Overview of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D CSYS/MATH 6701, 6713, & a pretend number, 2023–2024| @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont

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Outline

Something of a plan:

- Lecture 1: Overview; Background

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- Lecture 2: Random, Scale-free,
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- Lecture 1: Overview; Background
- Lecture 2: Random, Scale-free, and Small-World networks
- Lecture 3: Models of Contagion
- Lecture 4: Transportation networks; Discovering structure

Exciting details regarding these slides:

- Three versions (all in pdf):
  1. Presentation,
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- Brought to you by a concoction of LaTeX, Beamer, perl, and madness.

Bonus materials:

Graduate Course Websites:

- SFI Summer School Course (this one!):
  https://pdodds.w3.uvm.edu/teaching/courses/2023-2024pocsverse
- Principles of Complex Systems, University of Vermont
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Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction"
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World"

Bonus materials:

Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [4]
  Times cited: 1,028 (as of June 7, 2010)
- M. Newman
  "The structure and function of complex networks" [15]
  Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 3,995 (as of June 7, 2010)

Basic definitions:

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Basic definitions:

Complex System—Some ingredients:

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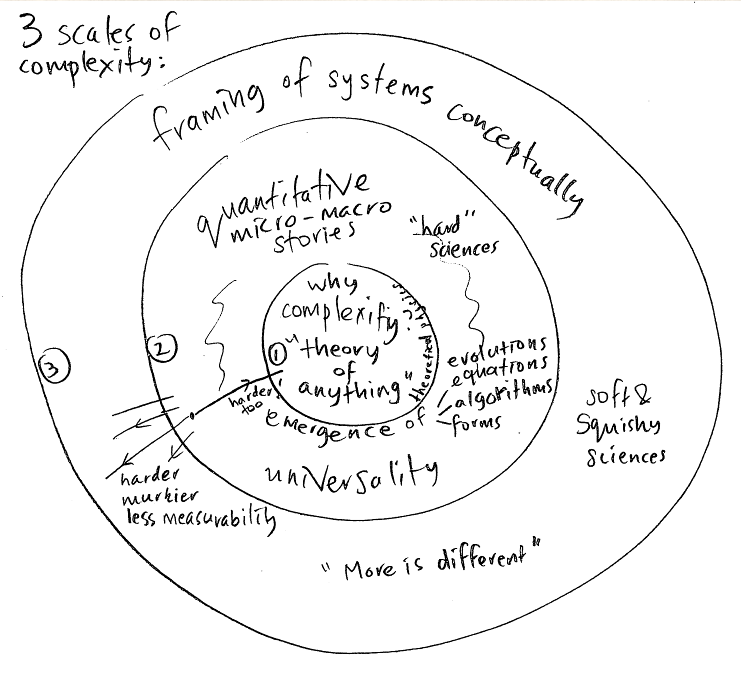
- Emergence—‘More is Different’ [2]


3 scales of complexity:
framing of systems conceptually
quantitative micro-macro stories
"hard" sciences
why complexify?
① "theory of anything"
theoretical physics
②
③
evolutions
equations
algorithms
forms
harder too
emergence of
universality
soft & squishy sciences
harder
murkier
less measurability
"More is different"

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation:

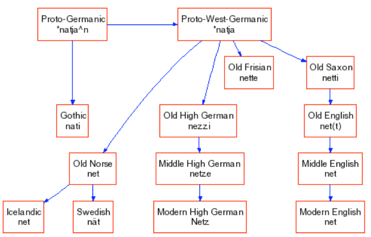
- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

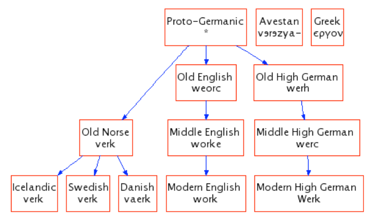
Ancestry:

Net and Work are venerable old words:

- ‘Net’ first used to mean spider web (King Ælfréd, 888).
- ‘Work’ appears to have long meant purposeful action.



The network of Germanic `net' words



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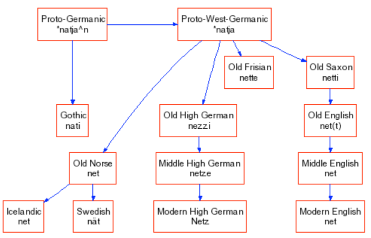

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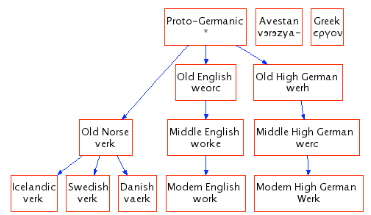

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- 'Network' = something built based on the idea of natural, flexible lattice or web.

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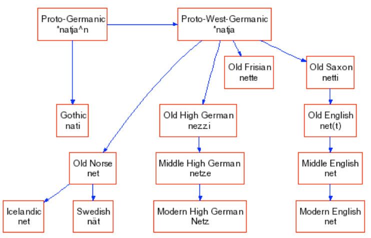

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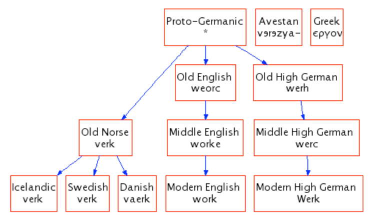

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- c.f., ironwork, stonework, fretwork.

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First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

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- Physical connections $\rightarrow$ Wire-less connections $\rightarrow$ abstract connections

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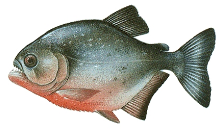

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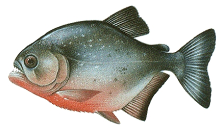

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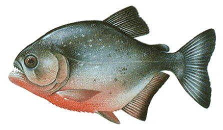


  - *Piranha physicus*
  - Hunt in packs.
  - Feast on new and interesting ideas (see chaos, cellular automata, ...)

Popularity (according to ISI Web of Knowledge)

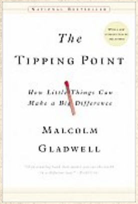
"Collective dynamics of 'small-world' networks" [21]

- Watts and Strogatz
  Nature, 1998
- Cited ≈ 4325 times (as of June 7, 2010)
- Over 1100 citations in 2008.

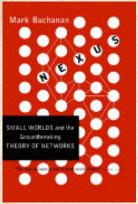
"Emergence of scaling in random networks" [3]

- Barabási and Albert
  Science, 1999
- Cited ≈ 4769 times (as of June 7, 2010)
- Over 1100 citations in 2008.

Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [10]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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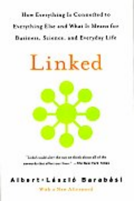
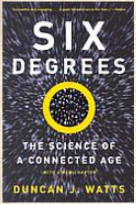


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [20]

Numerous others:

- Complex Social Networks—F. Vega-Redondo [19]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [16]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [6]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [9]

More observations

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**If this is upsetting, maybe string theory is for you...*

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But:

- For scientists, description is only part of the battle.
- We still need to understand.

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Other spiffing words: vertices and edges.

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- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

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- (n.b., for numerical work, we always use sparse matrices.)

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- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks

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- Neural networks

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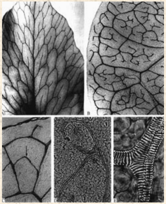
- River networks
- Neural networks
- Trees and leaves



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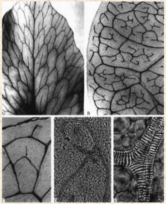

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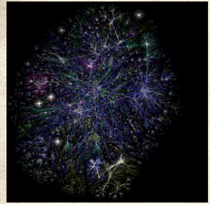

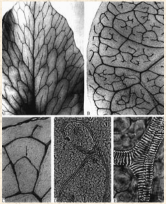

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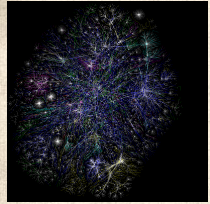

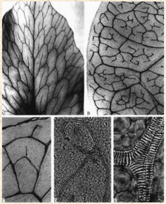

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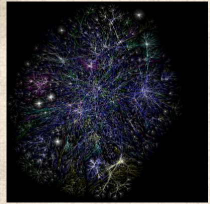

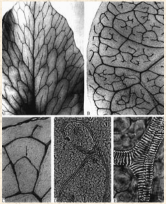

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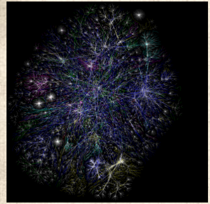

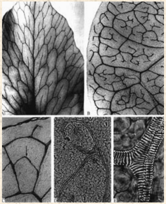

- Distribution (branching) vs. redistribution (cyclical)

Examples

Interaction networks

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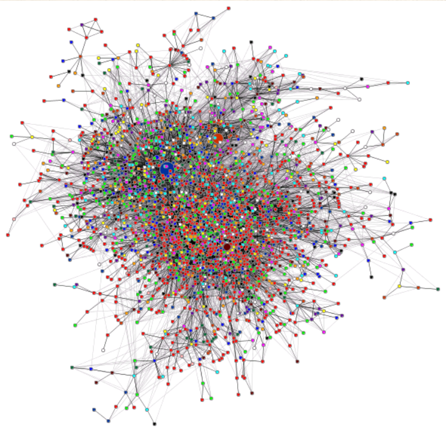

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Interaction networks: social networks

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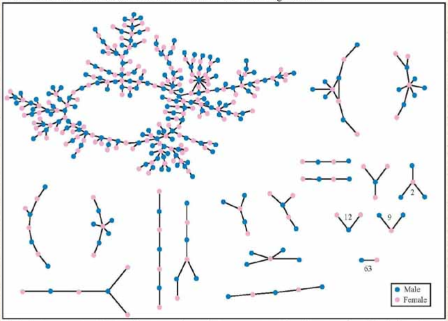
The Structure of Romantic and Sexual Relations at "Jefferson High School"



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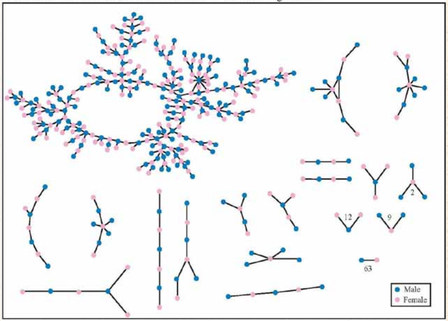

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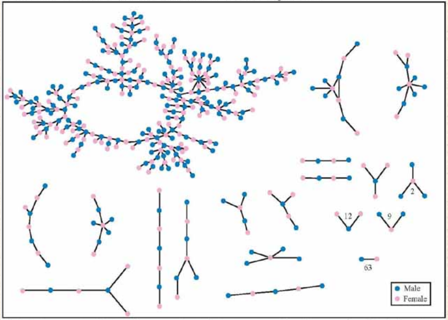

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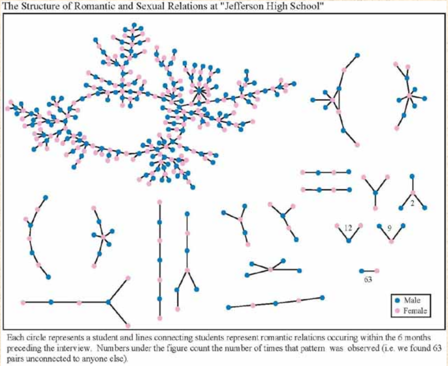

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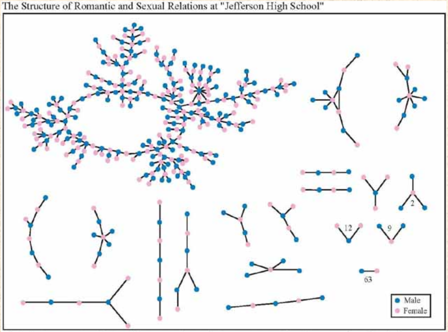

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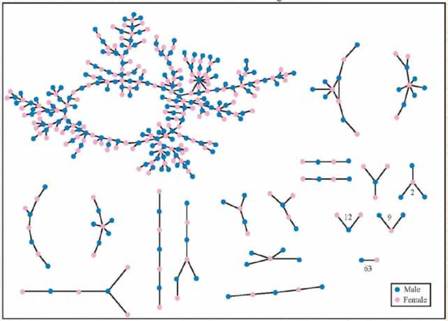

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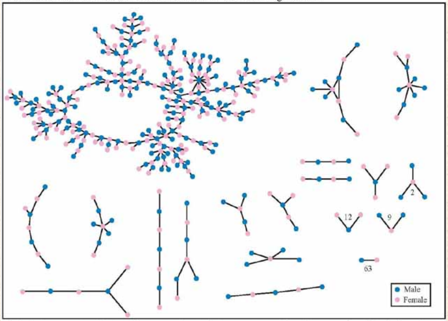

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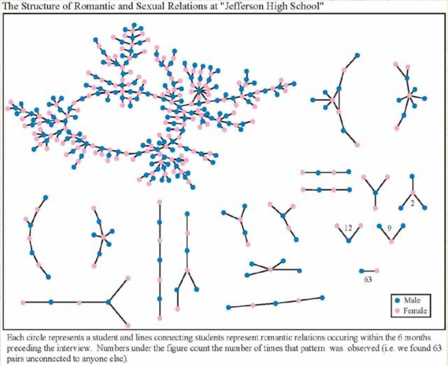

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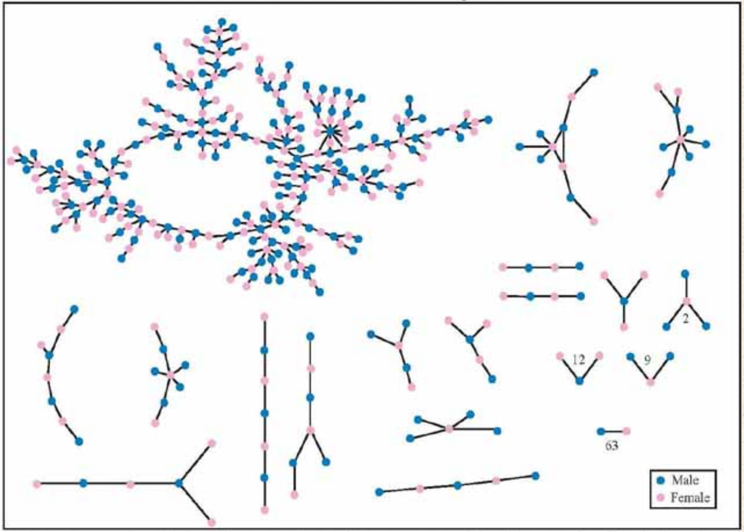

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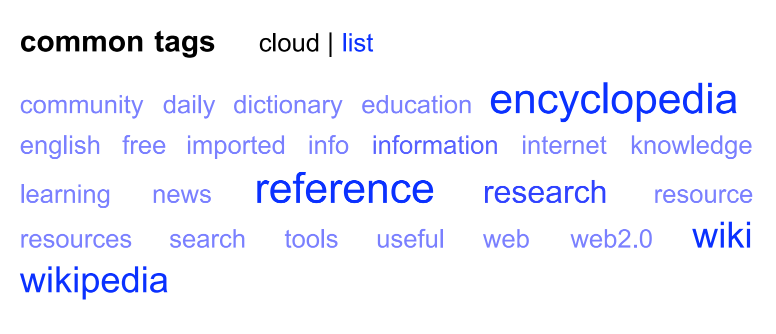
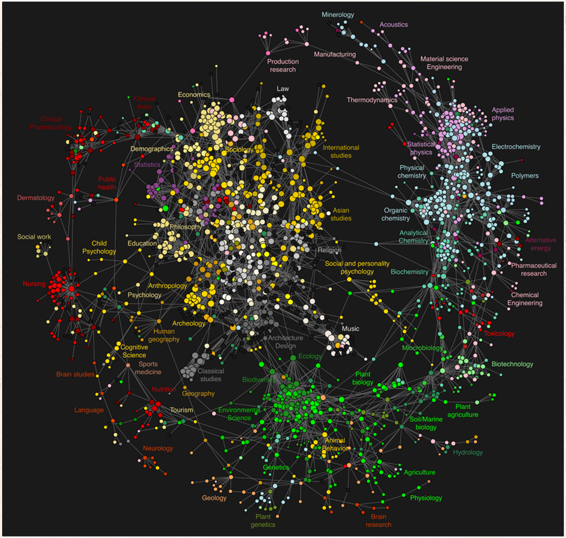
- Metadata—Tagging: del.icio.us, flickr



Clickworthy Science:



Bollen et al. [5]

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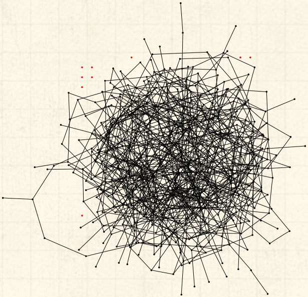

⇐ Typical hairball

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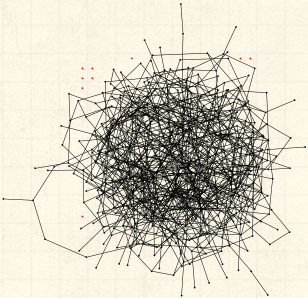

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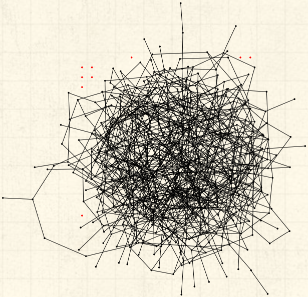

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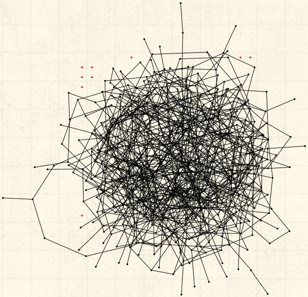

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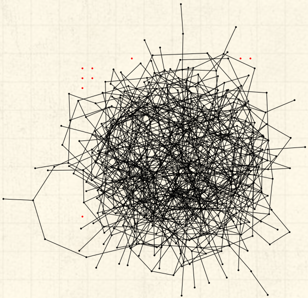

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Properties

Some key features of real complex networks:

- Degree distribution
- Assortativity
- Homophily
- Clustering
- Motifs
- Modularity
- Concurrency
- Hierarchical scaling
- Network distances
- Centrality
- Efficiency
- Robustness

- Coevolution of network structure and processes on networks.

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- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
- We'll come back to this business soon...

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  - *Often techological or biological: Internet, protein interactions, neural networks, food webs.*

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- Interpret C_2 as probability two of a node's friends know each other.

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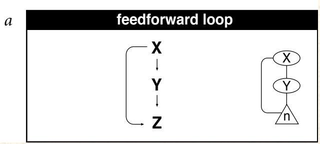
- e.g., Feed Forward Loop:



Shen-Orr, Uri Alon, *et al.* [17]

Properties

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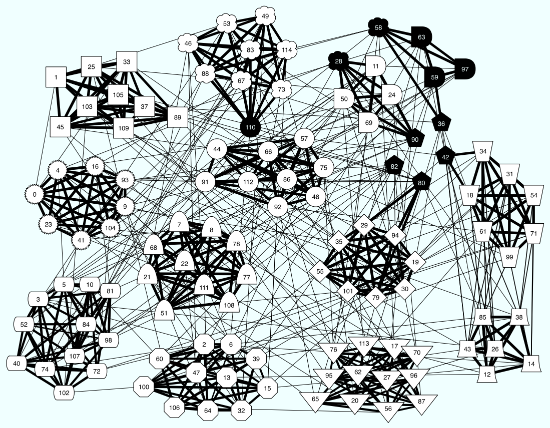

Clauset *et al.*, 2006 [7]: NCAA football

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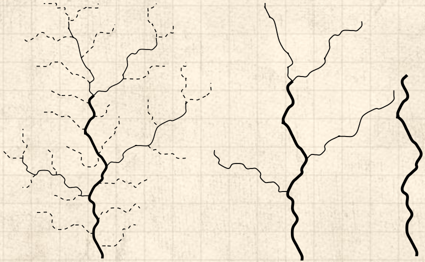
- Metrics for branching networks:



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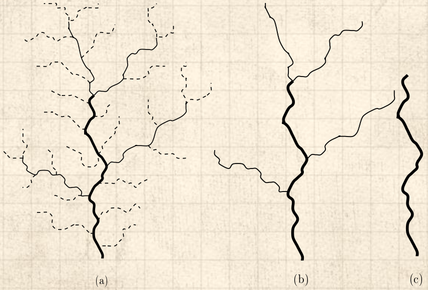

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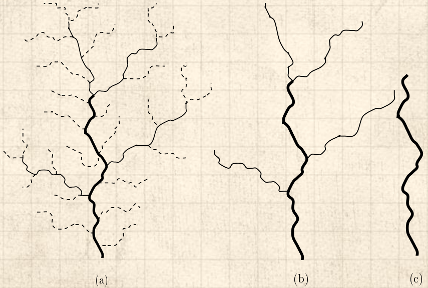

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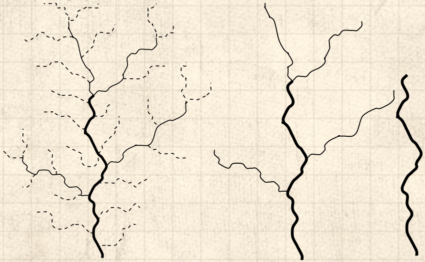

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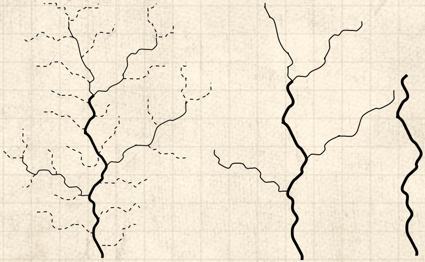

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- Beautifully described but poorly explained.

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- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg [12])

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- Three main (blurred) categories:
  1. Physical (e.g., river networks),
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- Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overall structure,...
- Still much work to be done, especially with respect to dynamics...

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