Random Networks Nutshell

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2023–2024| @pocsvox

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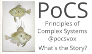

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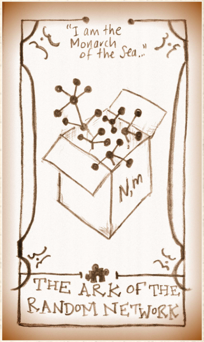
THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TAILS OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
It was the best of times,
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
PHOENIX
BIO HAZARD
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE ORDERS TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END


"I am the Monarch of the Sea.."
N,m
THE ARK OF THE RANDOM NETWORK

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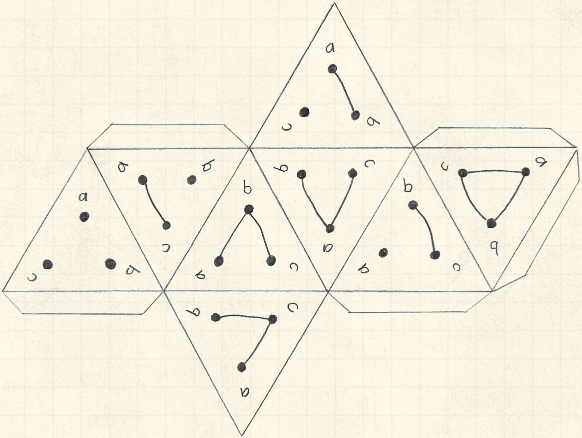
Random network generator for $N = 3$:



- Get your own exciting generator here.
- As $N \nearrow$, polyhedral die rapidly becomes a ball...

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Pure, abstract random networks:

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Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.

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- Standard random network = one randomly chosen network from this set.

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- To be clear: each network is equally probable.

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Pure, abstract random networks:

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- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

Random networks—basic features:

- Number of possible edges:

$$0 \leq m \leq \binom{N}{2} = \frac{N(N-1)}{2}$$

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- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2} N(N-1)}.$$

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- Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.

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- Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.

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- Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.
- Real world: links are usually costly so real networks are almost always sparse.

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Random networks

How to build standard random networks:

- Given N and m.

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Random networks

How to build standard random networks:

- Given N and m.
- Two probablistic methods

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How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)

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How to build standard random networks:

- Given N and m.
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1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.

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How to build standard random networks:

- Given N and m.
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1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.

2. Take N nodes and add exactly m links by selecting edges without replacement.

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   - Useful for theoretical work.
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   - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.

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    - Best for adding relatively small numbers of links (most cases).

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   - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
   - Best for adding relatively small numbers of links (most cases).
   - 1 and 2 are effectively equivalent for large N.

Random networks

A few more things:

- For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2}$$

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A few more things:

- For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

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$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

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$$= \frac{2}{N} p \frac{1}{2} N(N-1) = \frac{\cancel{2}}{\cancel{N}} p \frac{1}{\cancel{2}} \cancel{N}(N-1)$$

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- Which is what it should be...

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- Which is what it should be...
- If we keep $\langle k \rangle$ constant then $p \propto 1/N \to 0$ as $N \to \infty$.

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Random networks: examples

Next slides:

Example realizations of random networks

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Random networks: examples

Next slides:

Example realizations of random networks

- $N = 500$

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Next slides:

Example realizations of random networks

- $N = 500$
- Vary m, the number of edges from 100 to 1000.

Random networks: examples

Next slides:

Example realizations of random networks

- $N = 500$
- Vary m, the number of edges from 100 to 1000.
- Average degree $\langle k \rangle$ runs from 0.4 to 4.

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Random networks: examples

Next slides:

Example realizations of random networks

- $N = 500$
- Vary m, the number of edges from 100 to 1000.
- Average degree $\langle k \rangle$ runs from 0.4 to 4.
- Look at full network plus the largest component.

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Random networks: examples for N=500

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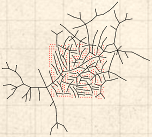
m = 100
$\langle k \rangle$ = 0.4

m = 200
$\langle k \rangle$ = 0.8



m = 230
$\langle k \rangle$ = 0.92

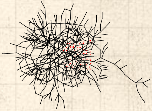
m = 240
$\langle k \rangle$ = 0.96

m = 250
$\langle k \rangle$ = 1

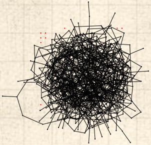
m = 260
$\langle k \rangle$ = 1.04

m = 280
$\langle k \rangle$ = 1.12

m = 300
$\langle k \rangle$ = 1.2

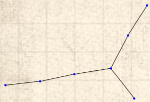


m = 500
$\langle k \rangle$ = 2



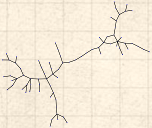
m = 1000
$\langle k \rangle$ = 4

Random networks: largest components

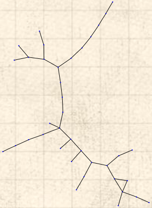


$m = 100$
$\langle k \rangle = 0.4$

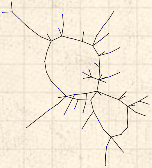
$m = 200$
$\langle k \rangle = 0.8$



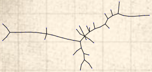
$m = 230$
$\langle k \rangle = 0.92$



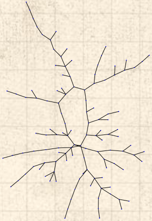
$m = 240$
$\langle k \rangle = 0.96$



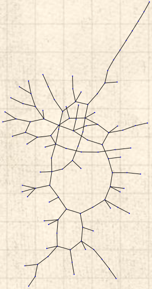
$m = 250$
$\langle k \rangle = 1$



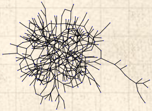
$m = 260$
$\langle k \rangle = 1.04$



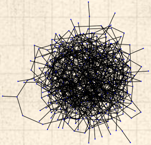
$m = 280$
$\langle k \rangle = 1.12$



$m = 300$
$\langle k \rangle = 1.2$



$m = 500$
$\langle k \rangle = 2$



$m = 1000$
$\langle k \rangle = 4$

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Random networks: examples for N=500

m = 250
$\langle k \rangle$ = 1

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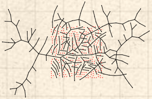
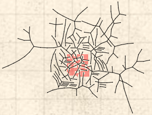
$\langle k \rangle$ = 1



m = 250
$\langle k \rangle$ = 1



m = 250
$\langle k \rangle$ = 1

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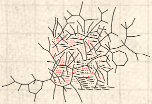
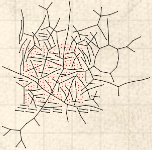


m = 250
$\langle k \rangle$ = 1



m = 250
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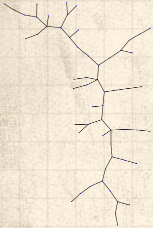
Random networks: largest components



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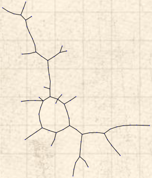
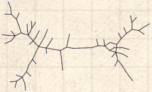
$\langle k \rangle$ = 1



m = 250
$\langle k \rangle$ = 1



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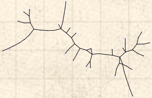

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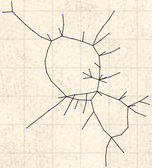

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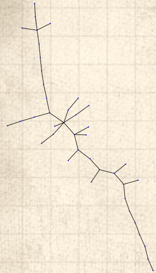

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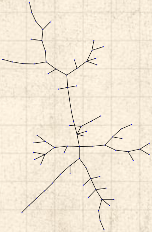

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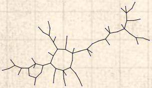

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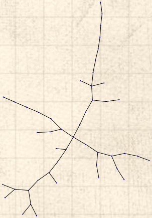

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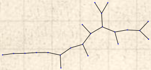

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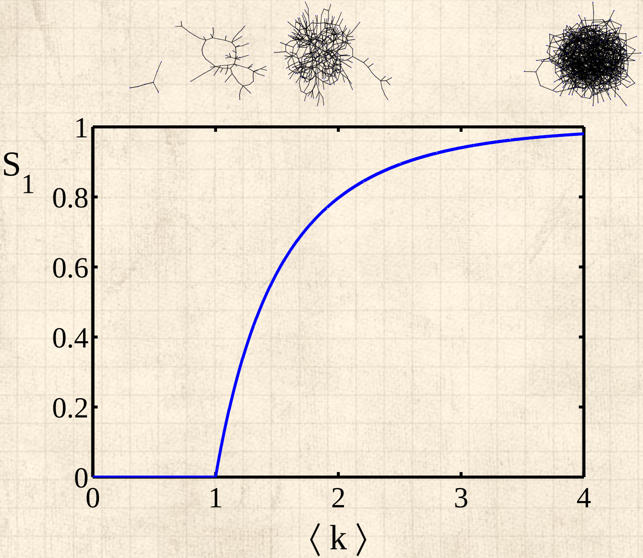

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Clustering in random networks:

- For construction method 1, what is the clustering coefficient for a finite network?

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Clustering in random networks:

- For construction method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient: [6]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

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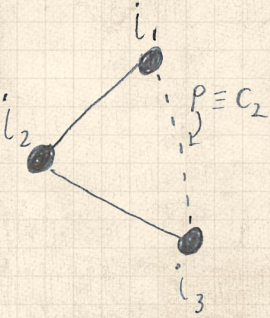

- Recall: C_2 = probability that two friends of a node are also friends.

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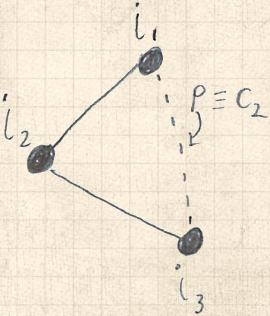

- Recall: C_2 = probability that two friends of a node are also friends.
- Or: C_2 = probability that a triple is part of a triangle.

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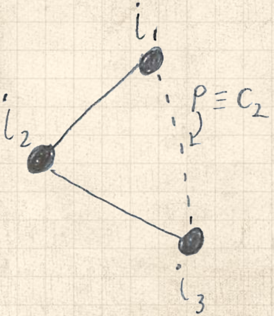

- Recall: C_2 = probability that two friends of a node are also friends.
- Or: C_2 = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p.$$

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Clustering in random networks:

- So for large random networks ($N \to \infty$), clustering drops to zero.

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- So for large random networks ($N \to \infty$), clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks

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Clustering in random networks:

- So for large random networks ($N \rightarrow \infty$), clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks
- No small loops.

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Degree distribution:

- Recall P_k = probability that a randomly selected node has degree k.

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- Consider method 1 for constructing random networks: each possible link is realized with probability p.

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- Each connection occurs with probability p, each non-connection with probability $(1-p)$.
- Therefore have a binomial distribution:

$$P(k;p,N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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Limiting form of $P(k; p, N)$:

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- What happens as $N \to \infty$?

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- What happens as $N \to \infty$?
- We must end up with the normal distribution right?

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- If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \to \infty$.

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- But we want to keep $\langle k \rangle$ fixed...
- So examine limit of $P(k;p,N)$ when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1)$ = constant.

$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k} \to \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

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- This is a Poisson distribution with mean $\langle k \rangle$.

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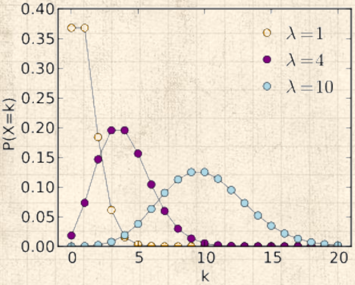
Poisson basics:

$$P(k;\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$



- $\lambda > 0$
- $k = 0, 1, 2, 3, \ldots$
- Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
- e.g.: phone calls/minute, horse-kick deaths.
- 'Law of small numbers'

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Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.

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Poisson basics:

- The variance of degree distributions for random networks turns out to be very important.
- Using calculation similar to one for finding $\langle k \rangle$ we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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- Variance is then

$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$$

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$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle .$$

- So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

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$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

- Variance is then

$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle.$$

- So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.
- Note: This is a special property of Poisson distribution and can trip us up...

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General random networks

- So... standard random networks have a Poisson degree distribution

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- Generalize to arbitrary degree distribution P_k.

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- So... standard random networks have a Poisson degree distribution
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$$P(\text{link between } i \text{ and } j) \propto w_i w_j.$$

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- But we'll be more interested in
  1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
  2. Examining mechanisms that lead to networks with certain degree distributions.

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Coming up:

Example realizations of random networks with power law degree distributions:

Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

- $N = 1000$.

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Random networks: examples

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- $P_k \propto k^{-\gamma}$ for $k \geq 1$.

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- $N = 1000$.
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- Vary exponent γ between 2.10 and 2.91.

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- Vary exponent γ between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

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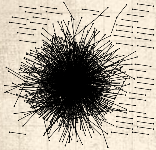
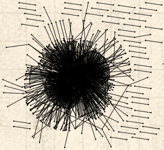
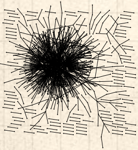
Random networks: examples for N=1000



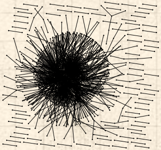
γ = 2.1
$\langle k \rangle$ = 3.448



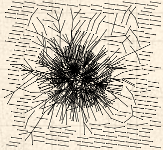
γ = 2.19
$\langle k \rangle$ = 2.986



γ = 2.28
$\langle k \rangle$ = 2.306

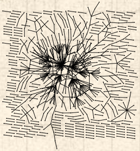


γ = 2.37
$\langle k \rangle$ = 2.504

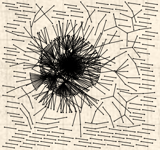


γ = 2.46
$\langle k \rangle$ = 1.856

γ = 2.55
$\langle k \rangle$ = 1.712



γ = 2.64
$\langle k \rangle$ = 1.6

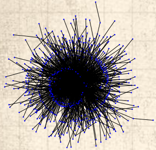


γ = 2.73
$\langle k \rangle$ = 1.862

γ = 2.82
$\langle k \rangle$ = 1.386

γ = 2.91
$\langle k \rangle$ = 1.49

Random networks: largest components



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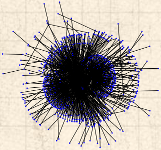

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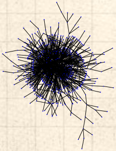

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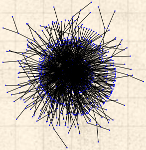

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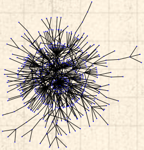
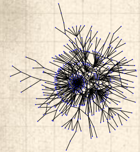


γ = 2.46
$\langle k \rangle$ = 1.856



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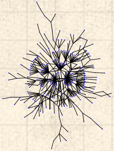

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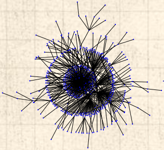

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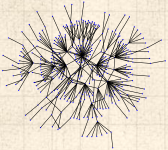

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Models

Generalized random networks:

- Arbitrary degree distribution P_k.

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- Arbitrary degree distribution P_k.
- Create (unconnected) nodes with degrees sampled from P_k.

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Models

Generalized random networks:

- Arbitrary degree distribution P_k.
- Create (unconnected) nodes with degrees sampled from P_k.
- Wire nodes together randomly.

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Generalized random networks:

- Arbitrary degree distribution P_k.
- Create (unconnected) nodes with degrees sampled from P_k.
- Wire nodes together randomly.
- Create ensemble to test deviations from randomness.

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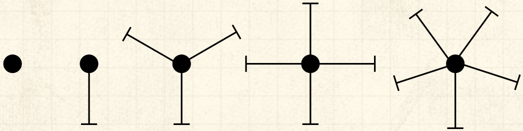
Building random networks: Stubs

Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):



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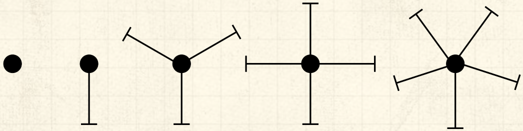

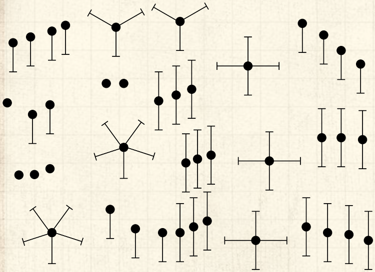

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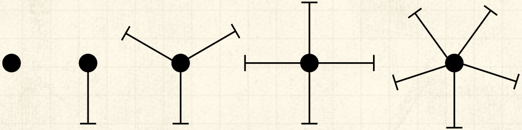

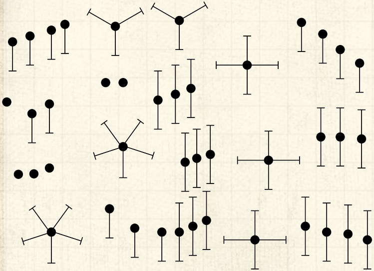

- Randomly select stubs (not nodes!) and connect them.

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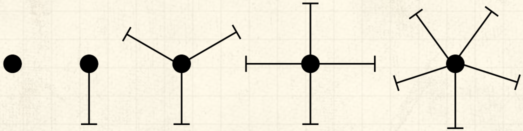

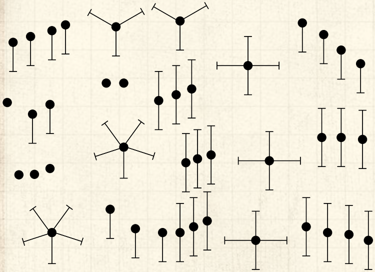

- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.

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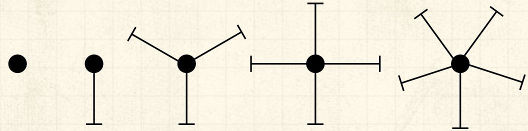

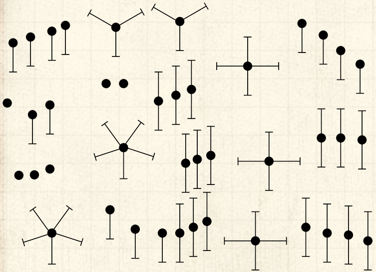

- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.
- Initially allow self- and repeat connections.

Building random networks: First rewiring

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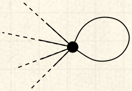
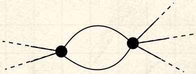
Phase 2:

- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.

(A) 

(B) 

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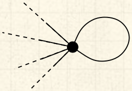
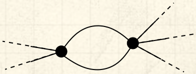
(A)  (B) 

- Being careful: we can't change the degree of any node, so we can't simply move links around.

Building random networks: First rewiring

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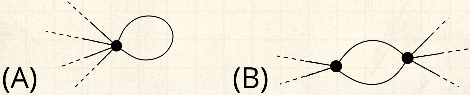
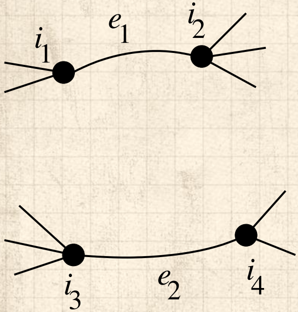


- Being careful: we can't change the degree of any node, so we can't simply move links around.
- Simplest solution: randomly rewire two edges at a time.

General random rewiring algorithm



- Randomly choose two edges. (Or choose problem edge and a random edge)

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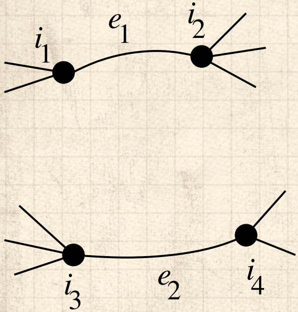

- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.

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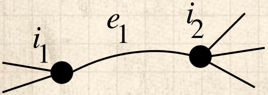

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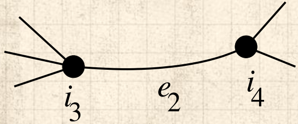

- Rewire one end of each edge.

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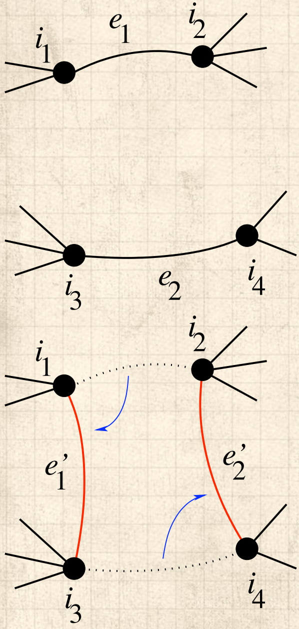
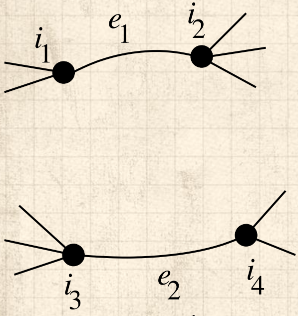
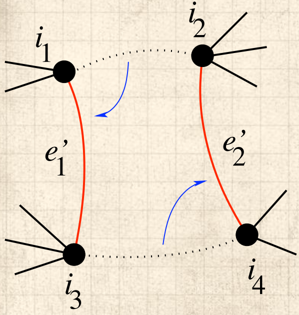
General random rewiring algorithm



- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.
- Rewire one end of each edge.
- Node degrees do not change.

General random rewiring algorithm





- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.

- Rewire one end of each edge.
- Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.

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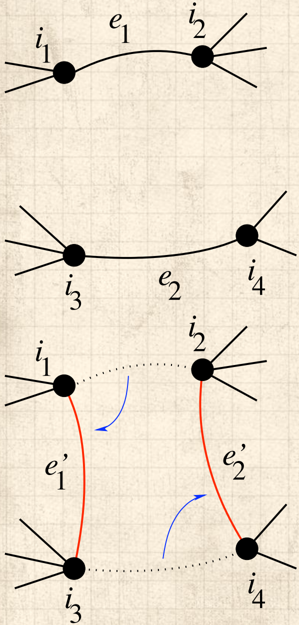

- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.

- Rewire one end of each edge.
- Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.
- Same as finding on/off/on/off 4-cycles. and rotating them.

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Sampling random networks

Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

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Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

- Randomize network wiring by applying rewiring algorithm liberally.

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Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

- Randomize network wiring by applying rewiring algorithm liberally.
- Rule of thumb: # Rewirings $\simeq 10 \times$ # edges [4].

Random sampling

- Problem with only joining up stubs is failure to randomly sample from all possible networks.

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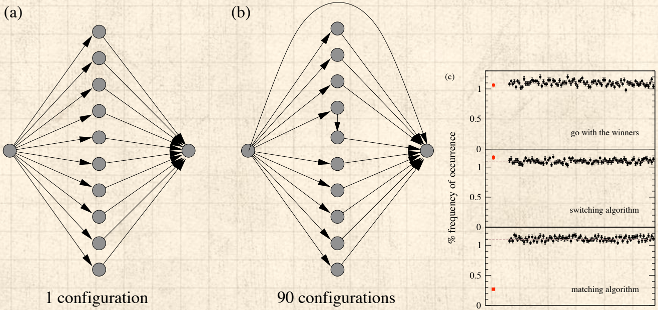
Random sampling

- **Problem** with only joining up stubs is **failure** to randomly sample from all possible networks.
- Example from Milo et al. (2003) [4]:



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Sampling random networks

- What if we have P_k instead of N_k?

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- What if we have P_k instead of N_k?
- Must now create nodes before start of the construction algorithm.

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Sampling random networks

- What if we have P_k instead of N_k?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k.
- Easy to do exactly numerically since k is discrete.

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- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k.
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- Note: not all P_k will always give nodes that can be wired together.

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Network motifs

- Idea of motifs [7] introduced by Shen-Orr, Alon et al. in 2002.

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- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k.
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

Network motifs

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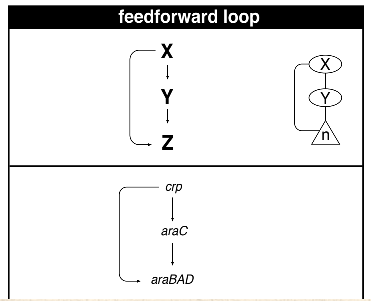

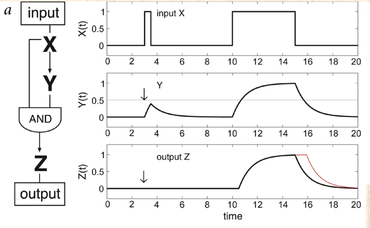

- Z only turns on in response to sustained activity in X.

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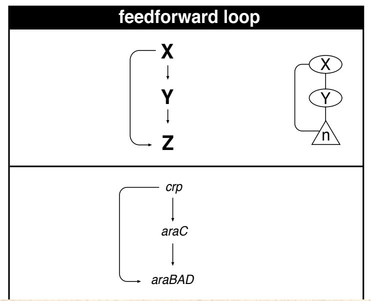
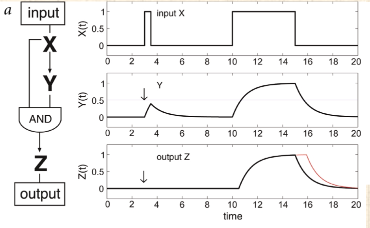
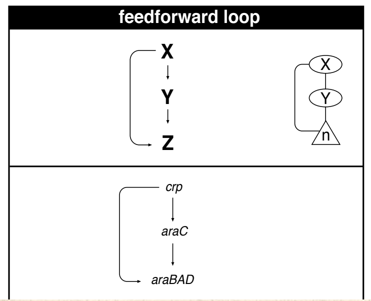
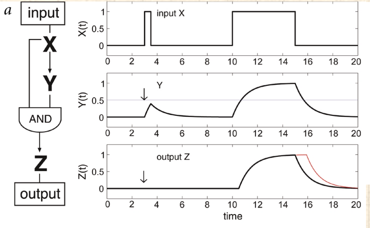




- Z only turns on in response to sustained activity in X.
- Turning off X rapidly turns off Z.

Network motifs





- Z only turns on in response to sustained activity in X.
- Turning off X rapidly turns off Z.
- Analogy to elevator doors.

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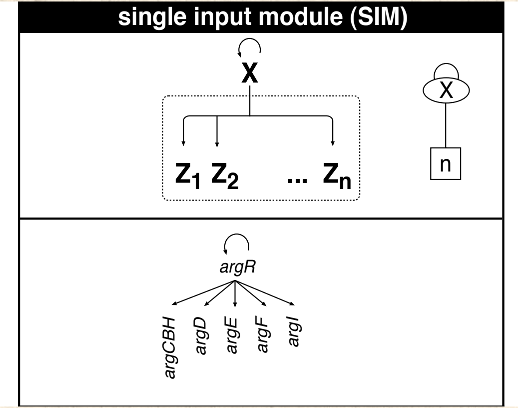
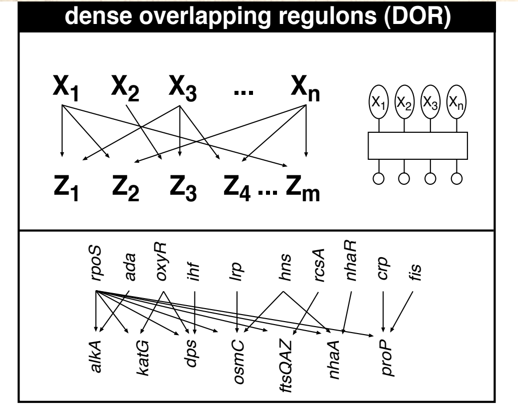


- Master switch.

Network motifs



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- Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

Network motifs

- Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
- For more, see work carried out by Wiggins *et al.* at Columbia.

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The edge-degree distribution:

- The degree distribution P_k is fundamental for our description of many complex networks

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- Now choosing nodes based on their degree (i.e., size):

$$\boxed{Q_k \propto k P_k}$$

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- Normalized form:

$$Q_k = \frac{k P_k}{\sum_{k'=0}^{\infty} k' P_{k'}}$$

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- Big deal: Rich-get-richer mechanism is built into this selection process.

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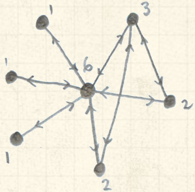

- Probability of randomly selecting a node of degree k by choosing from nodes:
$P_1 = 3/7$, $P_2 = 2/7$, $P_3 = 1/7$, $P_6 = 1/7$.

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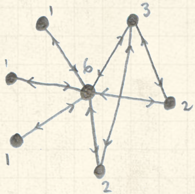

- Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7$, $P_2 = 2/7$, $P_3 = 1/7$, $P_6 = 1/7$.
- Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16$, $Q_2 = 4/16$, $Q_3 = 3/16$, $Q_6 = 6/16$.

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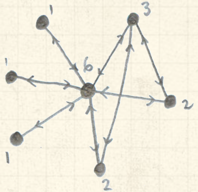

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- Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:
  $Q_1 = 3/16$, $Q_2 = 4/16$, $Q_3 = 3/16$, $Q_6 = 6/16$.
- Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:
  $R_0 = 3/16$ $R_1 = 4/16$, $R_2 = 3/16$, $R_5 = 6/16$.

The edge-degree distribution:

- For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.

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- For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
- Useful variant on Q_k:

  R_k = probability that a friend of a random node has k other friends.

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- $$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

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- Equivalent to friend having degree $k+1$.

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- Equivalent to friend having degree $k+1$.
- Natural question: what's the expected number of other friends that one friend has?

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The edge-degree distribution:

- Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\langle k \rangle_R = \sum_{k=0}^{\infty} k R_k$$

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$$\langle k \rangle_R = \sum_{k=0}^{\infty} k R_k = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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- Again, neatness of results is a special property of the Poisson distribution.
- So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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- #samesies.

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Two reasons why this matters

Reason #1:

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- Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

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  1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.

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  4. See also: class size paradoxes (nod to: Gelman)

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More on peculiarity #3:

- A node's average # of friends: $\langle k \rangle$

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$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

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- So only if everyone has the same degree (variance$= \sigma^2 = 0$) can a node be the same as its friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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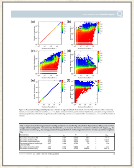

"Generalized friendship paradox in complex networks: The case of scientific collaboration"
Eom and Jo,
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Your friends really are ~~monsters~~ #winners:[1]

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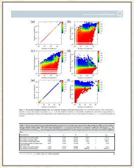

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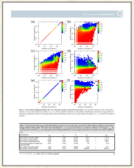

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- Go on, hurt me: Friends have more coauthors, citations, and publications.
- Other horrific studies: your connections on Twitter have more followers than you, your sexual partners more partners than you, ...

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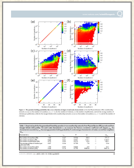

"Generalized friendship paradox in complex networks: The case of scientific collaboration"
Eom and Jo,
Nature Scientific Reports, **4**, 4603, 2014. [2]

Your friends really are ~~monsters~~ #winners:[1]

- **Go on, hurt me:** Friends have more coauthors, citations, and publications.
- **Other horrific studies:** your connections on Twitter have more followers than you, your sexual partners more partners than you, ...
- **The hope:** Maybe they have more enemies and diseases too.

[1] Some press here [MIT Tech Review].

Two reasons why this matters

(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.

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- Note: Component = Cluster

Outline

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- Definitions
- How to build theoretically
- Some visual examples
- Clustering
- Degree distributions

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- Configuration model
- How to build in practice
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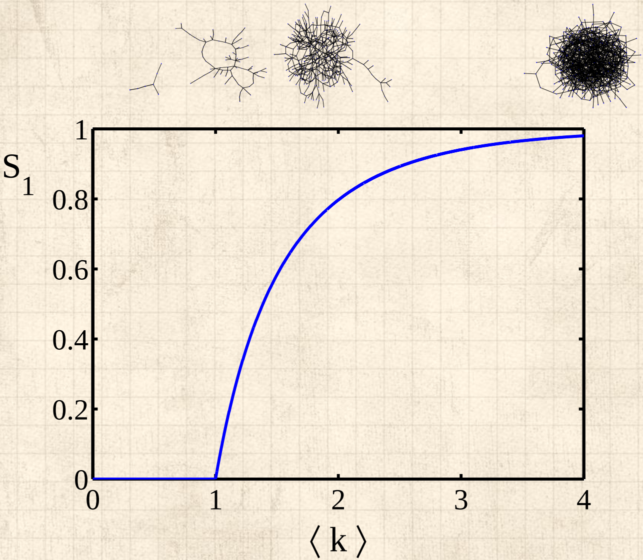
Giant component



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Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

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$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

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- Again, see that the second moment is an essential part of the story.
- Equivalent statement: $\langle k^2 \rangle > 2\langle k \rangle$

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Spreading on Random Networks

- For random networks, we know local structure is pure branching.

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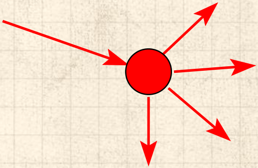
Success



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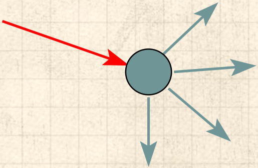

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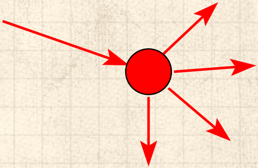

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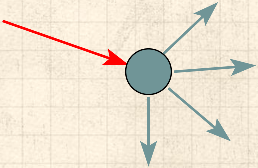

- Focus on binary case with edges and nodes either infected or not.

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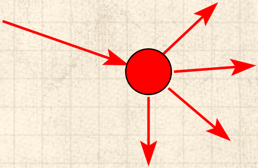

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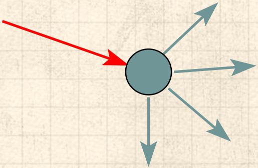

- Focus on binary case with edges and nodes either infected or not.
- First big question: for a given network and contagion process, can global spreading from a single seed occur?

Global spreading condition

- We need to find: [1]
  R = the average # of infected edges that one random infected edge brings about.
- Call **R** the gain ratio.

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$$\mathbf{R} = \sum_{k=0}^{\infty} \underbrace{\frac{kP_k}{\langle k \rangle}}_{\text{prob. of connecting to a degree } k \text{ node}}$$

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$$+ \sum_{k=0}^{\infty} \overbrace{\frac{kP_k}{\langle k \rangle}}$$

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Global spreading condition

- Our global spreading condition is then:

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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

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- Good: This is just our giant component condition again.

Global spreading condition

- Case 2—Simple disease-like:

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Global spreading condition

- Case 2—Simple disease-like: If $B_{k1} = \beta < 1$

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- A fraction (1-β) of edges do not transmit infection.

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- A fraction (1-β) of edges do not transmit infection.
- Analogous phase transition to giant component case but critical value of $\langle k \rangle$ is increased.

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- A fraction (1-β) of edges do not transmit infection.
- Analogous phase transition to giant component case but critical value of $\langle k \rangle$ is increased.
- Aka bond percolation.
- Resulting degree distribution $\tilde{P}_k$:

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

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Giant component for standard random networks:

- Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

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Giant component for standard random networks:

- Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.
- Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

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- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

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- Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.
- Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.
- When $\langle k \rangle < 1$, all components are finite.

Giant component for standard random networks:

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- Fine example of a continuous phase transition.

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- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.
- When $\langle k \rangle < 1$, all components are finite.
- Fine example of a continuous phase transition.
- We say $\langle k \rangle = 1$ marks the critical point of the system.

Random networks with skewed P_k:

- e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \geq 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

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- So giant component always exists for these kinds of networks.

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- So giant component **always exists** for these kinds of networks.
- Cutoff scaling is k^{-3}: if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

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- So giant component always exists for these kinds of networks.
- Cutoff scaling is k^{-3}: if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.
- How about $P_k = \delta_{k k_0}$?

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Giant component

And how big is the largest component?

- Define S_1 as the size of the largest component.

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And how big is the largest component?

- Define S_1 as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k \rangle$.

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And how big is the largest component?

- Define S_1 as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k \rangle$.
- Let's find S_1 with a back-of-the-envelope argument.

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- Let's find S_1 with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 - S_1$.

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- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

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- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.
- So

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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- Substitute in Poisson distribution...

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- Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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Giant component

- Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

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$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta}$$

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$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1-\delta)}.$$

- Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}.$$

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Giant component

- We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.

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Giant component

- We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.
- First, we can write $\langle k \rangle$ in terms of S_1:

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

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- As $\langle k \rangle \to 0$, $S_1 \to 0$.

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- As $\langle k \rangle \to 0$, $S_1 \to 0$.
- As $\langle k \rangle \to \infty$, $S_1 \to 1$.
- Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.

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- Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.
- Only solvable for $S_1 > 0$ when $\langle k \rangle > 1$.

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- Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.
- Only solvable for $S_1 > 0$ when $\langle k \rangle > 1$.
- Really a transcritical bifurcation. [8]

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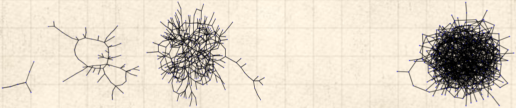

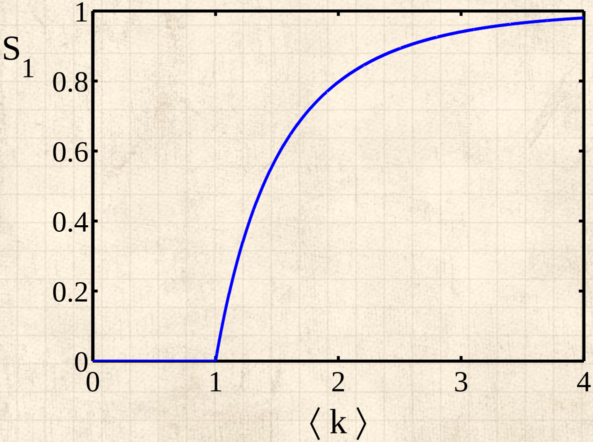

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Turns out we were lucky...

- Our dirty trick only works for ER random networks.

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Turns out we were lucky...

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- But we know our friends are different from us...

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- Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.

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- We can sort many things out with sensible probabilistic arguments...

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- We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a spot of Generatingfunctionology. [9]

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