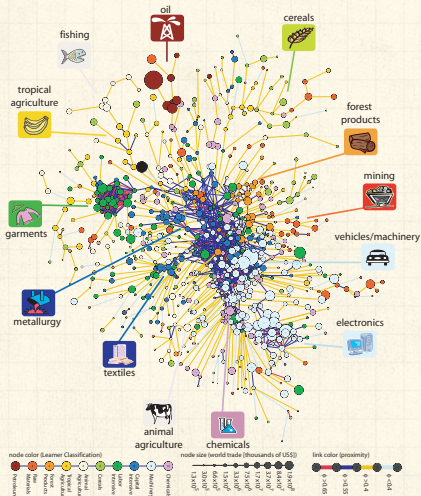
¹What was the World Wide Web, then the Internet, then the internet

topics:

 Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [10]

 How do products depend on each other, and how does this network evolve?

 How do countries depend on each other for water, energy, people (immigration), investments?



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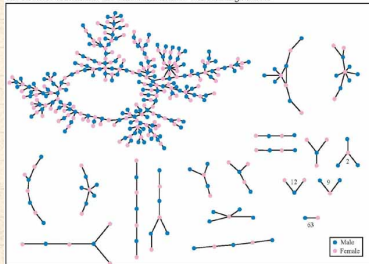
Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook ↗, twitter ↗,

‘Remotely sensed’ by: email activity, instant messaging, phone logs (*cough*).

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

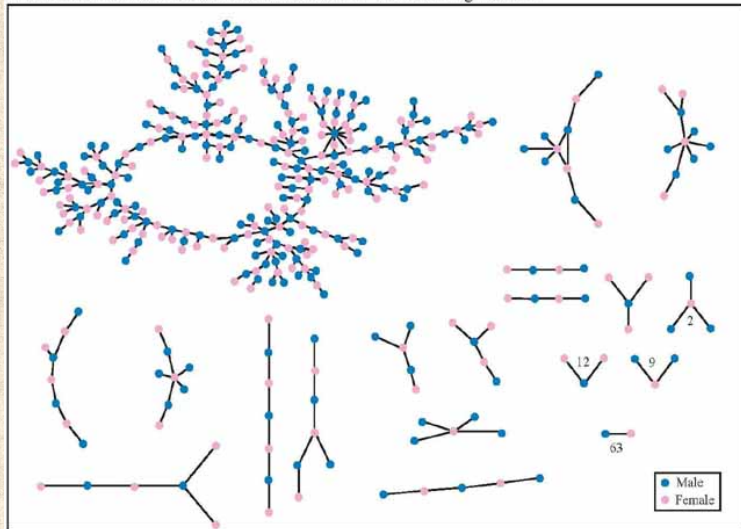
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The Structure of Romantic and Sexual Relations at "Jefferson High School"



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Examples

Relational networks



Consumer purchases
(Walmart, Target, Amazon, ...)





Thesauri: Networks of words generated by meanings



Knowledge/Databases/Ideas



Metadata—Tagging: [bit.ly](#)  [flickr](#) 

common tags cloud | [list](#)

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

[Etymology](#)

[Popularity](#)

[Graph theory?](#)

[Basic definitions](#)





[Physical networks](#)

[Interaction networks](#)

[Relational networks](#)



References I

- [1] R. Albert and A.-L. Barabási.
Statistical mechanics of complex networks.
Rev. Mod. Phys., 74:47–97, 2002. [pdf](#) 
- [2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. [pdf](#) 
- [3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang.
Complex networks: Structure and dynamics.
Physics Reports, 424:175–308, 2006. [pdf](#) 
- [4] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila.
Clickstream data yields high-resolution maps of science.
PLoS ONE, 4:e4803, 2009. [pdf](#) 






References II

- [5] S. Bornholdt and H. G. Schuster, editors.
Handbook of Graphs and Networks.
Wiley-VCH, Berlin, 2003.
- [6] S. N. Dorogovtsev and J. F. F. Mendes.
Evolution of Networks.
Oxford University Press, Oxford, UK, 2003.
- [7] D. Easley and J. Kleinberg.
Networks, crowds, and markets: Reasoning about a highly connected world.
Cambridge University Press, 2010.
- [8] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.



References III

- [9] A. Halevy, P. Norvig, and F. Pereira.
The unreasonable effectiveness of data.
IEEE Intelligent Systems, 24:8–12, 2009. [pdf](#) 
- [10] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and R. Hausman.
The product space conditions the development of nations.
Science, 317:482–487, 2007. [pdf](#) 
- [11] M. Newman.
Networks.
Oxford university press, 2nd edition, 2018.
- [12] M. E. J. Newman.
The structure and function of complex networks.
SIAM Rev., 45(2):167–256, 2003. [pdf](#) 



References IV

- [13] I. Rodríguez-Iturbe and A. Rinaldo.
Fractal River Basins: Chance and Self-Organization.
Cambridge University Press, Cambridge, UK, 1997.
- [14] F. Vega-Redondo.
Complex Social Networks.
Cambridge University Press, 2007.
- [15] D. J. Watts.
Six Degrees.
Norton, New York, 2003.
- [16] D. J. Watts and S. J. Strogatz.
Collective dynamics of ‘small-world’ networks.
Nature, 393:440–442, 1998. [pdf](#) 



[17] E. Wigner.

The unreasonable effectiveness of mathematics in the natural sciences.

[Communications on Pure and Applied Mathematics,](#)

13:1–14, 1960. pdf 