Mechanisms for Generating Power-Law Size Distributions, Part 3

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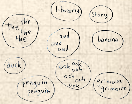
Principles of Complex Systems, Vols. 1, 2, & 3D CSYS/MATH 6701, 6713, & a pretend number, 2023–2024 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont



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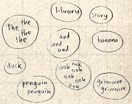

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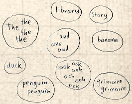
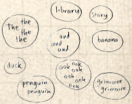


Outline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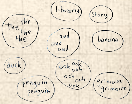
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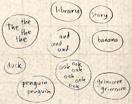


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF BEGATING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
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THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
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THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
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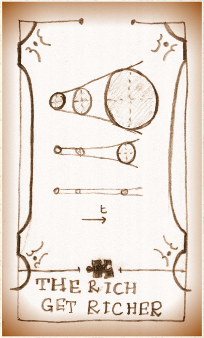
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END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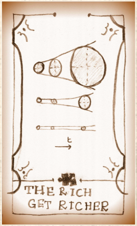
THE RICH GET RICHER

Aggregation:



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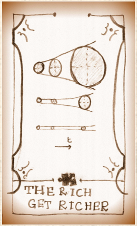

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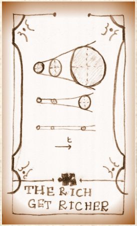

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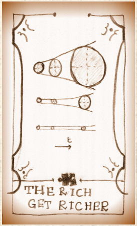

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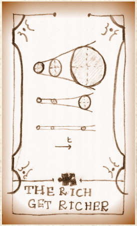

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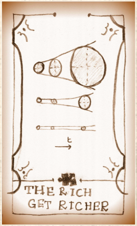
- Competing mechanisms (trickiness)



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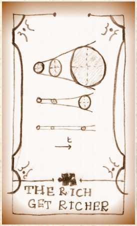

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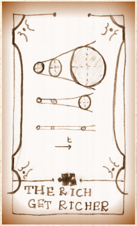

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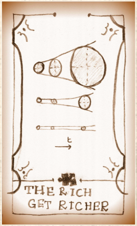

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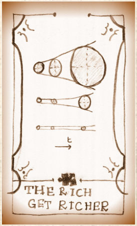

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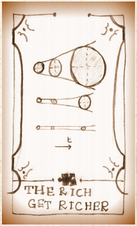

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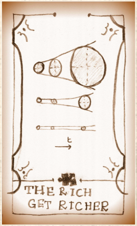

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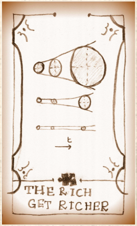

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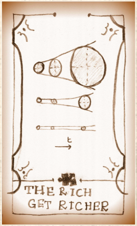

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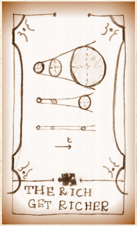
- 1999: Barabasi and Albert [2]:
  The World Wide Web, networks-at-large.



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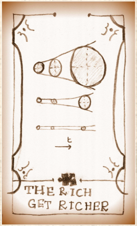

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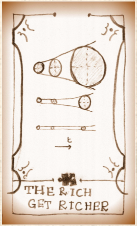

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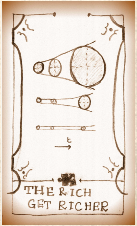

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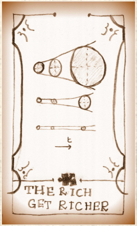

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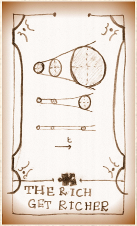

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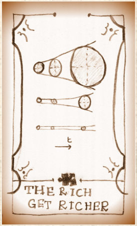
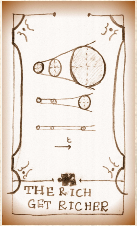
- 1978 Nobel Laureate in Economics (his Nobel bio is here).



Essential Extract of a Growth Model:



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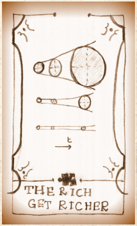

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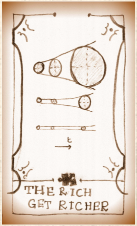

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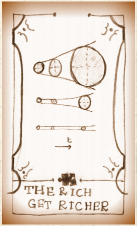

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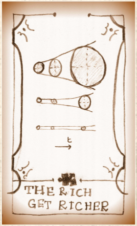

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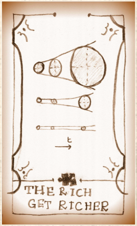

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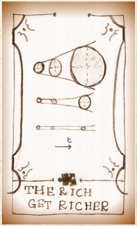

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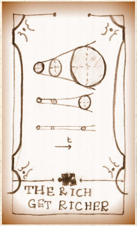

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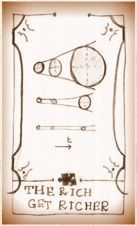

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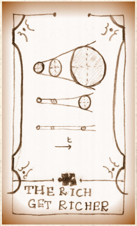

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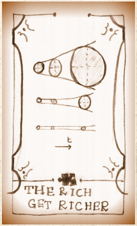

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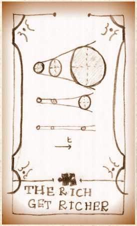

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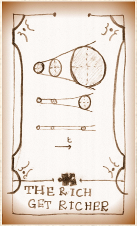
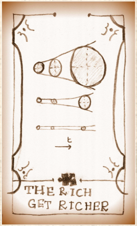
Note: This is a terrible way to write a novel.



For example:

- 21 words used
- next word is new with prob ρ
- next word is a copy with prob $1-\rho$

prob:	next word:
6/21	ook
4/21	the
3/21	and
2/21	penguin
⋮	
1/21	library



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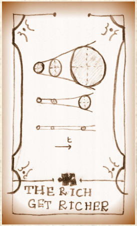

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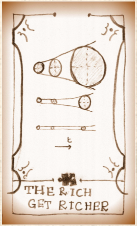

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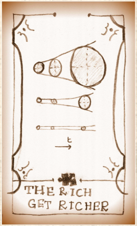

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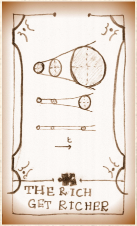

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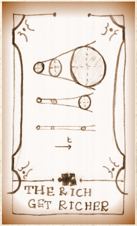

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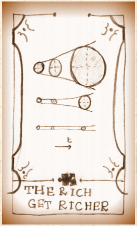

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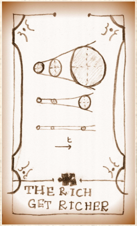
Your free set of tofu knives:



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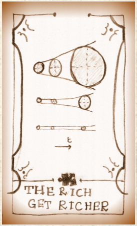
- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.



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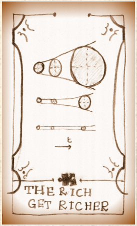
Your free set of tofu knives:

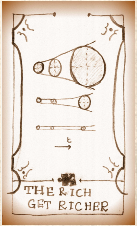
- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.



Random Competitive Replication:

Some observations:

- Steady growth of system: +1 elephant per unit time.



Random Competitive Replication:

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- Steady growth of system: +1 elephant per unit time.
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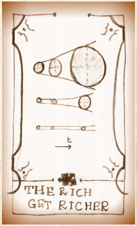

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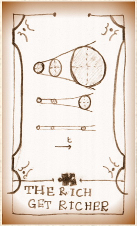

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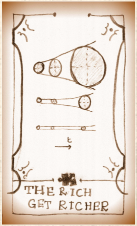

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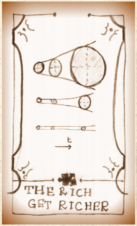

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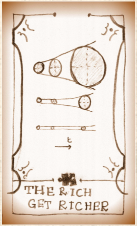
  3. Variable innovation rate ρ



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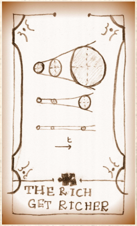

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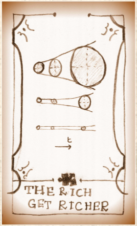
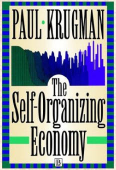
  2. Elephants moving between groups
  3. Variable innovation rate ρ
  4. Different selection based on group size
     (But mechanism for selection is not as simple...)





"The Self-Organizing Economy"
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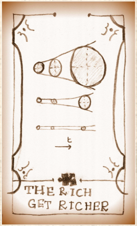

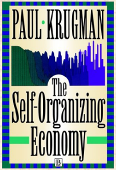

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Ch. 3: An Urban Mystery, p. 46

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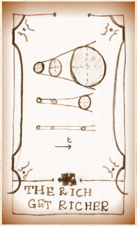

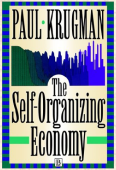

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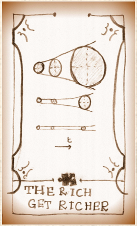
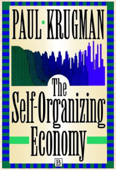


[1] Krugman's book was handed to the Deliverator by a certain Álvaro Cartea many years ago at the Santa Fe Institute Summer School.



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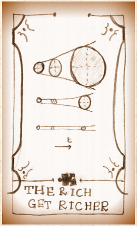
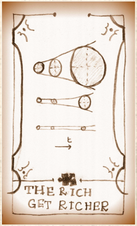


[1] Krugman's book was handed to the Deliverator by a certain Álvaro Cartea many years ago at the Santa Fe Institute Summer School.

[2] Let's use π for probability because π's not special, right guys?

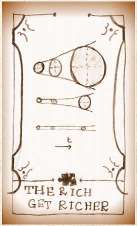
Outline



Random Competitive Replication:

Definitions:

- k_i = size of a group i



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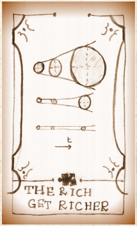
- k_i = size of a group i
- $N_{k,t}$ = # groups containing k elephants at time t.



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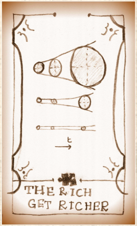
Basic question: How does $N_{k,t}$ evolve with time?



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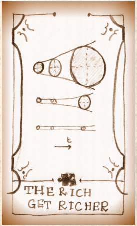
Basic question: How does $N_{k,t}$ evolve with time?

First: $\sum_k k N_{k,t} = t =$ number of elephants at time t



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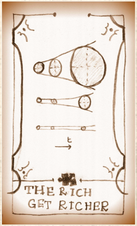
$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:



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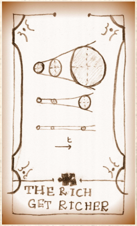

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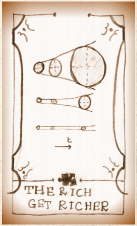

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$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

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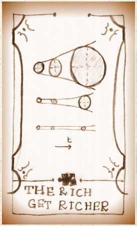
- t elephants overall



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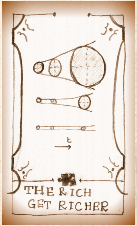
$$P_k(t) = \frac{kN_{k,t}}{t}.$$



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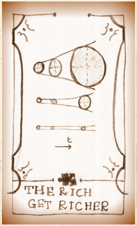
$N_{k,t}$, the number of groups with k elephants, changes at time t if



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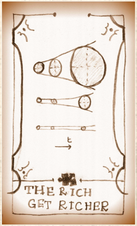

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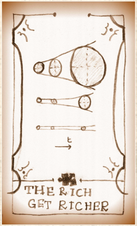

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$N_{k,t}$, the number of groups with k elephants, changes at time t if

1. An elephant belonging to a group with k elephants is replicated:
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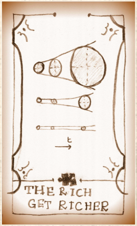

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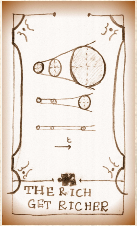

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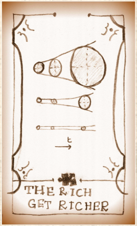

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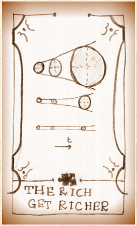
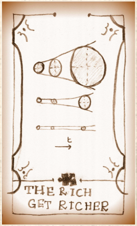


Random Competitive Replication:

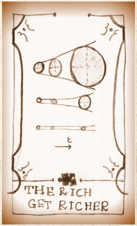
Special case for $N_{1,t}$:



Random Competitive Replication:

Special case for $N_{1,t}$:

1. The new elephant is a new flavor:

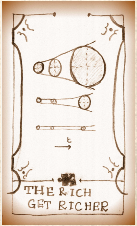


Random Competitive Replication:

Special case for $N_{1,t}$:

1. The new elephant is a new flavor:

2. A unique elephant is replicated:



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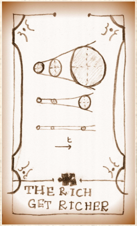

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2. A unique elephant is replicated:

Random Competitive Replication:

Special case for $N_{1,t}$:

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   Happens with probability ρ
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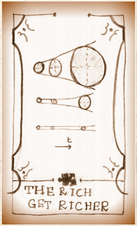

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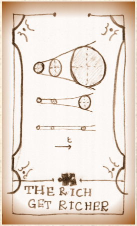

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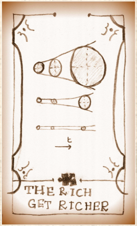


Special case for $N_{1,t}$:

1. The new elephant is a new flavor:
   $N_{1,t+1} = N_{1,t} + 1$
   Happens with probability ρ

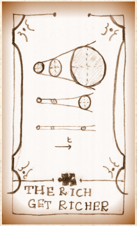
2. A unique elephant is replicated:
   $N_{1,t+1} = N_{1,t} - 1$
   Happens with probability $(1-\rho)N_{1,t}/t$

Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t} \right)$$



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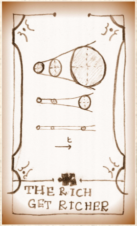

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Assume distribution stabilizes: $N_{k,t} = n_k t$

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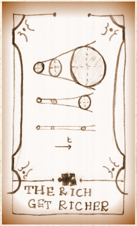

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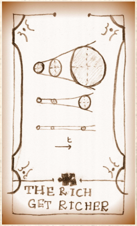

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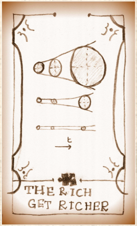

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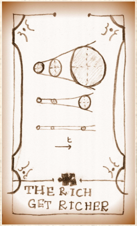

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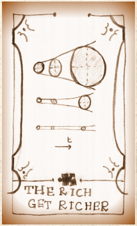
- For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k \cancel{t}}{\rho \cancel{t}} = \frac{n_k}{\rho}.$$



Random Competitive Replication:

Stochastic difference equation:

$$\langle N_{k,t+1} - N_{k,t} \rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t}\right)$$

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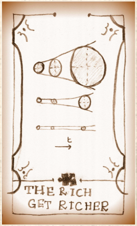

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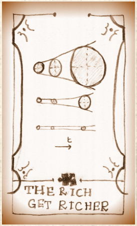

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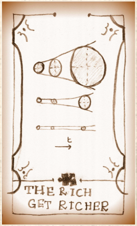

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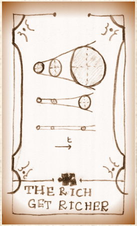
$$\Rightarrow n_k\left(1 + (1-\rho)k\right) = (1-\rho)(k-1)n_{k-1}$$



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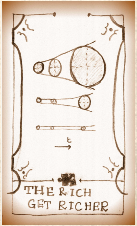

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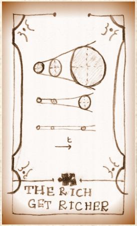

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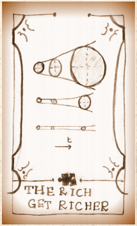

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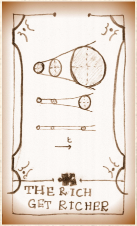

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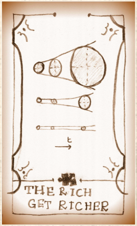

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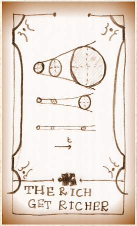

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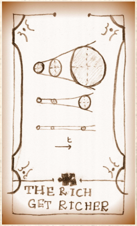
  We (okay, you) find

$$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$



Micro-to-Macro story with ρ and γ measurable.

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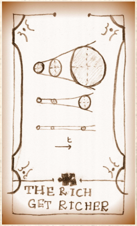

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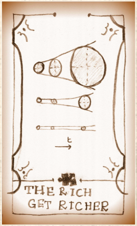

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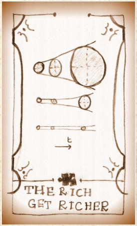

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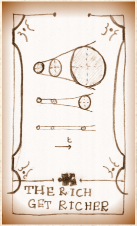

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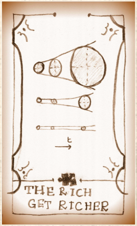

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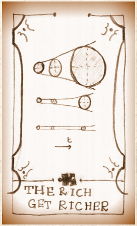

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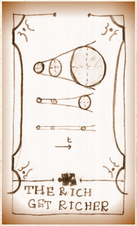
- Upshot: Tunable mechanism producing a family of universality classes.



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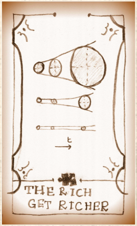

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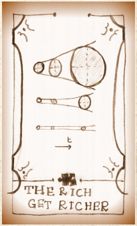

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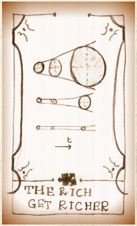

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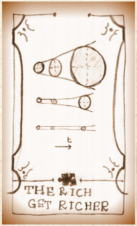

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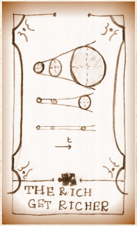

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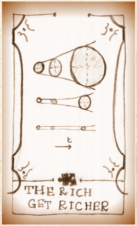

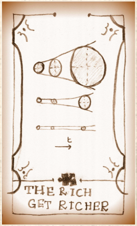

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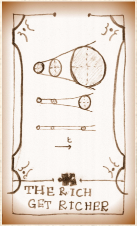

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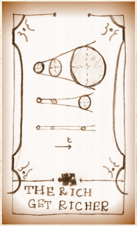

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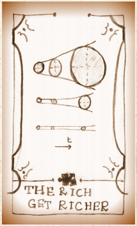

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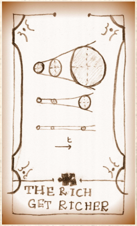
- Rearrange:
$$n_1 + (1-\rho) n_1 = \rho$$



What about small k?:

We had one other equation:

$$\langle N_{1,t+1} - N_{1,t} \rangle = \rho - (1-\rho) 1 \cdot \frac{N_{1,t}}{t}$$

- As before, set $N_{1,t} = n_1 t$ and drop expectations

$$n_1(t+1) - n_1 t = \rho - (1-\rho) 1 \cdot \frac{n_1 t}{t}$$

$$n_1 = \rho - (1-\rho) n_1$$

- Rearrange:

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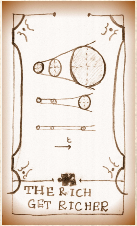
$$\boxed{n_1 = \frac{\rho}{2-\rho}}$$



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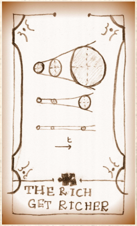
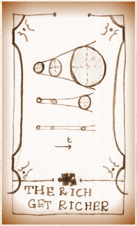
$$N_{1,t} = n_1 t = \frac{\rho t}{2-\rho}$$



So... $N_{1,t} = n_1 t = \frac{\rho t}{2-\rho}$

- Recall number of distinct elephants = ρt.

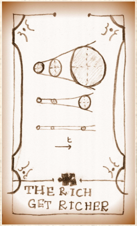


$$\text{So...} \qquad N_{1,t} = n_1 t = \frac{\rho t}{2 - \rho}$$

- Recall number of distinct elephants = ρt.
- Fraction of distinct elephants that are unique (belong to groups of size 1):

$$\frac{1}{\rho t} N_{1,t} = \frac{1}{\cancel{\rho t}} \frac{\cancel{\rho t}}{2 - \rho} = \frac{1}{2 - \rho}$$

(also = fraction of groups of size 1)



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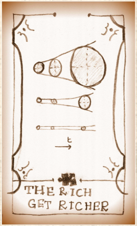

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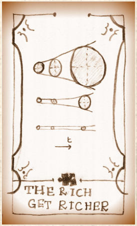

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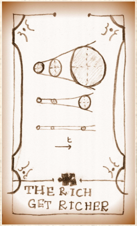

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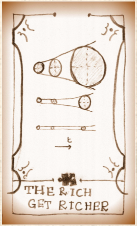
- ρ increases, fraction increases
- Can show fraction of groups with two elephants $\sim 1/6$



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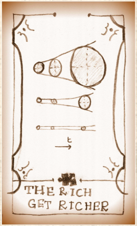

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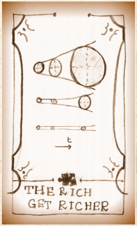
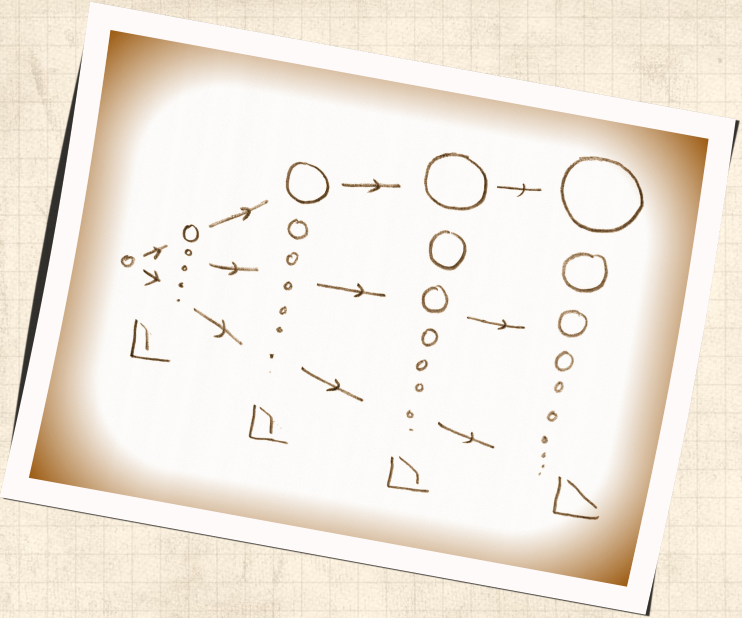
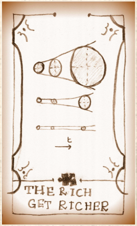
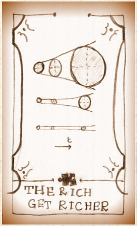





THE RICH
GET RICHER

Outline



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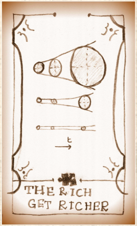

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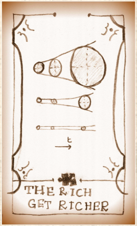

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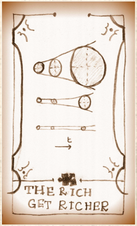
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870



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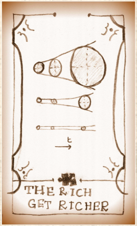

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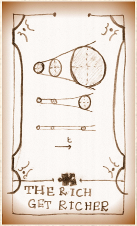

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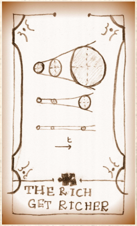

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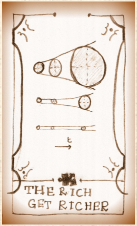

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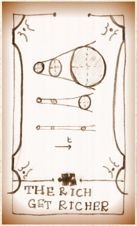

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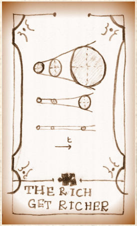

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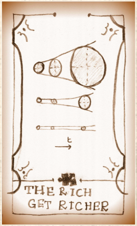

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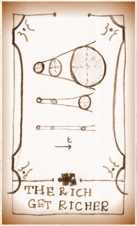

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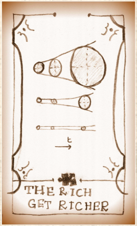

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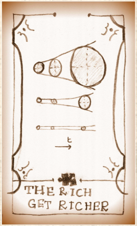

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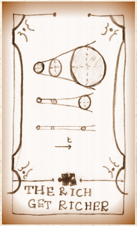

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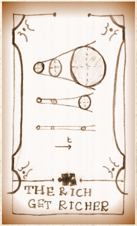

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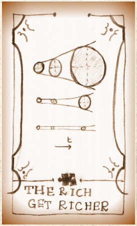

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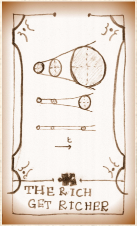

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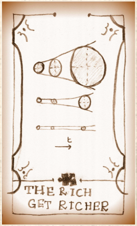
- Directed network
- Two (surmountable) problems:
    1. New papers have no citations
    2. Selection mechanism is more complicated



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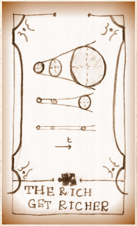

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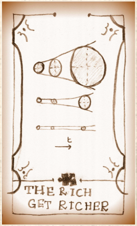

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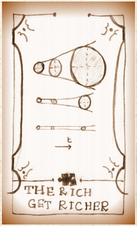

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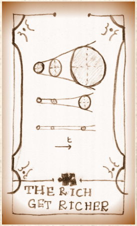

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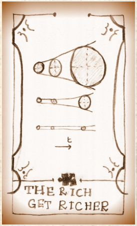

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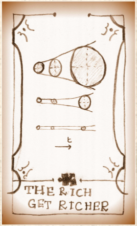

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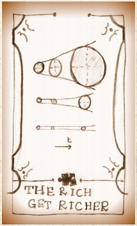
- (Hath = suggested unit of purchasing power.)
- Matilda effect: women's scientific achievements are often overlooked



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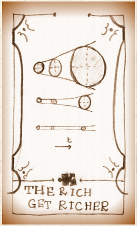
Merton was a catchphrase machine:



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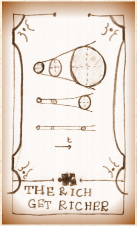

Evolution of catch phrases:

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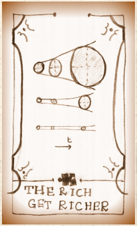
1. Self-fulfilling prophecy
2. Role model



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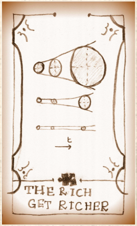

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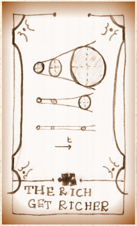

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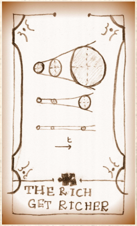

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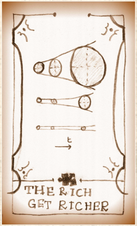

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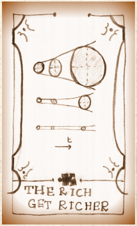
Merton's son, Robert C. Merton, won the Nobel Prize for Economics in 1997.



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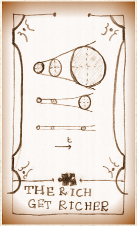

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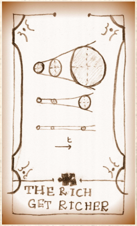

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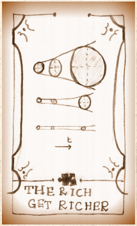

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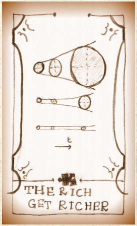

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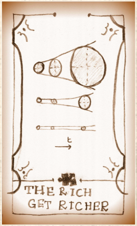

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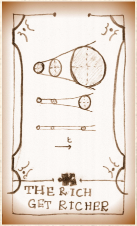

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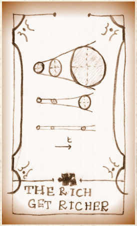

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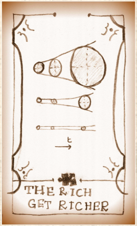
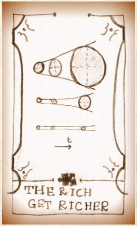
- "Scale-free networks" = food on the table for physicists

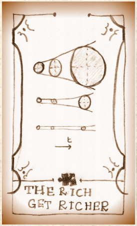


Outline



Another analytic approach: [5]

- Focus on how the nth arriving group typically grows.



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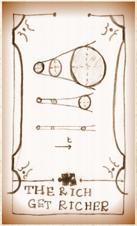
$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)} \left[\frac{1}{t}\right]^{-(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho} \left[\frac{n-1}{t}\right]^{-(1-\rho)} & \text{for } n \geq 2. \end{cases}$$



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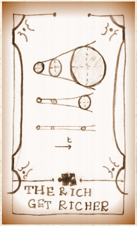
- First mover is a factor $1/\rho$ greater than expected.



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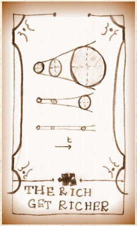
- Because ρ is usually close to 0, the first element is truly an elephant in the room.



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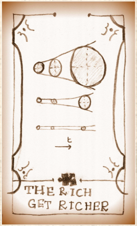
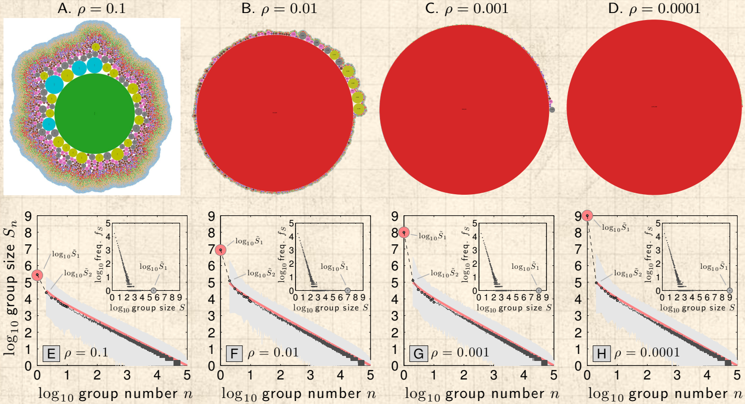
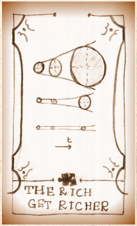
- Appears that this has been missed for 60 years ...



"Simon's fundamental rich-get-richer model entails a dominant first-mover advantage"
Dodds et al.,
Physical Review E, **95**, 052301, 2017. [5]



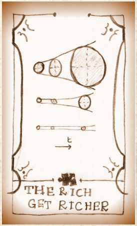


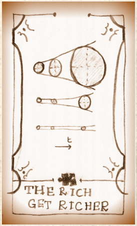
See visualization at paper's online app-endices

Alternate analysis:

- Evolution of the nth arriving group's size:

$$\left\langle S_{n,t+1} - S_{n,t} \right\rangle = (1 - \rho_t) \cdot \frac{S_{n,t}}{t} \cdot (+1).$$





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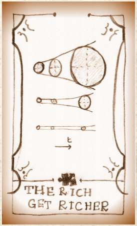
- For $t \geq t_n^{\textrm{init}}$, fix $\rho_t = \rho$ and shift t to $t-1$:

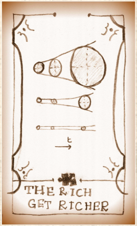
$$S_{n,t} = \left[1 + \frac{(1-\rho)}{t-1} \right] S_{n,t-1}.$$

where $S_{n,t_n^{\textrm{init}}} = 1$.

Betafication ensues:

$$
\begin{aligned}
S_{n,t} &= \left[1 + \frac{(1-\rho)}{t-1}\right]\left[1 + \frac{(1-\rho)}{t-2}\right] \cdots \left[1 + \frac{(1-\rho)}{t_n^{\text{init}}}\right] \cdot 1 \\
&= \left[\frac{t+1-\rho}{t-1}\right]\left[\frac{t-\rho}{t-2}\right] \cdots \left[\frac{t_n^{\text{init}}+1-\rho}{t_n^{\text{init}}}\right] \\
&= \frac{\Gamma(t+1-\rho)\Gamma(t_n^{\text{init}})}{\Gamma(t_n^{\text{init}}+1-\rho)\Gamma(t)} \\
&= \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}.
\end{aligned}
$$





The first mover is really different:

- The issue is t_n^{init} in

$$S_{n,t} = \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}$$

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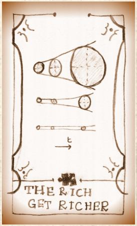

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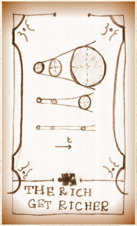

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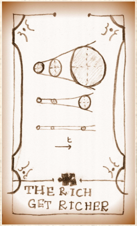

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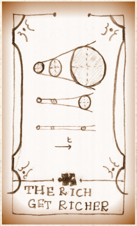
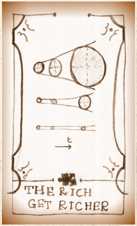
- Note: Does not apply to Barabási-Albert model.



Variability:

- The probability that the nth arriving group, if of size $S_{n,t} = k$ at time t, first replicates at time $t + \tau$:



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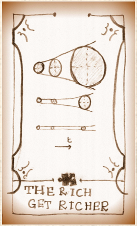
$$\textbf{Pr}(S_{n,t+\tau} = k + 1 \,|\, S_{n,t+i} = k \;\text{ for } i = 0, \dots, \tau - 1)$$

$$= \prod_{i=0}^{\tau-1} \left[1 - (1-\rho)\frac{k}{t+i}\right] \cdot (1-\rho)\frac{k}{t+\tau}$$

$$= k\frac{B(\tau, t)}{B\left(\tau, t - (1-\rho)\right)}\frac{1-\rho}{t+\tau} \propto \frac{\tau^{-(1-\rho)k}}{t+\tau} \sim \tau^{-(2-\rho)k}.$$



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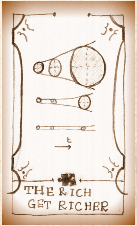
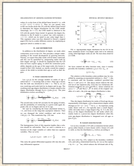
- Upshot: nth arriving group starting at size 1 will on average wait for an infinite time to replicate.

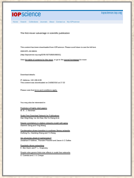


Related papers:



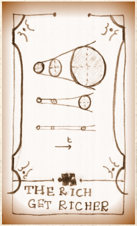
"Organization of Growing Random Networks"

Krapivsky and Redner,
Phys. Rev. E, **63**, 066123, 2001. [7]



"The first-mover advantage in scientific publication"

M. E. J. Newman,
Europhysics Letters, **86**, 68001, 2009. [11]



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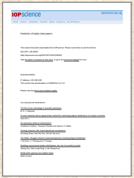
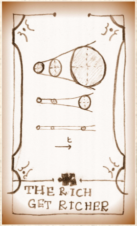

IOPscience

Prediction of highly cited papers

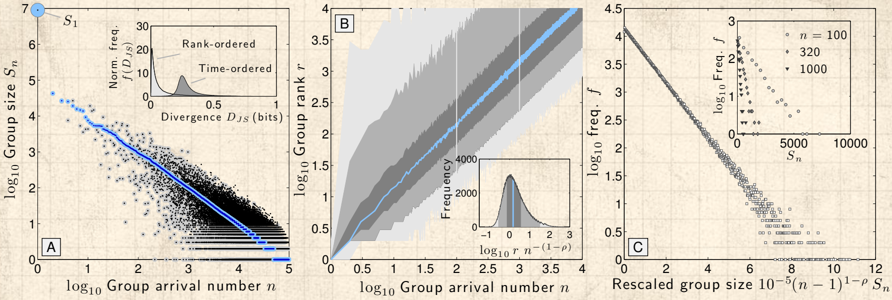
“Prediction of highly cited papers”
M. E. J. Newman,
Europhysics Letters, **105**, 28002, 2014. [12]


The effect of the initial network configuration on preferential attachment

“The effect of the initial network configuration on preferential attachment”
Berset and Medo,
The European Physical Journal B, **86**, 1–7, 2013. [3]

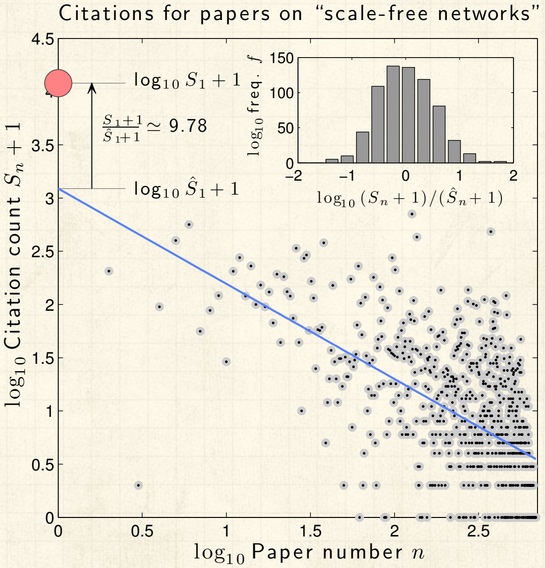


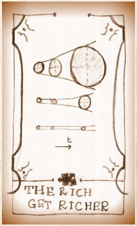
Arrival variability:



- Any one simulation shows a high amount of disorder.
- Two orders of magnitude variation in possible rank.
- Rank ordering creates a smooth Zipf distribution.
- Size distribution for the nth arriving group show exponential decay.

Self-referential citation data:

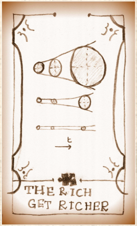




More mattering:

Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.



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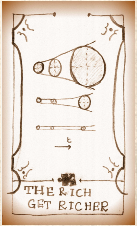
Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.
- People, wealth, sports, music, movies, books, schools, cities, countries, dogs (13/10), ...



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Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.
- People, wealth, sports, music, movies, books, schools, cities, countries, dogs (13/10), ...
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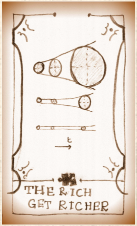

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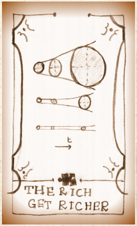

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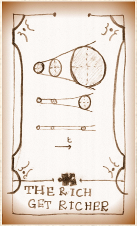

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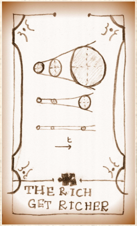

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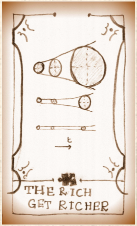

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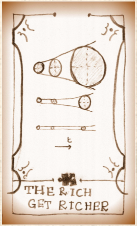

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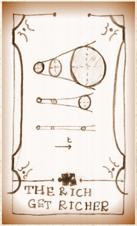
- What if a healthier Facebook is just ... Instagram? (hahahhaaha)



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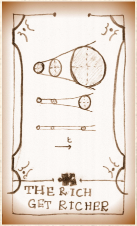

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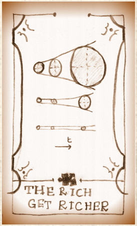

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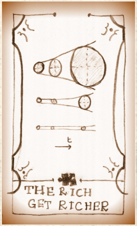

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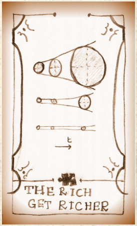

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