A Partial Overview of Complex **Networks**

Last updated: 2023/08/22, 11:48:25 EDT

Principles of Complex Systems, Vols. 1, 2, & 3D CSYS/MATH 6701, 6713, & a pretend number, 2023-2024 @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center Santa Fe Institute | University of Vermont

















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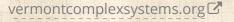
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Leveling up—Scaffolded educational mission:

🙈 Data Science Undergrad.

Graduate Certificate in Complex Systems and Data Science

3

Fall, 2015–: MS in Complex Systems and Data Science

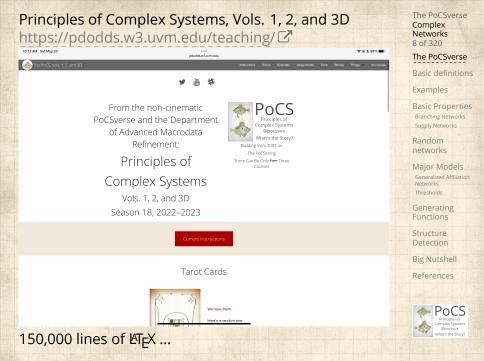
Fall, 2018–: PhD in The Study of Interesting Things Complex Systems and Data Science

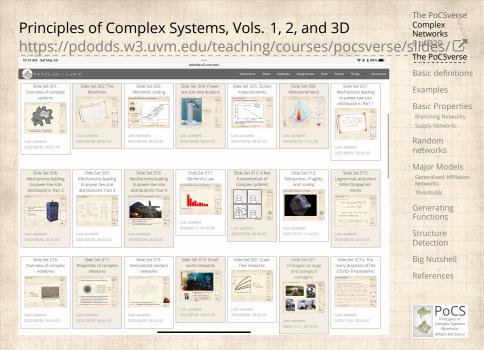


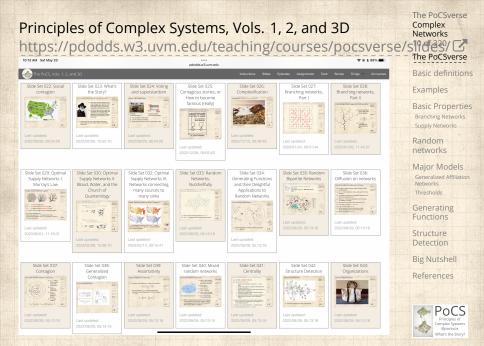
All the words: http://vermontcomplexsystems.org

Dipoloma-posters:









Principles of Complex Systems, Vols. 1, 2, and 3D

7:48 PM Sun May 21

https://pdodds.w3.uvm.edu/teaching/courses/pocsverse/slides/



Episode 1: The OG rich-get-richer model (1:52:03)

Clip 1: Intro to Simon vs Mandebrot and the mechanism of rich-get-richerness (6:35)

Clip 2: Observations of Zipfery, 1910 on (12:13)

Clip 3: Herbert Simon #awesomeness (2:18)

Clip 4: Toy model of rich-get-richer (14:51)

Clip 5: Observations about our toy model (7:10)

Clip 6: Krugman's math woes (1:34)

Clip 7: We work through an analysis (14:37)

Clip 8: What we find: Micro-to-macro story and surprising agreement with reality (8:30)

Clip 9: An appraisal of catchphrases (3:53)

Clip 10: Simon's model recap (3:47)

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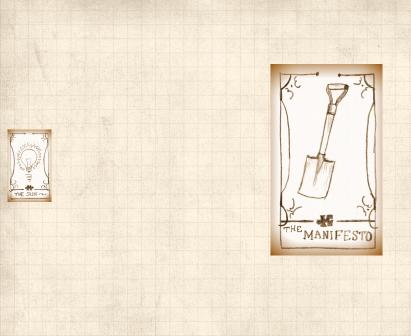
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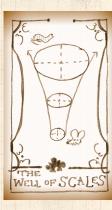
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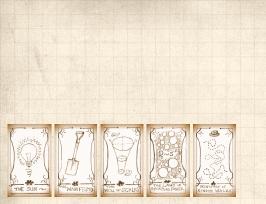
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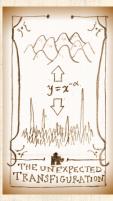
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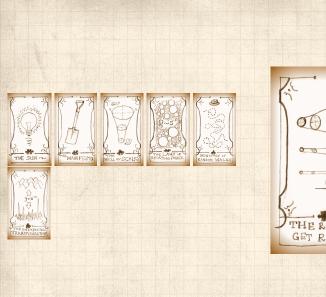
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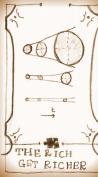
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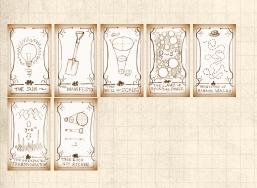
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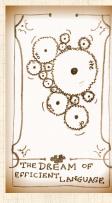
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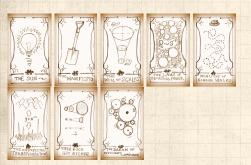
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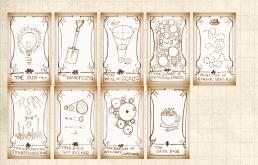
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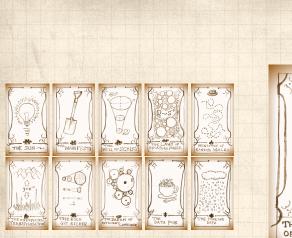
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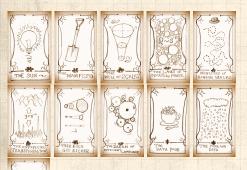
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THE EMERSENCE



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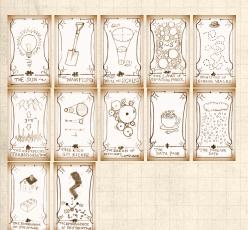
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OF DESTRUCTION



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It was the best of times, of Lh THE EMERGENCE OF STORIES

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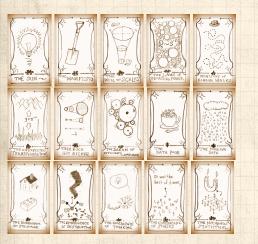
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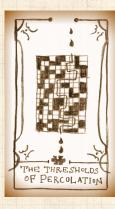
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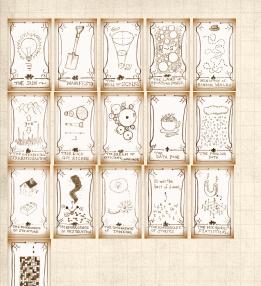
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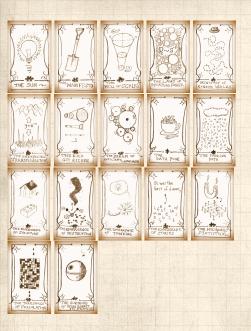
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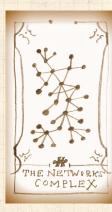
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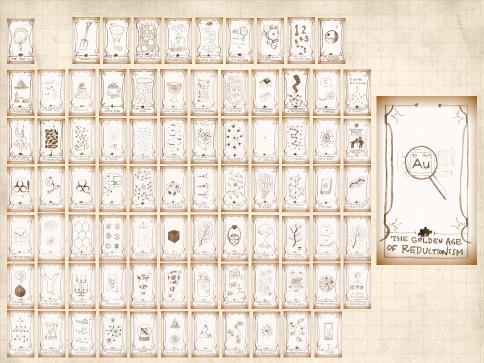
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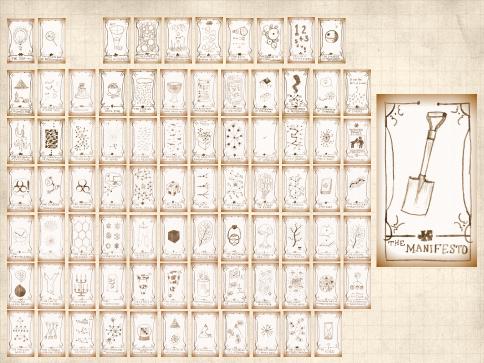
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1. Systems are ubiquitous and systems matter.

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- 1. Systems are ubiquitous and systems matter.
- 2. Consequently, much of science is about understanding how pieces dynamically fit together.



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- 1. Systems are ubiquitous and systems matter.
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- 3. 1700 to 2000 = Golden Age of Reductionism: Atoms!, sub-atomic particles, DNA, genes, people, ...

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- 1. Systems are ubiquitous and systems matter.
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- 1700 to 2000 = Golden Age of Reductionism: Atoms!, sub-atomic particles, DNA, genes, people, ...
- 4. Understanding and creating systems (including new 'atoms') is the greater part of science and engineering.

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- 5. Universality C: systems with quantitatively different micro details exhibit qualitatively similar macro behavior (fate, but real and limited)

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- 6. Computing advances make the Science of Complex Systems possible:

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 - 6.1 We can measure and record enormous amounts of data, research areas continue to transition from data scarce to data rich.
 - 6.2 We can simulate, model, and create complex systems in extraordinary detail.

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net•work |'net,wərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

• a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : a trade network.

- a group of people who exchange information, contacts, and experience for professional or social purposes : a support network.
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : the introduction of a second TV network | [as adj.] network television.
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network*.
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : the stock exchanges have proven to be resourceful in networking these deals.

• link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) networked workstations.

• [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Thesaurus deliciousness:

network

noun

 a network of arteries WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
 a network of lanes MAZE, labyrinth, warren, tangle.
 a network of friends SYSTEM, complex, nexus, web, webwork. The PoCSverse Complex Networks 18 of 320

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From Keith Briggs's etymological investigation:

 Opus reticulatum:
 A Latin origin?



[http://serialconsign.com/2007/11/we-put-net-

network]

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First known use: Geneva Bible, 1560 'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

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From the OED via Briggs:

🚳 1658–: reticulate structures in animals

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 1839-: rivers and canals

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1869-: railways

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- \lambda 1883–: distribution network of electrical cables
- 🗞 1914–: wireless broadcasting networks

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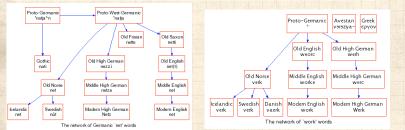
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888).

Work' appear to have long meant purposeful action.



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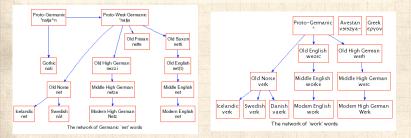
Structure Detection

Big Nutshell



Ancestry: Net and Work are venerable old words: 'Net' first used to mean spider web (King Ælfréd, 888).

Work' appear to have long meant purposeful action.



'Network' = something built based on the idea of natural, flexible lattice or web. The PoCSverse Complex Networks 21 of 320

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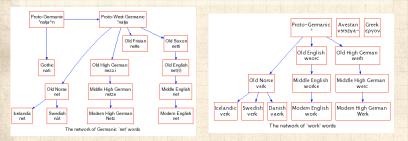
Structure Detection

Big Nutshell



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 Work' appear to have long meant purposeful action.



'Network' = something built based on the idea of natural, flexible lattice or web.

c.f., ironwork, stonework, fretwork.

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Many complex systems can be viewed as complex networks of physical or abstract interactions. The PoCSverse Complex Networks 22 of 320

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- Many complex systems can be viewed as complex networks of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.

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- Many complex systems can be viewed as complex networks of physical or abstract interactions.
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- Dominant approach of the first decade was of a theoretical-physics/stat-mechish flavor.

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Piranha physicus

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Piranha physicus
 Hunt in packs.

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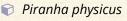
Structure Detection

Big Nutshell



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Hunt in packs.

Feast on new and interesting ideas (see chaos, cellular automata, ...)

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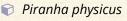
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- Hunt in packs.
 - Feast on new and interesting ideas (see chaos, cellular automata, ...)



Soo also: https://wkcd.com/702/

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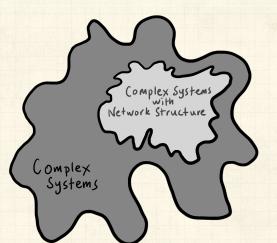
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Complex Systems is the Big Story:



Only sometimes a bit networky: Fluids-at-large (the atmosphere, oceans, ...), organism cells, ... The PoCSverse Complex Networks 23 of 320

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Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks" Watts and Strogatz, Nature, **393**, 440–442, 1998.^[112]

Times cited: C (as of May 19, 2023)



"Emergence of scaling in random networks" Barabási and Albert, Science, **286**, 509–511, 1999.^[8]

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"Collective dynamics of 'small-world' networks" Watts and Strogatz, Nature, **393**, 440–442, 1998.^[112]

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"Collective dynamics of 'small-world' networks" Watts and Strogatz, Nature, **393**, 440–442, 1998.^[112]

Times cited: ~ 51,622 C (as of May 19, 2023)



"Emergence of scaling in random networks" Barabási and Albert, Science, **286**, 509–511, 1999.^[8]

Times cited: ~ 43,853 🖸 (as of May 19, 2023)

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Review articles:



"Complex Networks: Structure and Dynamics" C Boccaletti et al., Physics Reports, **424**, 175–308, 2006. ^[14]

Times cited: ~ 12,318 🖸 (as of May 9, 2023)



"The structure and function of complex networks" M. E. J. Newman, SIAM Rev., **45**, 167–256, 2003.^[77]

Times cited: ~ 23,611 🖸 (as of May 9, 2023)



"Statistical mechanics of complex networks" Albert and Barabási, Rev. Mod. Phys., **74**, 47–97, 2002. ^[3]

Times cited: ~ 26,636 C (as of May 9, 2023)

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Popularity according to textbooks:

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Popularity according to textbooks:

Textbooks:

- Mark Newman (Physics, Michigan) "Networks: An Introduction"
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell) "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" ^C

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Popularity according to popular books:



GLADWELL

(A set of particular and a set of the set of

The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell^[43]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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Popularity according to popular books:

Haw Everything In Connected to Exceptions Else and What Is Means for Datasets, Science, and Everyday Life



Albert-Lészlő Barabési

Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts^[107]

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Numerous others ...

- Complex Social Networks—F. Vega-Redondo^[105]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo^[84]
- 🗞 Random Graph Dynamics—R. Durette
- 🚳 Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- 🚳 Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster^[19]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes^[34]

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But surely networks aren't new ...

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🚳 But surely networks aren't new ... 🗞 Graph theory was well established ... The PoCSverse Complex Networks 30 of 320

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But surely networks aren't new ...
 Graph theory was well established ...
 Study of social networks started in the 1930's ...

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- But surely networks aren't new ...
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- line why all this 'new' research on networks?

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But surely networks aren't new ...
Graph theory was well established ...
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So why all this 'new' research on networks?
Answer: Oodles of Easily Accessible Data.



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But surely networks aren't new ...
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We can now inform (alas) our theories with a much more measurable reality.*

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But surely networks aren't new ... 🚳 Graph theory was well established ... 🚓 Study of social networks started in the 1930's ... So why all this 'new' research on networks? Answer: Oodles of Easily Accessible Data. 🚳 We can now inform (alas) our theories with a much more measurable reality.* Graph theory missed "becoming": Stories = Characters + Time

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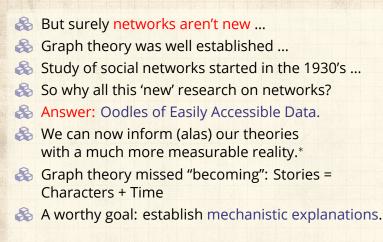
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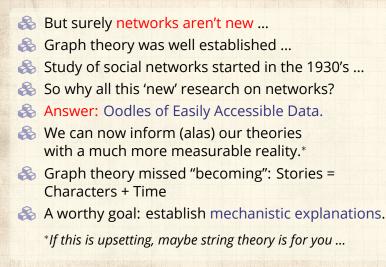
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lnternet-scale data sets can be overly exciting.

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Internet-scale data sets can be overly exciting.

Witness:

The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) The PoCSverse Complex Networks 31 of 320

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But:

For scientists, description is only part of the battle.

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But:

For scientists, description is only part of the battle.
We still need to understand.

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Nodes = A collection of entities which have properties that are somehow related to each other

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Nodes = A collection of entities which have properties that are somehow related to each other

local e.g., people, forks in rivers, proteins, webpages, organisms, ...

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e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

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Nodes = A collection of entities which have properties that are somehow related to each other

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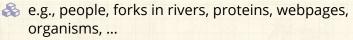
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Big Nutshell



Nodes = A collection of entities which have properties that are somehow related to each other



Links = Connections between nodes Links may be directed or undirected. Links may be binary or weighted.

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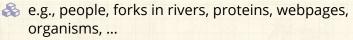
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Big Nutshell



Nodes = A collection of entities which have properties that are somehow related to each other



Links = Connections between nodes

Links may be directed or undirected.
 Links may be binary or weighted.

Other spiffing words: vertices and edges.

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Node degree = Number of links per node

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Node degree = Number of links per node Notation: Node *i*'s degree = k_i .

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Node degree = Number of links per node

Solution: Node *i*'s degree = k_i . $k_i = 0,1,2,...$ The PoCSverse Complex Networks 33 of 320

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Node degree = Number of links per node

- \aleph Notation: Node *i*'s degree = k_i .
- $k_i = 0, 1, 2, \dots$
- \aleph Notation: the average degree of a network = $\langle k \rangle$

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- $k_i = 0, 1, 2, \dots$
- Solution: the average degree of a network = $\langle k \rangle$ (and sometimes z)

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- $\& k_i = 0, 1, 2,$
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes *z*)
- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

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- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

 \mathfrak{S} Defn: \mathcal{N}_i = the set of *i*'s k_i neighbors

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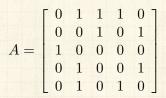
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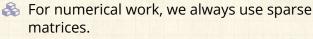


Adjacency matrix:

💑 e.g.,

We can represent a network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).





 \mathfrak{F} For many real networks, A is a function of time.

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So what passes for a complex network?

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So what passes for a complex network?

Complex networks are large (in node number)
 Complex networks are sparse (low edge to node ratio)

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So what passes for a complex network?

- lin node number) 🚳
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving

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So what passes for a complex network?

- lin node number) 🚳
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

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Physical networks



🙈 River networks Neural networks A Trees and leaves Blood networks

The internet (pipes) 3 Road networks \lambda Power grids







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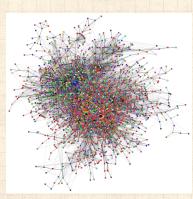
References



Distribution (branching) versus redistribution (cyclical)

Interaction networks

- 🚳 The Blogosphere (RIP)
 - Biochemical networks
 - Sene-protein networks
- Food webs: who eats whom
- 🚳 Airline networks
 - 🗞 Call networks (AT&T)
- 🚳 The Media
- The internet (World Wide Web)



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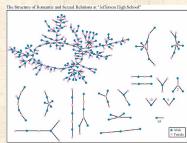
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Interaction networks: social networks



🚳 Snogging



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman et al., 2004)

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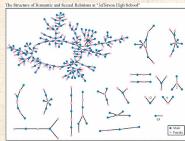
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Interaction networks: social networks



🚳 Snogging 🚳 Friendships



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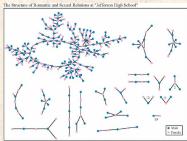
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Interaction networks: social networks



- \delta Snogging
- \lambda Friendships
- \lambda Acquaintances



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(Bearman et al., 2004)

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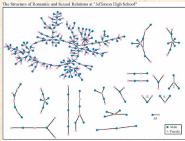
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Interaction networks: social networks



- 🗞 Snogging
- \lambda Friendships
- \lambda Acquaintances
- Boards and directors



preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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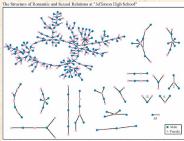
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Interaction networks: social networks



- 🗞 Snogging
- \lambda Friendships
- \lambda Acquaintances
 - Boards and directors
 - Organizations



preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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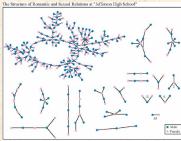


Interaction networks: social networks



- 🚳 Snogging
- \lambda Friendships
- \lambda Acquaintances
 - Boards and directors

Organizations 🚳 facebook 🗹 twitter 📿 ,



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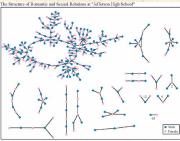
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Interaction networks: social networks



- 🚳 Snogging
 - Friendships
 - Acquaintances
 - Boards and directors
- Organizations 🔧 facebook 🖸 twitter 🖸



Each circle represents a student and lines connecting students represent remantic relations occuring within the 6 month preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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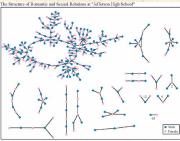


🚳 'Remotely sensed' by: email activity, instant messaging, phone logs

Interaction networks: social networks



- 🚳 Snogging
 - Friendships
 - Acquaintances
 - Boards and directors
- Organizations 🔧 facebook 🖸 twitter 🖸



Each circle represents a student and lines connecting students represent remantic relations occuring within the 6 month preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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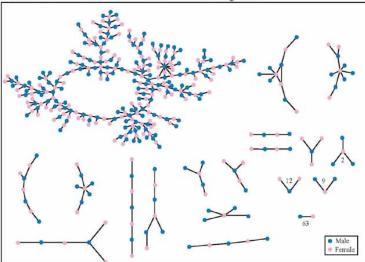
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References



🚳 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else). The PoCSverse Complex Networks 39 of 320

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Relational networks

🚳 Consumer purchases

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Relational networks

Consumer purchases
 (Walmart, Target, Amazon, ...)

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Relational networks

Consumer purchases (Walmart, Target, Amazon, ...)

Thesauri: Networks of words generated by meanings

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Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- 🗞 Knowledge/Databases/Ideas

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Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- 🗞 Knowledge/Databases/Ideas
- 🚷 Metadata—Tagging, Keywords bit.ly 🗹 flickr 🗹

common tags cloud | list

community daily dictionary education **encyclopedia** english free imported info information internet knowledge learning news **reference** research resource resources search tools useful web web2.0 **Wiki wikipedia** The PoCSverse Complex Networks 40 of 320

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Relational networks

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- 🚳 Large Language Models

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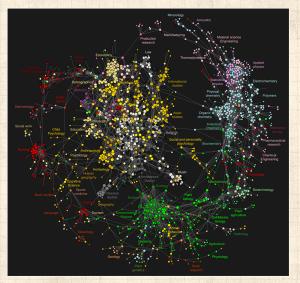
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Clickworthy Science:



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"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. ^[18], 2009.

Neural reboot (NR):

Dog has fun.

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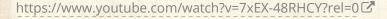
Random networks

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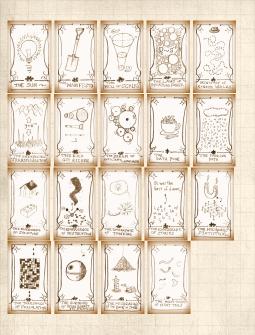
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🚳 Graphical renderings are often just a big mess.

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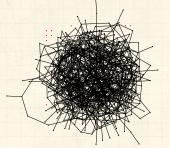
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🚳 Graphical renderings are often just a big mess.



⇐ Typical hairball lacktriangleright number of nodes N = 500number of edges m = 1000

average degree $\langle k \rangle = 4$

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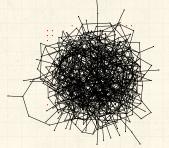
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🚳 Graphical renderings are often just a big mess.



 $\Leftarrow Typical hairball$ rightarrow Typical hairball rightarrow Typical hairball

And even when renderings somehow look good:

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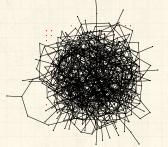
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🚳 Graphical renderings are often just a big mess.



 $\Leftarrow Typical hairball$ number of nodes N = 500
number of edges m = 1000
average degree $\langle k \rangle = 4$

And even when renderings somehow look good: "That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way" said Ponder [Stibbons] —*Making Money*, T. Pratchett. The PoCSverse Complex Networks 44 of 320

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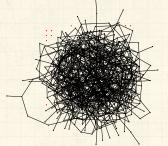
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🚳 Graphical renderings are often just a big mess.



⇐ Typical hairball number of nodes N = 500number of edges m = 1000

average degree $\langle k \rangle = 4$

And even when renderings somehow look good: "That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way" said Ponder [Stibbons] - Making Money, T. Pratchett. We need to extract digestible, meaningful aspects.

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Some key aspects of real complex networks:

degree distribution*
 assortativity
 homophily
 clustering
 motifs
 modularity

concurrency
 hierarchical scaling
 network distances
 centrality
 efficiency
 interconnectedness
 robustness

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Plus coevolution of network structure and processes on networks.

* Degree distribution is the elephant in the room that we are now all very aware of ...

Properties

1. degree distribution P_k

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Properties

1. degree distribution P_k

P_k is the probability that a randomly selected node has degree k.

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Properties

1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k.
- k =node degree = number of connections.

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1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k.
- k =node degree = number of connections.
- ex 1: Erdős-Rényi random networks have Poisson degree distributions:

$$P_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

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 \clubsuit ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'.

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1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k.
- & k = node degree = number of connections.
- ex 1: Erdős-Rényi random networks have Poisson degree distributions:

$$P_{k} = e^{-\langle k \rangle} \frac{\langle k \rangle^{k}}{k!}$$

♦ ex 2: "Scale-free" networks: P_k ∝ k^{-γ} ⇒ 'hubs'.
 ♦ link cost controls skew.
 ♦ hubs may facilitate or impede contagion.

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Note:

Erdős-Rényi random networks are a mathematical construct. The PoCSverse Complex Networks 47 of 320

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Note:

- Erdős-Rényi random networks are a mathematical construct.
- Scale-free' networks are growing networks that form according to a plausible mechanism.

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Note:

- Erdős-Rényi random networks are a mathematical construct.
- Scale-free' networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.

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Note:

- Erdős-Rényi random networks are a mathematical construct.
- Scale-free' networks are growing networks that form according to a plausible mechanism.
- Randomness is out there, just not to the degree of a completely random network.
- 🚳 "Becoming": Stories = Characters + Time

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2. Assortativity/3. Homophily: Social networks: Homophily = birds of a feather

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2. Assortativity/3. Homophily:

 Social networks: Homophily = birds of a feather
 e.g., degree is standard property for sorting: measure degree-degree correlations. The PoCSverse Complex Networks 48 of 320

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2. Assortativity/3. Homophily:

 Social networks: Homophily = birds of a feather
 e.g., degree is standard property for sorting: measure degree-degree correlations.
 Assortative network: ^[74] similar degree nodes

connecting to each other.

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2. Assortativity/3. Homophily:

- Social networks: Homophily = birds of a feather
 e.g., degree is standard property for sorting: measure degree-degree correlations.
 Assortative network: ^[74] similar degree nodes
- connecting to each other.

Disassortative network: high degree nodes connecting to low degree nodes. The PoCSverse Complex Networks 48 of 320

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2. Assortativity/3. Homophily:

 Social networks: Homophily = birds of a feather
 e.g., degree is standard property for sorting: measure degree-degree correlations.
 Assortative network: ^[74] similar degree nodes connecting to each other. *Often social: company directors, coauthors, actors.* Disassortative network: high degree nodes

connecting to low degree nodes.

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2. Assortativity/3. Homophily:

🚳 Social networks: Homophily 🗹 = birds of a feather line and a standard property for sorting: measure degree-degree correlations. Assortative network: ^[74] similar degree nodes connecting to each other. Often social: company directors, coauthors, actors. Disassortative network: high degree nodes connecting to low degree nodes. Often techological or biological: internet, WWW, protein interactions, neural networks, food webs.

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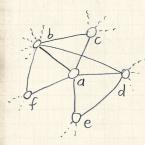
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Local socialness:

4. Clustering:



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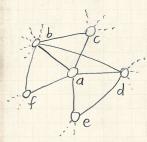


Local socialness:

4. Clustering:



A Your friends tend to know each other.



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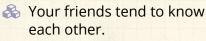
Big Nutshell



Local socialness:

a

4. Clustering:



- Two measures (explained on following slides):
 - 1. Watts & Strogatz^[112]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2} \right\rangle$$

2. Newman^[77]

$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$$

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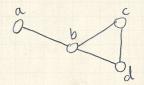
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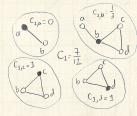
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Calculation of C_1 :



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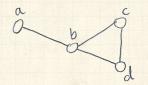
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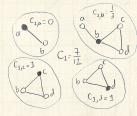
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Calculation of C_1 :



C₁ is the average fraction of pairs of neighbors who are connected.

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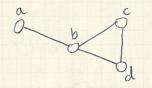
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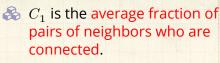
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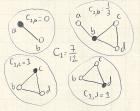




Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1j_2\in\mathcal{N}_i}a_{j_1j_2}}{k_i(k_i-1)/2}$$

Calculation of
$$C_1$$
:



where k_i is node *i*'s degree, and \mathcal{N}_i is the set of *i*'s neighbors. The PoCSverse Complex Networks 50 of 320

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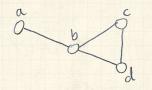
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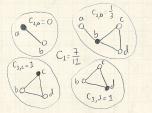
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Calculation of C_1 :



 $rightarrow C_1$ is the average fraction of pairs of neighbors who are connected.

Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1j_2\in\mathcal{N}_i}a_{j_1j_2}}{k_i(k_i-1)/2}$$

where k_i is node *i*'s degree, and \mathcal{N}_i is the set of *i*'s neighbors.

Averaging over all nodes, we have:

$$C_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2}$$

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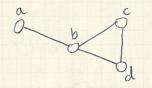
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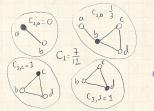
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Calculation of C_1 :



 $rightarrow C_1$ is the average fraction of pairs of neighbors who are connected.

Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1j_2\in\mathcal{N}_i}a_{j_1j_2}}{k_i(k_i-1)/2}$$

where k_i is node *i*'s degree, and \mathcal{N}_i is the set of *i*'s neighbors.

Averaging over all nodes, we have:

$$\begin{split} C_1 &= \frac{1}{n} \sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2} \\ \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i (k_i - 1)/2} \right\rangle_i \end{split}$$

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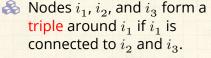
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Example network:



Triangles:

Triples:



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Example network:









Nodes i₁, i₂, and i₃ form a triple around i₁ if i₁ is connected to i₂ and i₃.
 Nodes i₁, i₂, and i₃ form a triangle if each pair of nodes is connected

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Example network:

Triangles:



Triples:

Nodes i₁, i₂, and i₃ form a triple around i₁ if i₁ is connected to i₂ and i₃.
 Nodes i₁, i₂, and i₃ form a triangle if each pair of nodes is connected
 The definition C₂ = 3×#triangles measures the fraction of closed triples

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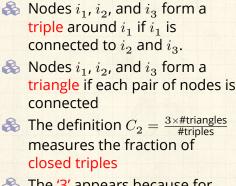
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Example network:

Triangles:

Triples:



The '3' appears because for each triangle, we have 3 closed triples. The PoCSverse Complex Networks 51 of 320

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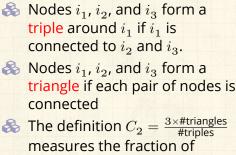


Example network:

Triangles:

, add

Triples:



closed triples

- The '3' appears because for each triangle, we have 3 closed triples.
- Social Network Analysis (SNA): fraction of transitive triples.

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Sneaky counting for undirected, unweighted networks:

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Sneaky counting for undirected, unweighted networks:

 \mathfrak{R} If the path i-j- ℓ exists then $a_{ij}a_{j\ell} = 1$.



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Sneaky counting for undirected, unweighted networks:

 \mathfrak{R} If the path $i-j-\ell$ exists then $a_{ij}a_{j\ell} = 1$.

$$\ref{black}$$
 Otherwise, $a_{ij}a_{j\ell}=0.$

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Sneaky counting for undirected, unweighted networks:

- \mathfrak{R} If the path i-j- ℓ exists then $a_{ij}a_{j\ell} = 1$.
- 3 Otherwise, $a_{ij}a_{j\ell} = 0$.
- \mathfrak{S} We want $i \neq \ell$ for good triples.

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Sneaky counting for undirected, unweighted networks:

- \mathfrak{R} If the path i-j- ℓ exists then $a_{ij}a_{j\ell} = 1$.
- 3 Otherwise, $a_{ij}a_{j\ell} = 0$.
- \mathfrak{S} We want $i \neq \ell$ for good triples.
- So In general, a path of n edges between nodes i_1 and i_n travelling through nodes i_2 , i_3 , ... i_{n-1} exists

$$\Leftrightarrow a_{i_1i_2}a_{i_2i_3}a_{i_3i_4}\cdots a_{i_{n-2}i_{n-1}}a_{i_{n-1}i_n} =$$

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2

Sneaky counting for undirected, unweighted networks:

- \mathfrak{R} If the path i-j- ℓ exists then $a_{ij}a_{j\ell} = 1$.
- \bigotimes Otherwise, $a_{ij}a_{j\ell} = 0$.
- \mathfrak{S} We want $i \neq \ell$ for good triples.
- $\begin{cases} & \text{In general, a path of } n \text{ edges between nodes } i_1 \\ & \text{and } i_n \text{ travelling through nodes } i_2, i_3, \dots i_{n-1} \text{ exists} \\ & \Leftrightarrow a_{i_1i_2}a_{i_2i_3}a_{i_3i_4}\cdots a_{i_{n-2}i_{n-1}}a_{i_{n-1}i_n} = 1. \end{cases}$

$$\# \text{triples} = \frac{1}{2} \left(\sum_{i=1}^{N} \sum_{\ell=1}^{N} \left[A^2 \right]_{i\ell} - \text{Tr}A^2 \right)$$

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2

R

Sneaky counting for undirected, unweighted networks:

- \mathfrak{R} If the path i-j- ℓ exists then $a_{ij}a_{j\ell} = 1$.
- \bigotimes Otherwise, $a_{ij}a_{j\ell} = 0$.
- \mathfrak{S} We want $i \neq \ell$ for good triples.
- $\begin{cases} & \text{In general, a path of } n \text{ edges between nodes } i_1 \\ & \text{and } i_n \text{ travelling through nodes } i_2, i_3, \dots i_{n-1} \text{ exists} \\ & \Leftrightarrow a_{i_1i_2}a_{i_2i_3}a_{i_3i_4}\cdots a_{i_{n-2}i_{n-1}}a_{i_{n-1}i_n} = 1. \end{cases}$

$$\# \text{triples} = \frac{1}{2} \left(\sum_{i=1}^{N} \sum_{\ell=1}^{N} \left[A^2 \right]_{i\ell} - \text{Tr}A^2 \right)$$

$$\#$$
triangles $=$ $\frac{1}{6}$ Tr A^3

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5. motifs:

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5. motifs:

small, recurring functional subnetworks

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5. motifs:

line small, recurring functional subnetworks 🚳 e.g., Feed Forward Loop:

a

feedforward loop

7

Shen-Orr, Uri Alon, et al. [89]

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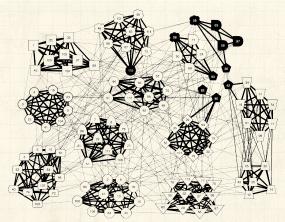
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6. modularity and structure/community detection:



Clauset et al., 2006 [24]: NCAA football

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7. concurrency:

transmission of a contagious element only occurs during contact

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7. concurrency:

- transmission of a contagious element only occurs during contact
- 🚳 rather obvious but easily missed in a simple model

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7. concurrency:

- transmission of a contagious element only occurs during contact
- 🗞 rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough

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7. concurrency:

- transmission of a contagious element only occurs during contact
- line a simple model as a simple model a simple model line a simple model line as a simple model as a simple model with the second secon
- dynamic property—static networks are not enough
- 🗞 knowledge of previous contacts crucial

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- 🚳 beware cumulated network data

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7. concurrency:

- transmission of a contagious element only occurs during contact
- line a simple model 🗞 🗞
- dynamic property—static networks are not enough
- 🗞 knowledge of previous contacts crucial
- 🚳 beware cumulated network data
- line text and Morris, 1996 [58] Kretzschmar and Morris, 1996

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7. concurrency:

- transmission of a contagious element only occurs during contact
- line a simple model as a simple model a simple model line a simple model line as a simple model as a simple model with the second secon
- dynamic property—static networks are not enough
- 🗞 knowledge of previous contacts crucial
- 🚳 beware cumulated network data
- 🗞 Kretzschmar and Morris, 1996 [58]
- "Temporal networks" become a concrete area of study for Piranha Physicus in 2013.

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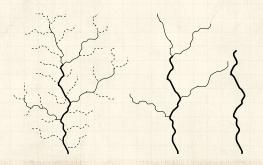
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8. Horton-Strahler ratios:

Metrics for branching networks:



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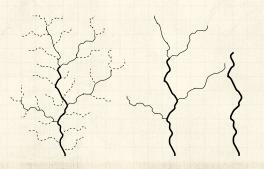
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8. Horton-Strahler ratios:

Metrics for branching networks: Method for ordering streams hierarchically



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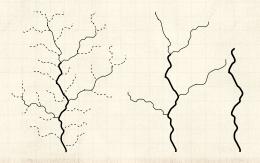
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8. Horton-Strahler ratios:

Metrics for branching networks:

Method for ordering streams hierarchically
Number: $R_n = N_\omega / N_{\omega+1}$



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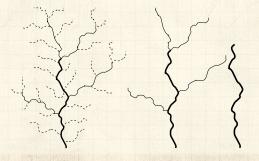
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8. Horton-Strahler ratios:



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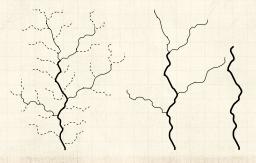
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8. Horton-Strahler ratios:

 $\begin{array}{l} & \underset{()}{\otimes} & \text{Metrics for branching networks:} \\ & \underset{()}{\otimes} & \text{Method for ordering streams hierarchically} \\ & \underset{()}{\otimes} & \text{Number: } R_n = N_\omega/N_{\omega+1} \\ & \underset{()}{\otimes} & \text{Segment length: } R_l = \langle l_{\omega+1} \rangle / \langle l_{\omega} \rangle \\ & \underset{()}{\otimes} & \text{Area/Volume: } R_a = \langle a_{\omega+1} \rangle / \langle a_{\omega} \rangle \\ \end{array}$



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9. network distances:

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9. network distances:

(a) shortest path length d_{ij} :

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9. network distances:

(a) shortest path length d_{ij} :

Fewest number of steps between nodes i and j.

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9. network distances:

(a) shortest path length d_{ij} :

Fewest number of steps between nodes *i* and *j*.
 (Also called the chemical distance between *i* and *j*.)

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9. network distances:

(a) shortest path length d_{ij} :

Fewest number of steps between nodes *i* and *j*.
 (Also called the chemical distance between *i* and *j*.)

(b) average path length $\langle d_{ij} \rangle$:

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9. network distances:

(a) shortest path length d_{ij} :

Fewest number of steps between nodes *i* and *j*.
 (Also called the chemical distance between *i* and *j*.)

(b) average path length $\langle d_{ij} \rangle$:

🗞 Average shortest path length in whole network.

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9. network distances:

(a) shortest path length d_{ij} :

Fewest number of steps between nodes *i* and *j*.
 (Also called the chemical distance between *i* and *j*.)

(b) average path length $\langle d_{ij} \rangle$:

Average shortest path length in whole network.
 Good algorithms exist for calculation.

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9. network distances:

(a) shortest path length d_{ij} :

Fewest number of steps between nodes *i* and *j*.
 (Also called the chemical distance between *i* and *j*.)

(b) average path length $\langle d_{ij} \rangle$:

Average shortest path length in whole network.
 Good algorithms exist for calculation.
 Weighted links can be accommodated.

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9. network distances:

network diameter d_{max}: Maximum shortest path length between any two nodes.

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9. network distances:

- \clubsuit network diameter d_{max} : Maximum shortest path length between any two nodes.
- \clubsuit closeness $d_{\mathsf{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$: Average 'distance' between any two nodes.

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9. network distances:

- network diameter d_{max}: Maximum shortest path length between any two nodes.
- $\begin{array}{l} \textcircled{l} & \mbox{closeness } d_{\rm cl} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1} \\ & \mbox{Average 'distance' between any two nodes.} \\ & \mbox{Closeness handles disconnected networks} \\ & (d_{ij} = \infty) \end{array} \end{array}$

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9. network distances:

- network diameter d_{max}: Maximum shortest path length between any two nodes.
- Solution closeness $d_{cl} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$: Average 'distance' between any two nodes.
- Solution Closeness handles disconnected networks $(d_{ij} = \infty)$
- $d_{cl} = \infty$ only when all nodes are isolated.
- Closeness perhaps compresses too much into one number

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10. centrality:

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10. centrality:

🚳 Many such measures of a node's 'importance.'

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10. centrality:

Many such measures of a node's 'importance.' k_i . The PoCSverse Complex Networks 59 of 320

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10. centrality:

🙈 Many such measures of a node's 'importance.'

 \bigotimes ex 1: Degree centrality: k_i .

ex 2: Node i's betweenness

= fraction of shortest paths that pass through *i*.

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10. centrality:

- 🚳 Many such measures of a node's 'importance.'
- \bigotimes ex 1: Degree centrality: k_i .
- ex 2: Node i's betweenness
 = fraction of shortest paths that pass through i.
- ex 3: Edge l's betweenness
 = fraction of shortest paths that travel along l.

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10. centrality:

- 🚳 Many such measures of a node's 'importance.'
- \bigotimes ex 1: Degree centrality: k_i .
- ex 2: Node i's betweenness
 = fraction of shortest paths that pass through i.
- ex 3: Edge l's betweenness
 = fraction of shortest paths that travel along l.
- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg^[56])

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Interconnected networks and robustness (two for one deal):

"Catastrophic cascade of failures in interdependent networks"^[21]. Buldyrev et al., Nature 2010.

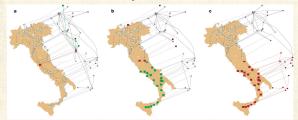


Figure 11 Modelling a blackout in thay. Illustration of an incrative processor of a scacado of failures using real-world kind from a power network (located on the may of 11ab) and an internet network, (shifted above the map) that were 2000°. The networks are drawn using the real georgraphical locations and every internet server is connected to the georgraphical locations and every internet server is into increased (of nodo cen map) from the power station. A Cone power station is removed (for dono de namp) from the power station. A cone power station is removed (for dono de namp) from the power the laterest network (red moles above the map). The nucles that will be disconnected from the gain cluster (a cluster that spants the remover) at the next step are marked in green, b, Additional modes that were disconnected from the Internet communication network given in component are removed (red nodes above map). As a result the power stations (depending on them are removed from the power attwork, ref. ondes on map). Again, the nodes that will be disconnected from the giant cluster at the from the giant component of the power activeds. It could be also from the giant component of the power activeds, ref. removed from holes and map), a well as the nodes in the Internet network that depend on them (red nodes above map). The PoCSverse Complex Networks 60 of 320

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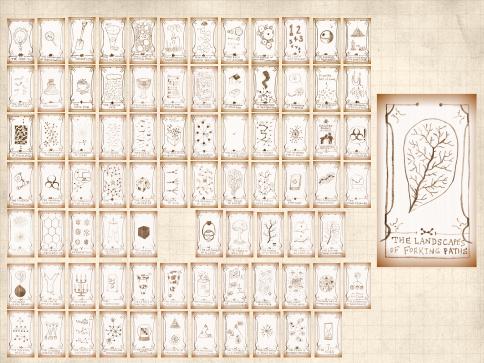
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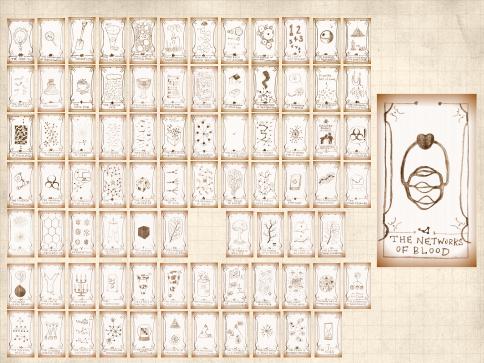
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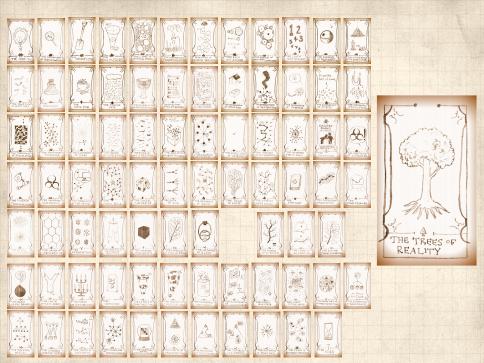
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Big Nutshell









Branching networks are useful things:

- lacktrian section and collection and
- Supply: From one source to many sinks in 2- or 3-d.
- Collection: From many sources to one sink in 2- or 3-d.
- Typically observe hierarchical, recursive self-similar structure

Examples:

- 🗞 River networks
- 🚳 Cardiovascular networks
- 🚳 Plants
- 🚳 Evolutionary trees
 - Organizations (only in theory ...)

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Branching networks are everywhere ...

HydroSHEDS Amazon Basin

River network derived from SRTM elevation data at 500 m resolution



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http://hydrosheds.cr.usgs.gov/

Only maior

rivers and streams are

visualized

River line width proportional to

upstream basin area

500 Kilometers 1000

Branching networks are everywhere ...



http://en.wikipedia.org/wiki/Image:Applebox.JPG

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An early thought piece: Extension and Integration



"The Development of Drainage Systems: A Synoptic View" Waldo S. Glock, The Geographical Review, **21**, 475–482, 1931. ^[45]







Initiation, Elongation Elaboration, Piracy. Abstraction, Absorption. The PoCSverse Complex Networks 68 of 320

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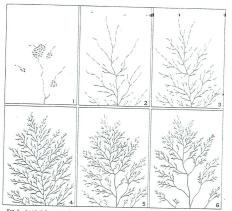


FIG. 3—An ideal diagrammatic summary of the development of a drainage system given for purposes of comparison only. The first four parts show extension, thus: 1, initiation; 2, elongation; 3, elaboration; and 4, maximum extension. Parts 3 and 6 represent steps during integration.

The sequential stages recognized in the evolution of a drainage system are "extension" and "integration"; the first, a stage of increasing complexity; the second, of simplification.

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Allometry

🚳 Isometry:

dimensions scale linearly with each other.

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Allometry

🚳 Isometry:

dimensions scale linearly with each other.

Allometry: dimensions scale nonlinearly.

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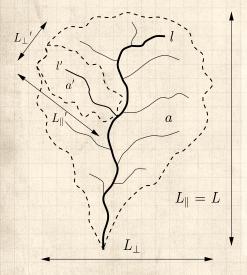
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Allometric relationships:

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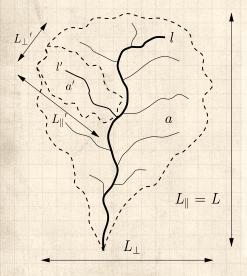
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Allometric relationships:

3

 $\ell \propto a^h$

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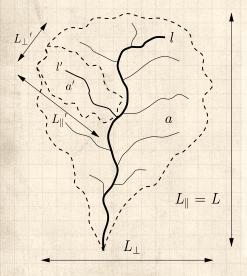
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Allometric relationships:

2

8

 $\ell \propto a^h$

 $\ell \propto L^d$

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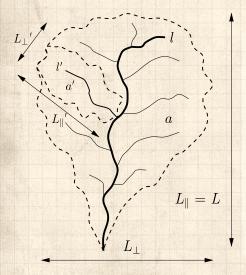
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Allometric relationships:

2

3

 $\ell \propto a^h$

 $\ell \propto L^d$

🚳 Combine above:

$$a \propto L^{d/h} \equiv L^D$$

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'Laws'

🙈 Hack's law (1957)^[50]:

 $\ell \propto a^h$

reportedly 0.5 < h < 0.7



🚳 Hack's law (1957)^[50]:

 $\ell \propto a^h$

reportedly 0.5 < h < 0.7

🙈 Scaling of main stream length with basin size:

 $\ell \propto L^d_{\parallel}$

reportedly 1.0 < d < 1.1



🚳 Hack's law (1957)^[50]:

 $\ell \propto a^h$

reportedly 0.5 < h < 0.7

Scaling of main stream length with basin size:

 $\ell \propto L^d_{\parallel}$

reportedly 1.0 < d < 1.1

🚳 Basin allometry:

 $L_{\parallel} \propto a^{h/d} \equiv a^{1/D}$

 $D < 2 \rightarrow$ basins elongate.

There are a few more 'laws': [31]

Relation: Name or description:

nples $T_{k} = T_{1}(R_{T})^{k-1}$ Tokunaga's law c Properties $\ell \sim L^d$ self-affinity of single channels hing Networks $n_{\omega}/n_{\omega+1}=R_n$ Horton's law of stream numbers $\ell_{\omega+1}/\ell_{\omega} = R_{\ell}$ Horton's law of main stream lengths /orks Horton's law of basin areas $\bar{a}_{\omega+1}/\bar{a}_{\omega} = R_a$ or Models alized Affiliation $\bar{s}_{\omega+1}/\bar{s}_{\omega} = R_s$ Horton's law of stream segment lengths $L_{\perp} \sim L^H$ scaling of basin widths erating $P(a) \sim a^{-\tau}$ probability of basin areas cture probability of stream lengths $P(\ell) \sim \ell^{-\gamma}$ $\ell \sim a^h$ Hack's law $a \sim L^D$ scaling of basin areas $\Lambda \sim a^{\beta}$ Langbein's law variation of Langbein's law $\lambda \sim L^{\varphi}$ PoCS

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Reported parameter values: [31]

Parameter:	Real networks:
R_n	3.0-5.0
R_a	3.0-6.0
$R_{\ell} = R_T$	1.5-3.0
T_1	1.0–1.5
d	1.1 ± 0.01
D	1.8 ± 0.1
h	0.50-0.70
au	1.43 ± 0.05
γ	1.8 ± 0.1
H	0.75-0.80
β	0.50-0.70
arphi	1.05 ± 0.05

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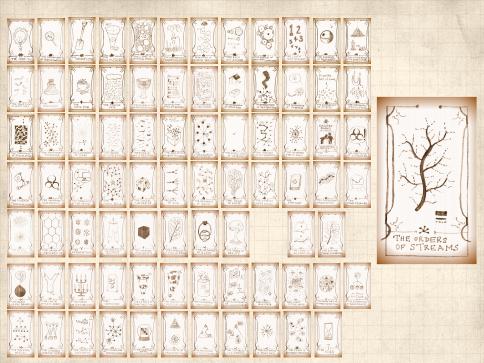
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1. Label all source streams as order $\omega = 1$ and remove.

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1. Label all source streams as order $\omega = 1$ and remove.

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- 1. Label all source streams as order $\omega = 1$ and remove.
- 2. Label all new source streams as order $\omega = 2$ and remove.

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- 1. Label all source streams as order $\omega = 1$ and remove.
- 2. Label all new source streams as order $\omega = 2$ and remove.

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- 1. Label all source streams as order $\omega = 1$ and remove.
- 2. Label all new source streams as order $\omega = 2$ and remove.
- 3. Repeat until one stream is left (order = Ω)

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- 1. Label all source streams as order $\omega = 1$ and remove.
- 2. Label all new source streams as order $\omega = 2$ and remove.
- 3. Repeat until one stream is left (order = Ω)
- 4. Basin is said to be of the order of the last stream removed.

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- 1. Label all source streams as order $\omega = 1$ and remove.
- 2. Label all new source streams as order $\omega = 2$ and remove.
- 3. Repeat until one stream is left (order = Ω)
- Basin is said to be of the order of the last stream removed.
- 5. Example above is a basin of order $\Omega = 3$.

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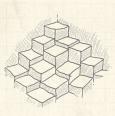
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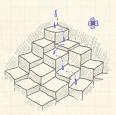
Structure Detection

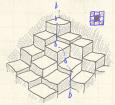
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Basic algorithm for extracting networks from Digital Elevation Models (DEMs):









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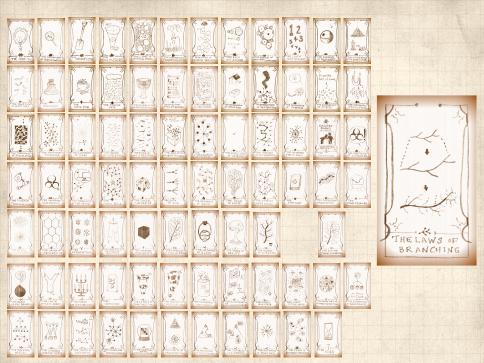
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🚳 Also:

/Users/dodds/work/rivers/1998dems/kevinlakewaster.c



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Horton's laws

Self-similarity of river networks

First quantified by Horton (1945)^[53], expanded by Schumm (1956)^[88] The PoCSverse Complex Networks 79 of 320

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Horton's laws

Self-similarity of river networks

First quantified by Horton (1945)^[53], expanded by Schumm (1956)^[88]

Three laws:

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First quantified by Horton (1945)^[53], expanded by Schumm (1956) [88]

Three laws:

Horton's law of stream numbers:

$$n_{\omega}/n_{\omega+1}=R_n>1$$

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First guantified by Horton (1945)^[53], expanded by Schumm (1956) [88]

Three laws:

Horton's law of stream numbers:

$$n_{\omega}/n_{\omega+1}=R_n>1$$

Horton's law of stream lengths:

$$\boxed{\bar{\ell}_{\omega+1}/\bar{\ell}_{\omega}=R_{\ell}>1}$$

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First quantified by Horton (1945)^[53], expanded by Schumm (1956) [88]

Three laws:

Horton's law of stream numbers:

$$n_{\omega}/n_{\omega+1}=R_n>1$$

Horton's law of stream lengths:

$$\bar{\ell}_{\omega+1}/\bar{\ell}_{\omega}=R_{\ell}>1$$



\Lambda Horton's law of basin areas:

$$\bar{a}_{\omega+1}/\bar{a}_{\omega}=R_a>1$$

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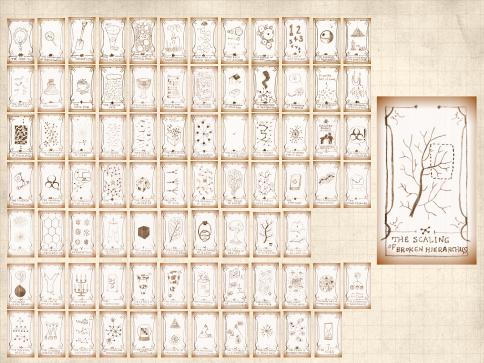
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Network Architecture Tokunaga's law^[101, 102, 103]

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Property 1: Scale independence—depends only on difference between orders:

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Property 1: Scale independence—depends only on difference between orders:

$$T_{\mu,\nu} = T_{\mu-\nu}$$

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Property 1: Scale independence—depends only on difference between orders:

$$T_{\mu,\nu} = T_{\mu-\nu}$$

Property 2: Number of side streams grows exponentially with difference in orders:

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Property 1: Scale independence—depends only on difference between orders:

$$T_{\mu,\nu} = T_{\mu-\nu}$$

Property 2: Number of side streams grows exponentially with difference in orders:

 $T_{\mu,\nu} = T_1 (R_T)^{\mu - \nu - 1}$

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Property 1: Scale independence—depends only on difference between orders:

$$T_{\mu,\nu} = T_{\mu-\nu}$$

Property 2: Number of side streams grows exponentially with difference in orders:

 $T_{\mu,\nu} = T_1 (R_T)^{\mu-\nu-1}$

We usually write Tokunaga's law as:

 $T_k = T_1(R_T)^{k-1}$ where $R_T \simeq 2$

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Connecting exponents

Only 3 parameters are independent: e.g., take d, R_n , and R_s

relation:	scaling relation/parameter: ^[31]
$\ell \sim L^d$	d
$T_k = T_1 (R_T)^{k-1}$	$T_1 = R_n - R_s - 2 + 2R_s/R_n$
	$R_T = \frac{R_s}{R_s}$
$n_{\omega}/n_{\omega+1}=R_n$	R_n
$\bar{a}_{\omega+1}/\bar{a}_{\omega}=R_a$	$R_a = R_n$
$\ell_{\omega+1}/\ell_\omega=R_\ell$	$R_{\ell} = R_s$
$\ell \sim a^h$	$h = \ln R_s / \ln R_n$
$a \sim L^D$	D = d/h
$L_{\perp} \sim L^H$	H = d/h - 1
$P(a) \sim a^{-\tau}$	$\tau=2-h$
$P(\ell) \sim \ell^{-\gamma}$	$\gamma = 1/h$
$\Lambda \sim a^\beta$	$\beta = 1 + h$
$\lambda \sim L^{\varphi}$	$\varphi = d$

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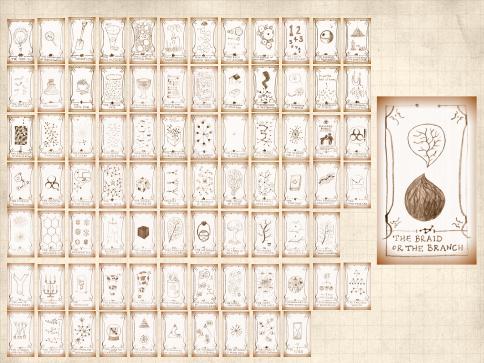
Major Models Generalized Affiliation Networks Thresholds

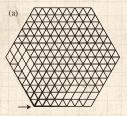
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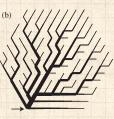
Structure Detection

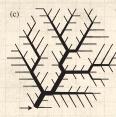
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(a) γ > 1: Braided (bulk) flow
(b) γ < 1: Local minimum: Branching flow
(c) γ < 1: Global minimum: Branching flow
Note: This is a single source supplying a region.

From Bohn and Magnasco^[16] See also Banavar *et al.*^[6]: "Topology of the Fittest Transportation Network"; focus is on presence or absence of loops—same story The PoCSverse Complex Networks 85 of 320

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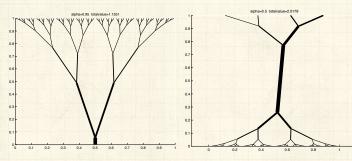
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Optimal paths related to transport (Monge) problems C:



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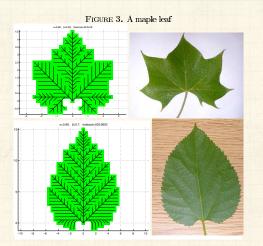
References



ar Water, Karles Balantes, Na Marian ar Maria (2000) Roy (2000) Ro

"Optimal paths related to transport problems" Qinglan Xia, Communications in Contemporary Mathematics, **5**, 251–279, 2003.^[116]

Growing networks: [117]



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δ Top: $\alpha = 0.66$, $\beta = 0.38$; Bottom: $\alpha = 0.66$, $\beta = 0.70$

An immensely controversial issue ...

The form of natural branching networks: Random, optimal, or some combination?^[55, 113, 7, 33, 27] The PoCSverse Complex Networks 88 of 320

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An immensely controversial issue ...

- The form of natural branching networks: Random, optimal, or some combination? ^[55, 113, 7, 33, 27]
- 🚳 River networks, blood networks, trees, ...

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An immensely controversial issue ...

- The form of natural branching networks: Random, optimal, or some combination? ^[55, 113, 7, 33, 27]
- 🚳 River networks, blood networks, trees, ...

Two observations:

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An immensely controversial issue ...

- The form of natural branching networks: Random, optimal, or some combination? ^[55, 113, 7, 33, 27]
- 🚓 River networks, blood networks, trees, ...

Two observations:

Self-similar networks appear everywhere in nature for single source supply/single sink collection. The PoCSverse Complex Networks 88 of 320

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An immensely controversial issue ...

- The form of natural branching networks: Random, optimal, or some combination? ^[55, 113, 7, 33, 27]
 - 🗞 River networks, blood networks, trees, ...

Two observations:

- Self-similar networks appear everywhere in nature for single source supply/single sink collection.
- Real networks differ in details of scaling but reasonably agree in scaling relations.

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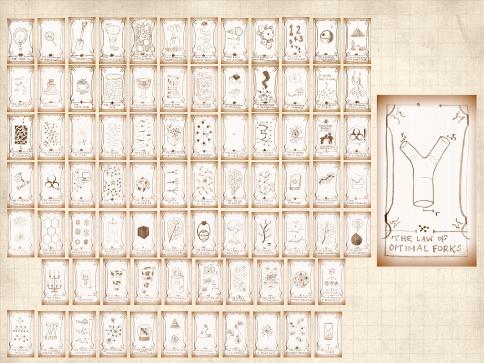
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Murray's law (1926) connects branch radii at forks: ^[72, 71, 73, 59, 100] The PoCSverse Complex Networks 90 of 320

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 $r_{\rm parent}^3 = r_{\rm offspring1}^3 + r_{\rm offspring2}^3 r_{\rm supply Networks}^{\rm Basic Properties}$

where r_{parent} = radius of 'parent' branch, and $r_{\text{offspring1}}$ and $r_{\text{offspring2}}$ are radii of the two 'offspring' sub-branches. Random networks

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Murray's law (1926) connects branch radii at forks: ^[72, 71, 73, 59, 100]

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radii of the two 'offspring'

'parent' branch, and

sub-branches.

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 $r_{\rm parent}^3 = r_{\rm offspring1}^3 + r_{\rm offspring2}^3 {\rm Basic Properties} {\rm Basic Properties} {\rm Properties}$

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Holds up well for outer branchings of blood networks^[90].

Murray's law (1926) connects branch radii at forks: ^[72, 71, 73, 59, 100]

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'parent' branch, and

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 $r_{\rm parent}^3 = r_{\rm offspring1}^3 + r_{\rm offspring2}^3 {\rm Basic Properties} {\rm Basic Properties} {\rm Properties}$

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 Holds up well for outer branchings of blood networks ^[90].
 Also found to hold for troos ^[73, 66] when yulom

Also found to hold for trees ^[73, 66] when xylem is not a supporting structure ^[67].

Murray's law (1926) connects branch radii at forks: ^[72, 71, 73, 59, 100] The PoCSverse Complex Networks 90 of 320

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 $r_{\rm parent}^3 = r_{\rm offspring1}^3 + r_{\rm offspring2}^{\rm Basic Properties}_{\rm supply Networks}$

where r_{parent} = radius of 'parent' branch, and $r_{\text{offspring1}}$ and $r_{\text{offspring2}}$ are radii of the two 'offspring' sub-branches.

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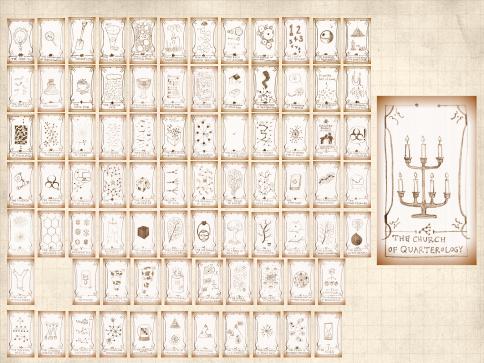
Big Nutshell

References



Holds up well for outer branchings of blood networks^[90].

- Also found to hold for trees ^[73, 66] when xylem is not a supporting structure ^[67].
- See D'Arcy Thompson's "On Growth and Form" for background and general inspiration ^[99, 100].



Animal power

Fundamental biological and ecological constraint:

 $P = c \, M^{\,\alpha}$

P = basal metabolic rate M = organismal body mass





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Animal power

Fundamental biological and ecological constraint:

 $P = c \, M^{\,\alpha}$

P = basal metabolic rate M = organismal body mass







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Stories—The Fraction Assassin:





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- 🚳 1964: Troon, Scotland.
- 🚳 3rd Symposium on Energy Metabolism.



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- 🚳 1964: Troon, Scotland.
- 🚳 3rd Symposium on Energy Metabolism.
- $\approx \alpha = 3/4$ made official ...



... 29 to zip.

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- 🚳 1964: Troon, Scotland.
- 🚳 3rd Symposium on Energy Metabolism.



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But the Cabal slipped up by publishing the conference proceedings ...

- 🚳 1964: Troon, Scotland.
- 🚳 3rd Symposium on Energy Metabolism.



But the Cabal slipped up by publishing the conference proceedings ...

"Energy Metabolism; Proceedings of the 3rd symposium held at Troon, Scotland, May 1964," Ed. Sir Kenneth Blaxter^[13]

... 29 to zip.

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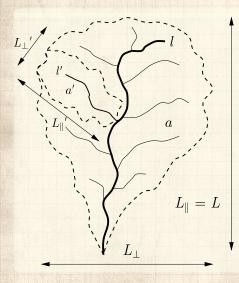
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Somehow, optimal river networks are connected:



 $\begin{array}{l} \bigotimes \ a = \text{drainage} \\ \text{basin area} \\ \end{array} \\ \begin{array}{l} \bigotimes \ \ell = \text{length of} \\ \text{longest (main)} \\ \text{stream} \\ \end{array} \\ \begin{array}{l} \bigotimes \ L = L_{\parallel} = \end{array}$

 $L - L_{\parallel}$ longitudinal length of basin The PoCSverse Complex Networks 95 of 320

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1957: J. T. Hack^[50] "Studies of Longitudinal Stream Profiles in Virginia and Maryland"

> $\ell \sim a^h$ $h \sim 0.6$

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1957: J. T. Hack^[50] "Studies of Longitudinal Stream Profiles in Virginia and Maryland"

 $h \sim 0.6$

 $\ell \sim a^h$

 \clubsuit Anomalous scaling: we would expect $h = 1/2 \dots$

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1957: J. T. Hack^[50] "Studies of Longitudinal Stream Profiles in Virginia and Maryland"

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Subsequent studies: $0.5 \leq h \leq 0.6$

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1957: J. T. Hack^[50] "Studies of Longitudinal Stream Profiles in Virginia and Maryland"

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Subsequent studies: $0.5 \leq h \leq 0.6$

Another quest to find universality/god ...

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1957: J. T. Hack^[50] "Studies of Longitudinal Stream Profiles in Virginia and Maryland"

 $h \sim 0.6$

 $\ell \sim a^h$

Anomalous scaling: we would expect h = 1/2 ...
Subsequent studies: $0.5 \leq h \leq 0.6$ Another quest to find universality/god ...
A catch: studies done on small scales.

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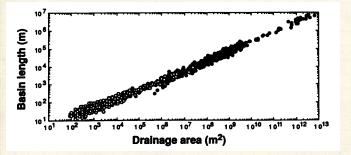
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Large-scale networks: (1992) Montgomery and Dietrich^[69]:



 Composite data set: includes everything from unchanneled valleys up to world's largest rivers.
 Estimated fit:

$$L \simeq 1.78a^{0.49}$$

Mixture of basin and main stream lengths.

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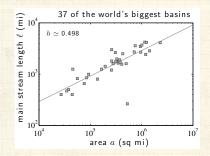
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World's largest rivers only:



Data from Leopold (1994)^[60, 32]
 Estimate of Hack exponent: $h = 0.50 \pm 0.06$

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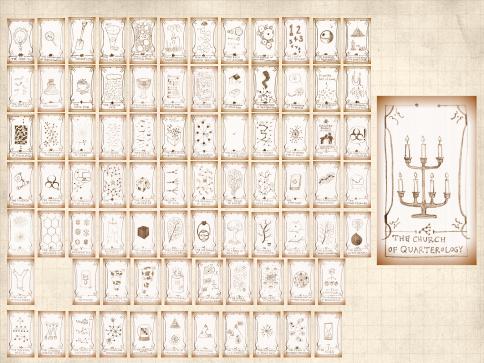
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Nutrient delivering networks:

1960's: Rashevsky considers blood networks and finds a 2/3 scaling. The PoCSverse Complex Networks 100 of 320

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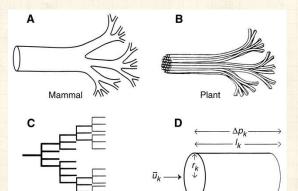
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Nutrient delivering networks:

- 1960's: Rashevsky considers blood networks and finds a 2/3 scaling.
- 1997: West *et al.* ^[113] use a network story to find 3/4 scaling.



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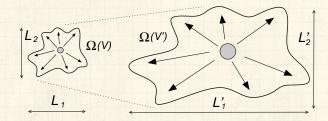
Model

k = 0

Parameters

Geometric argument

Allometrically growing regions:



 \mathfrak{S} Have d length scales which scale as

$$L_i \propto V^{\gamma_i}$$
 where $\gamma_1 + \gamma_2 + \ldots + \gamma_d = 1$.

For isometric growth, \(\gamma_i = 1/d\).
For allometric growth, we must have at least two of the \{\(\gamma_i\)\} being different

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Spherical cows and pancake cows:

Assume an isometrically Scaling family of cows:

Extremes of allometry: The pancake cows-



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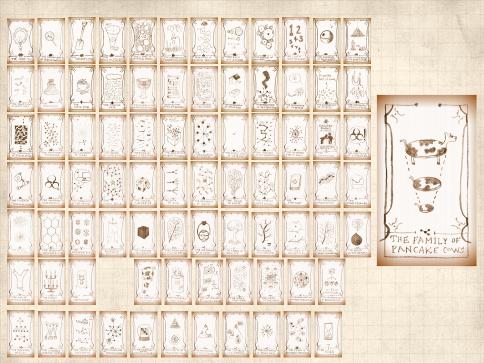
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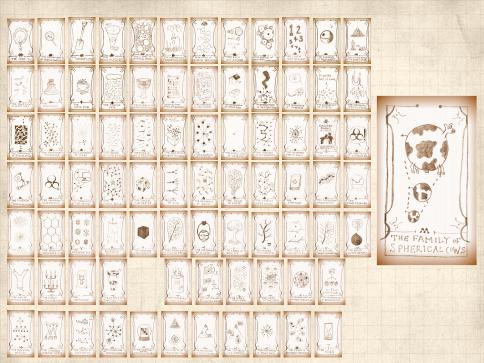
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Minimal network volume:

Real supply networks are close to optimal:

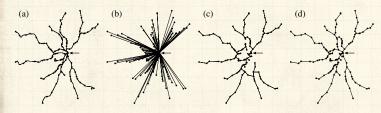


Figure 1. (a) Commuter rail network in the Boston area. The arrow marks the assumed root of the network. (b) Star graph. (c) Minimum spanning tree. (d) The model of equation (3) applied to the same set of stations.

Gastner and Newman (2006): "Shape and efficiency in spatial distribution networks" ^[41]

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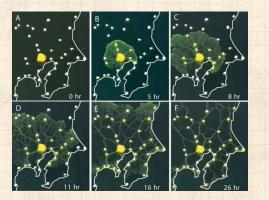
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And a second sec

"Rules for Biologically Inspired Adaptive Network Design" Tero et al., Science, **327**, 439-442, 2010.^[98]



Urban deslime in action: https://www.youtube.com/watch?v=GwKuFREOgmo The PoCSverse Complex Networks 106 of 320

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Then *P*, the rate of overall energy use in Ω, can at most scale with volume as

 $P\propto \rho V$

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Then *P*, the rate of overall energy use in Ω, can at most scale with volume as

 $P\propto \rho V\propto \rho\,M$

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Then *P*, the rate of overall energy use in Ω, can at most scale with volume as

 $P\propto \rho V\propto \rho\,M\propto M^{\,(d-1)/d}$

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Then *P*, the rate of overall energy use in Ω, can at most scale with volume as

 $P\propto \rho V\propto \rho\,M\propto M^{\,(d-1)/d}$

$rac{2}{8}$ For d = 3 dimensional organisms, we have

$$P \propto M^{2/3}$$

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Then *P*, the rate of overall energy use in Ω, can at most scale with volume as

 $P\propto \rho V\propto \rho\,M\propto M^{\,(d-1)/d}$

 $rac{2}{8}$ For d = 3 dimensional organisms, we have

 $P \propto M^{\,2/3}$

Including other constraints may raise scaling exponent to a higher, less efficient value.

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Exciting bonus: Scaling obtained by the supply network story and the surface-area law only match for isometrically growing shapes.

The surface area-supply network mismatch for allometrically growing shapes:

SerV1-

P oc Ninks of V

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Volume of water in river network can be calculated by adding up basin areas

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Volume of water in river network can be calculated by adding up basin areas
 Flows sum in such a way that

 $V_{\mathsf{net}} = \sum a_{\mathsf{pixel } i}$ all pixels

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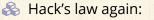
Structure Detection

Big Nutshell



Volume of water in river network can be calculated by adding up basin areas
 Flows sum in such a way that

$$V_{\sf net} = \sum_{\sf all \ pixels} a_{\sf pixels}$$



$$\ell \sim a^h$$

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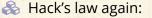
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Big Nutshell



Volume of water in river network can be calculated by adding up basin areas
 Flows sum in such a way that

$$V_{\mathsf{net}} = \sum_{\mathsf{all pixels}} a_{\mathsf{pixel } i}$$



$$\ell \sim a^h$$

Can argue

$$V_{\rm net} \propto V_{\rm basin}^{1+h} = a_{\rm basin}^{1+h}$$

where h is Hack's exponent.

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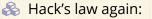
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$$\ell \sim a^h$$

🚳 Can argue

$$V_{\rm net} \propto V_{\rm basin}^{1+h} = a_{\rm basin}^{1+h}$$

$$h = 1/2$$

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3

Banavar et al.'s approach^[7] is okay because ρ really is constant.

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Banavar et al.'s approach^[7] is okay because ρ really is constant.

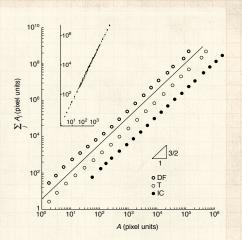


Figure 2 Allometric scaling in river networks. Double logarithmic plot of $C \propto \Sigma_{xe_x} A_x$ versus A for three river networks characterized by different climates, geology and geographic locations (Dry Fork, West Virginia, 586 km², digital terrain map (DTM) size 30 × 30 m²; Island Creek, Idaho, 260 km², DTM size 30 × 30 m²; Tirso, Italy, 2,024 km², DTM size 237 × 237 m²). The experimental points are obtained by binning total contributing areas, and computing the ensemble average of the sum of the inner areas for each sub-basin within the binned interval. The figure uses pixel units in which the smallest area element is assigned a unit value. Also plotted is the predicted scaling relationship with slope 3/2. The inset shows the raw data from the Tirso basin before any binning has been done

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Banavar et al.'s approach^[7] is okay because ρ really is constant.
 The irony: shows optimal basins are isometric

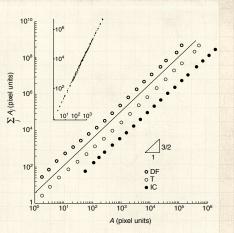


Figure 2 Allometric scaling in river networks. Double logarithmic plot of $C \propto \Sigma_{xeA} A_x$ versus A for three river networks characterized by different climates, geology and geographic locations (Dv Fork, West Viriginia, 568 km², digital terrain map (DTM) size 30 × 30 m²; Island Creek, Idaho, 260 km², DTM size 30 × 30 m²; Tirso, Italy, 2,024 km², DTM size 237 × 237 m³). The experimental points are obtained by binning total contributing areas, and computing the ensemble average of the sum of the inner areas for each sub-basin within the binned interval. The figure uses pixel units in which the smallest area element is assigned a unit value. Also plotted is the predicted scaling relationship with slope 3/2. The inset shows the raw data from the Tirso basin before any binning base base does does and the sum of the inner areas the raw shows the raw data from the Tirso basin before any binning base base does does and the sum of the sum of the first base base base base based based based bases are element is assigned a unit value. Also plotted is the predicted scaling relationship with slope 5/2. The inset shows the raw data from the Tirso basin before any binning base based based

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Banavar et al.'s approach^[7] is okay because ρ really is constant.

The irony: shows optimal basins are isometric

Solution Optimal Hack's law: $\ell \sim a^h$ with h = 1/2

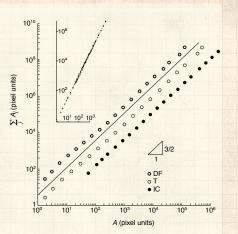


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Solution Optimal Hack's law: $\ell \sim a^h$ with h = 1/2(Zzzz)

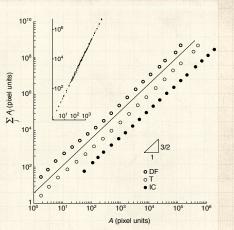


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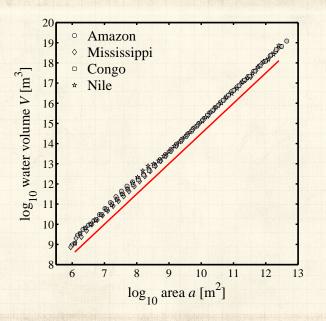
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Even better—prefactors match up:



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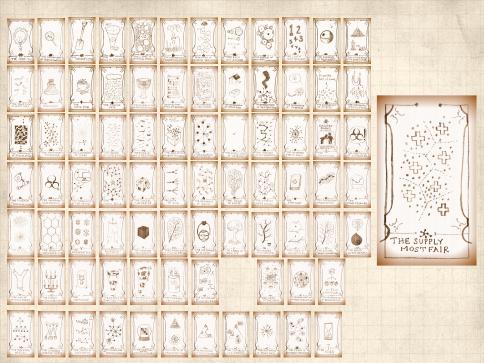
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"Optimal design of spatial distribution networks" Gastner and Newman, Phys. Rev. E, **74**, 016117, 2006. ^[40] The PoCSverse Complex Networks 113 of 320

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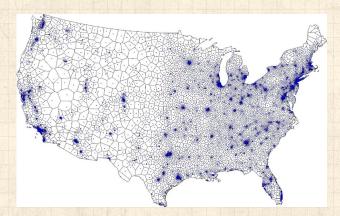
Major Models Generalized Affiliation Networks Thresholds

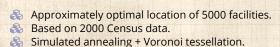
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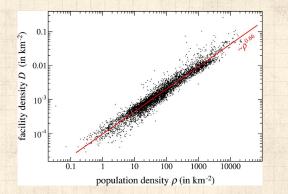
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Optimal source allocation



Solution density ρ_{fac} vs. population density ρ_{pop} .

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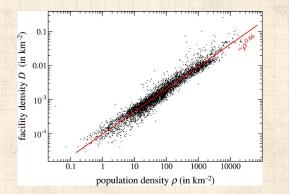
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Optimal source allocation



Optimal facility density \(\rho_{fac}\) vs. population density \(\rho_{pop}\).
 Fit is \(\rho_{fac}\) \(\phi\) \(\rho_{pop}\)^{0.66} with \(r^2 = 0.94\).

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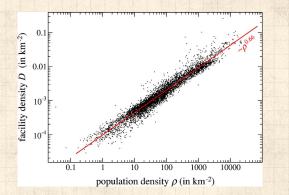
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Optimal source allocation



Optimal facility density \(\rho_{fac}\) vs. population density \(\rho_{pop}\).
 Fit is \(\rho_{fac}\) \(\phi\) \(\rho_{pop}\)^{0.66} with \(r^2 = 0.94\).
 Looking good for a 2/3 power ...

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Basic idea: Minimize the average distance from a random individual to the nearest facility.^[40]

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Basic idea: Minimize the average distance from a random individual to the nearest facility. ^[40]
 Accume given a fixed population density of the second secon

Solution Assume given a fixed population density ρ_{pop} defined on a spatial region Ω .

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- Basic idea: Minimize the average distance from a random individual to the nearest facility. ^[40]
- Solution Assume given a fixed population density ρ_{pop} defined on a spatial region Ω .
- Sources $\{\vec{x}_1, \dots, \vec{x}_n\}$ that minimizes the cost function

$$F(\{\vec{x}_1,\ldots,\vec{x}_n\}) = \int_{\Omega} \rho_{\mathsf{pop}}(\vec{x}) \min_i ||\vec{x} - \vec{x}_i|| \mathsf{d}\vec{x}$$

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Deriving the optimal source distribution:

- Basic idea: Minimize the average distance from a random individual to the nearest facility. ^[40]
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Also known as the p-median problem, and connected to cluster analysis. The PoCSverse Complex Networks 115 of 320

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 Also known as the p-median problem, and connected to cluster analysis.
 Not easy ...in fact this one is an NP-hard

problem.^[40]

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Deriving the optimal source distribution:

- Basic idea: Minimize the average distance from a random individual to the nearest facility. ^[40]
- \mathfrak{F} Assume given a fixed population density ρ_{pop} defined on a spatial region Ω .
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🚳 Also known as the p-median problem, and connected to cluster analysis.

- Not easy ... in fact this one is an NP-hard problem.^[40]
- Approximate solution originally due to 3 Gusein-Zade^[49].

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One more thing:

How do we supply these facilities?

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One more thing:

- low do we supply these facilities?
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One more thing:

- How do we supply these facilities?
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- 🚳 How do we get beer to the pubs?

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One more thing:

- How do we supply these facilities?
- 🗞 How do we best redistribute mail? People?
- \lambda How do we get beer to the pubs?
- Gastner and Newman model: cost is a function of basic maintenance and travel time:

$$C_{\mathsf{maint}} + \gamma C_{\mathsf{travel}}$$

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Travel time is more complicated: Take 'distance' between nodes to be a composite of shortest path distance ℓ_{ij} and number of legs to journey:

 $(1-\delta)\ell_{ij} + \delta(\#hops).$

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Travel time is more complicated: Take 'distance' between nodes to be a composite of shortest path distance ℓ_{ij} and number of legs to journey:

 $(1-\delta)\ell_{ij} + \delta(\#hops).$

& When $\delta = 1$, only number of hops matters.

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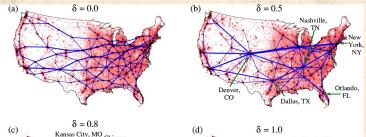
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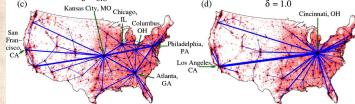
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From Gastner and Newman (2006)^[40]

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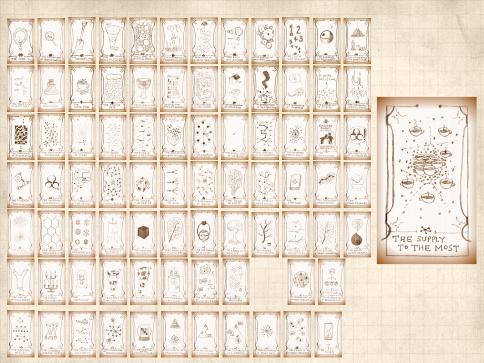
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Beyond minimizing distances:

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Beyond minimizing distances:

"Scaling laws between population and facility densities" by Um *et al.*, Proc. Natl. Acad. Sci., 2009.^[104] The PoCSverse Complex Networks 119 of 320

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Beyond minimizing distances:

- "Scaling laws between population and facility densities" by Um *et al.*, Proc. Natl. Acad. Sci., 2009.^[104]
- Um et al. find empirically and argue theoretically that the connection between facility and population density

 $ho_{
m fac} \propto
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does not universally hold with $\alpha = 2/3$.

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Beyond minimizing distances:

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m fac} \propto
ho_{
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does not universally hold with $\alpha = 2/3$. **Two idealized limiting classes:** 1. For-profit, commercial facilities: $\alpha = 1$; The PoCSverse Complex Networks 119 of 320

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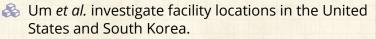
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 $ho_{\rm fac} \propto
ho_{
m pop}^{lpha}$

does not universally hold with $\alpha = 2/3$. Solution Weightson We

- 1. For-profit, commercial facilities: $\alpha = 1$;
- 2. Pro-social, public facilities: $\alpha = 2/3$.



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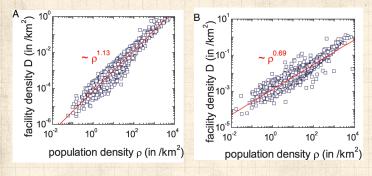
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Public versus private facilities: evidence



Left plot: ambulatory hospitals in the U.S.
 Right plot: public schools in the U.S.

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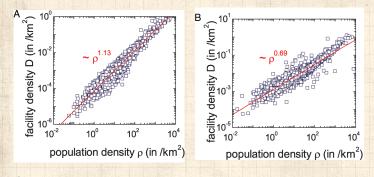
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Public versus private facilities: evidence



Left plot: ambulatory hospitals in the U.S.
Right plot: public schools in the U.S.
Note: break in scaling for public schools. Transition from \$\alpha\$ \approx 2/3 to \$\alpha\$ = 1 around \$\rho_{pop}\$ \approx 100. The PoCSverse Complex Networks 120 of 320 The PoCSverse

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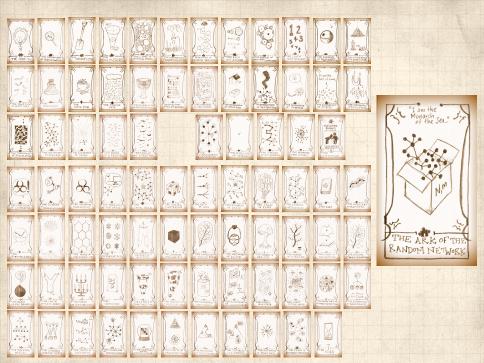


Public versus private facilities: evidence

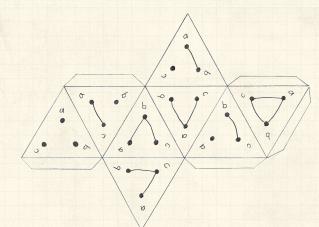
US facility	α (SE)	R ²	
Ambulatory hospital	1.13(1)	0.93	
Beauty care	1.08(1)	0.86	
Laundry	1.05(1)	0.90	
Automotive repair	0.99(1)	0.92	
Private school	0.95(1)	0.82	
Restaurant	0.93(1)	0.89	
Accommodation	0.89(1)	0.70 R	20
Bank	0.88(1)	0.80	
Gas station	0.86(1)	0.94 C	pet
Death care	0.79(1)	0.80 a	in
* Fire station	0.78(3)	0.93	
* Police station	0.71(6)	0.75 C	χ -
Public school	0.69(1)	0.87	
SK facility	α (SE)	_{R2} N	10
Bank	1.18(2)	_{0.96} a	ina
Parking place	1.13(2)	0.91 c	ta
* Primary clinic	1.09(2)	1.00	la
* Hospital	0.96(5)	0.97	ev
* University/college	0.93(9)	0.89	-
Market place	0.87(2)	0.90 C	0
* Secondary school	0.77(3)	0.98	
* Primary school	0.77(3)	0.97	
Social welfare org.	0.75(2)	0.84	
* Police station	0.71(5)	0.94	
Government office	0.70(1)	0.93	
* Fire station	0.60(4)	0.93	
* Public health center	0.09(5)	0.19	

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Random network generator for N = 3:



Set your own exciting generator here ☑.
 As N ↗, polyhedral die rapidly becomes a ball ...

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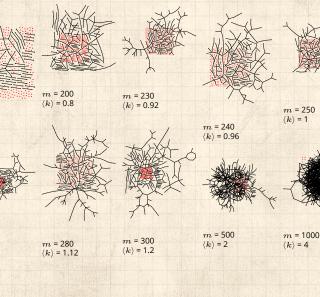
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Random networks: examples for N=500



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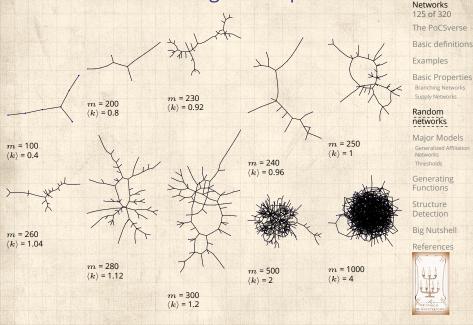


m = 260 $\langle k \rangle = 1.04$

m = 100

(k) = 0.4

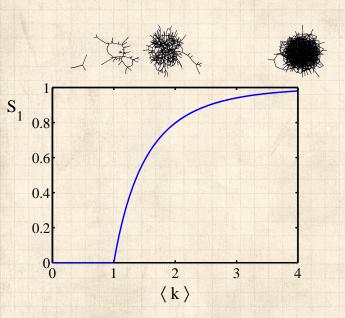
Random networks: largest components



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Complex

Giant component



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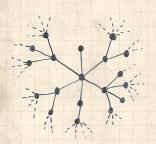
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Clustering in random networks:



So for large random networks ($N \rightarrow \infty$), clustering drops to zero.

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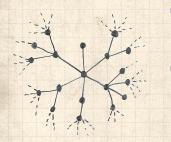
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Clustering in random networks:



So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks

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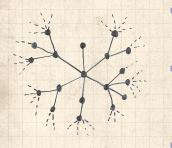
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Clustering in random networks:



So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks
 No small loops.

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Recall P_k = probability that a randomly selected node has degree k.

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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.

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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.

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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1-p).

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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1-p).
- Therefore have a binomial distribution 🗹:

$$P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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Solution: $P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$ What happens as $N \to \infty$? The PoCSverse Complex Networks 129 of 320

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- Solution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$
- \mathfrak{S} What happens as $N \to \infty$?

We must end up with the normal distribution right? The PoCSverse Complex Networks 129 of 320

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- We must end up with the normal distribution right?
- Solution If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \rightarrow \infty$.
- \mathfrak{S} But we want to keep $\langle k \rangle$ fixed ...

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- \bigotimes But we want to keep $\langle k \rangle$ fixed ...
- So examine limit of P(k; p, N) when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1)$ = constant.

$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \rightarrow \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

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 \mathfrak{B} This is a Poisson distribution \mathfrak{C} with mean $\langle k \rangle$.

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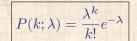
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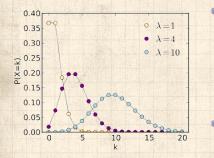
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Poisson basics:







 $\lambda > 0$ $k = 0, 1, 2, 3, \dots$ 🚳 Classic use: probability that an event occurs ktimes in a given time period, given an average rate of occurrence. 3 e.g.:

phone calls/minute, horse-kick deaths. 'Law of small numbers' The PoCSverse Complex Networks 130 of 320

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Generalized random networks:

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Generalized random networks: Arbitrary degree distribution P_k .

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Generalized random networks:

- \mathfrak{S} Arbitrary degree distribution P_k .
- Create (unconnected) nodes with degrees sampled from P_k.

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Generalized random networks:

- \mathfrak{S} Arbitrary degree distribution P_k .
- Solution Create (unconnected) nodes with degrees sampled from P_k .
- 🚳 Wire nodes together randomly.

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Generalized random networks:

- & Arbitrary degree distribution P_k .
- Solution Create (unconnected) nodes with degrees sampled from P_k .
- 🚳 Wire nodes together randomly.
- Create ensemble to test deviations from randomness.

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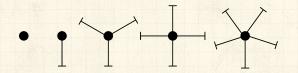
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Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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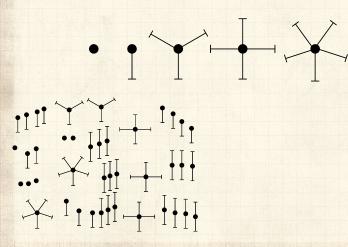
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Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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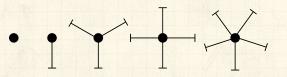
Structure Detection

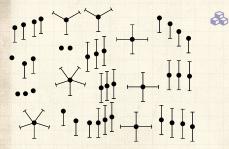
Big Nutshell



Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. The PoCSverse Complex Networks 132 of 320

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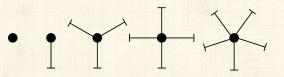
Structure Detection

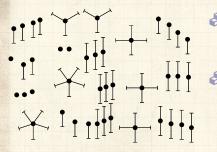
Big Nutshell



Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs. The PoCSverse Complex Networks 132 of 320

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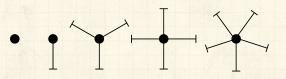
Structure Detection

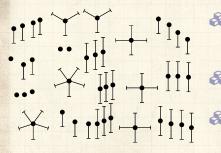
Big Nutshell



Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- and repeat connections. The PoCSverse Complex Networks 132 of 320

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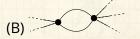


Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





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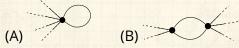
Big Nutshell



Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



Being careful: we can't change the degree of any node, so we can't simply move links around. The PoCSverse Complex Networks 133 of 320

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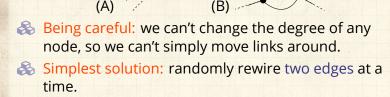
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Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



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e1

Randomly choose two edges. (Or choose problem edge and a random edge) The PoCSverse Complex Networks 134 of 320

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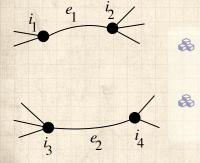
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 Randomly choose two edges.
 (Or choose problem edge and a random edge)

 Check to make sure edges are disjoint. The PoCSverse Complex Networks 134 of 320

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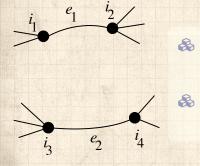
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Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.



Rewire one end of each edge.

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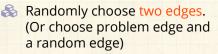
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 e_1'



Check to make sure edges are disjoint.

Rewire one end of each edge.
 Node degrees do not change.

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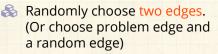
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e'



Check to make sure edges are disjoint.

Rewire one end of each edge.

- Node degrees do not change.
- Works if e₁ is a self-loop or repeated edge.

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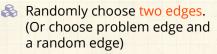
Generating Functions

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Big Nutshell



e'



Check to make sure edges are disjoint.

- Rewire one end of each edge.
 - Node degrees do not change.
 - Works if e₁ is a self-loop or repeated edge.
 - Same as finding on/off/on/off 4-cycles. and rotating them.

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Sampling random networks

Phase 2:

Use rewiring algorithm to remove all self and repeat loops. The PoCSverse Complex Networks 135 of 320

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Sampling random networks

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally. The PoCSverse Complex Networks 135 of 320

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Sampling random networks

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Rule of thumb: # Rewirings $\simeq 10 \times \#$ edges^[68].

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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks. The PoCSverse Complex Networks 136 of 320

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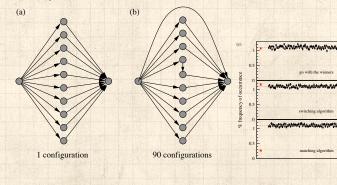
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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.
 Example from Milo et al. (2003)^[68]:



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Idea of motifs^[89] introduced by Shen-Orr, Alon et al. in 2002.

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- Idea of motifs^[89] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.

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- Idea of motifs^[89] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.

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Big Nutshell



- Idea of motifs^[89] introduced by Shen-Orr, Alon et al. in 2002.
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- Directed network with 577 interactions (edges) and 424 operons (nodes).

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- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .

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Big Nutshell



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- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

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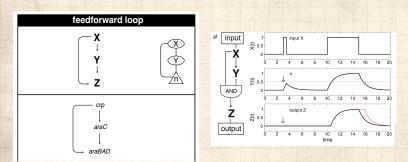
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 \mathbb{R} Z only turns on in response to sustained activity in X.

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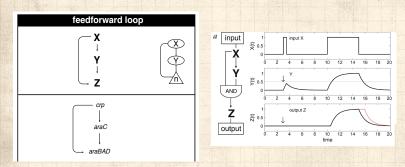
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Network motifs



Z only turns on in response to sustained activity in X.

 \mathfrak{S} Turning off X rapidly turns off Z.

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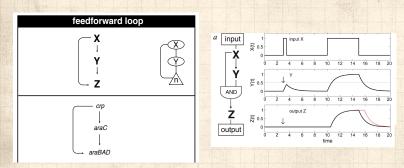
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Network motifs



rightarrow Z only turns on in response to sustained activity in X.

- \mathfrak{S} Turning off X rapidly turns off Z.
- \lambda Analogy to elevator doors.

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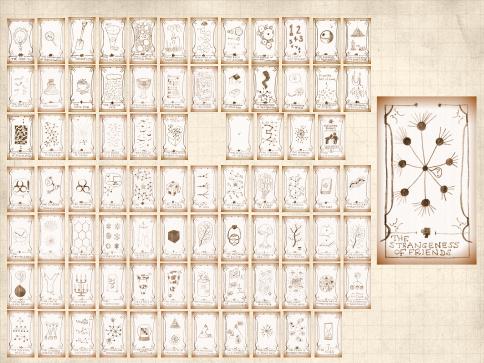
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The degree distribution P_k is fundamental for our description of many complex networks

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- The degree distribution P_k is fundamental for our description of many complex networks
- rightarrow Again: P_k is the degree of randomly chosen node.



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- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
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- Solution Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.

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- Solution Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$

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- Solution Again: P_k is the degree of randomly chosen node.
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- Solution Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- 🚳 Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$

Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

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Big Nutshell



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- Solution Again: P_k is the degree of randomly chosen node.
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- Solution Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- 🚳 Now choosing nodes based on their degree (i.e., size):

Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty}k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

 $Q_k \propto k P_k$

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Big Nutshell



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- Solution Again: P_k is the degree of randomly chosen node.
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- Solution Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- 🚳 Now choosing nodes based on their degree (i.e., size):

Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

 $Q_k \propto k P_k$

Big deal: Rich-get-richer mechanism is built into this selection process.

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For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends. The PoCSverse Complex Networks 141 of 320

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For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

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For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

R

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R

For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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R

For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.

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R

For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.
 Natural question: what's the expected number of other friends that one friend has?

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Probability of randomly selecting a node of degree kby choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$ The PoCSverse Complex Networks 142 of 320

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Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16, Q_2 = 4/16,$ $Q_3 = 3/16, Q_6 = 6/16.$

Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel: $R_0 = 3/16 R_1 = 4/16$,

 $R_2 = 3/16, R_5 = 6/16.$

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Reason #1:

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Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

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Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right)$$

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Reason #1:

Average # friends of friends per node is

$$\langle k_2
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ight) = \langle k^2
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Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

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🚳 Three peculiarities:

1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.

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🚳 Three peculiarities:

- 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
- 2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big.

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Reason #1:

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Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

🚳 Three peculiarities:

- 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
- 2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big. (e.g., in the case of a power-law distribution)

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Reason #1:

Average # friends of friends per node is

$$egin{aligned} &\langle k_2
angle = \langle k
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angle_R = \langle k
angle rac{1}{\langle k
angle} \left(\langle k^2
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 Your friends really are different from you ...^[37, 76]

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- 4. See also: class size paradoxes (nod to: Gelman)

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"Generalized friendship paradox in complex networks: The case of scientific collaboration" Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014. ^[35]

Your friends really are monsters #winners:¹

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¹Some press here C [MIT Tech Review].



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- Other horrific studies: your connections on Twitter have more followers than you, are happier than you^[17], more sexual partners than you, ...
- The hope: Maybe they have more enemies and diseases too.
- 🗞 Research possibility: The Frenemy Paradox.

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Spreading on Random Networks



🙈 For random networks, we know local structure is pure branching.

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- For random networks, we know local structure is pure branching.
- Successful spreading is .. contingent on single edges infecting nodes.

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- For random networks, we know local structure is pure branching.
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Success Failure:

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- For random networks, we know local structure is pure branching.
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Success Failure:

Focus on binary case with edges and nodes either infected or not.

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- For random networks, we know local structure is pure branching.
- Successful spreading is .. contingent on single edges infecting nodes.

Success Failure:

Focus on binary case with edges and nodes either infected or not.

First big question: for a given network and contagion process, can global spreading from a single seed occur? The PoCSverse Complex Networks 145 of 320

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We need to find: ^[30]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.

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- We need to find: ^[30]
 R = the average # of infected edges that one random infected edge brings about.
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- Solution Define B_{k1} as the probability that a node of degree k is infected by a single infected edge.

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 $\mathbf{R} = \sum$

2

 kP_k $\langle k \rangle$

prob. of connecting to a degree k node The PoCSverse Complex Networks 146 of 320

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 kP_k

prob. of connecting to a degree k node

(k - 1)

outgoing infected edges The PoCSverse Complex Networks 146 of 320

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 $\mathbf{R} = \sum_{i=1}^{n}$

3

 $\frac{kP_k}{\langle k\rangle}$

prob. of connecting to a degree k node

outgoing infected edges

Prob. of infection

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 kP_k

$$+\sum_{k=0}^\infty \frac{\widehat{kP_k}}{\langle k\rangle}$$

outgoing infected edges

Prob. of infection

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prob. of connecting to a degree k node

 $+\sum_{k=0}^{\infty}\frac{kP_k}{\langle k\rangle}$

outgoing
infected
edges

```
# outgoing
infected
edges
```

 B_{k1} Prob. of infection

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 R = the average # of infected edges that one random infected edge brings about.
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 $\mathbf{R} = \sum$

prob. of connecting to a degree *k* node

(k - 1)

outgoing infected edges

 $\underbrace{B_{k1}}_{\text{Prob. of infection}}$

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 $+\sum_{k=0}^{\infty}\frac{\widehat{kP_k}}{\langle k\rangle}$

outgoing infected edges

 $(1 - B_{k1})$

Prob. of no infection

Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

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Our global spreading condition is then:

$$\label{eq:R} \mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

🗞 Case 1–Rampant spreading:

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🚳 Our global spreading condition is then:

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Solution Case 1–Rampant spreading: If $B_{k1} = 1$



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🚳 Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

Solution Case 1–Rampant spreading: If $B_{k1} = 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

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Our global spreading condition is then:

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Good: This is just our giant component condition again.

Case 2—Simple disease-like:

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So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$



 \bigotimes Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

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So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1$$

\mathfrak{S} A fraction (1- β) of edges do not transmit infection.

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So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1$$

A fraction (1-β) of edges do not transmit infection.
 Analogous phase transition to giant component case but critical value of (k) is increased.

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So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

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 Aka bond percolation C.

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So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1$$

A fraction (1-β) of edges do not transmit infection.
 Analogous phase transition to giant component case but critical value of (k) is increased.

Aka bond percolation C.

Resulting degree distribution \tilde{P}_k :

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

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Random directed networks:

- So far, we've largely studied networks with undirected, unweighted edges.
- 🚳 Now consider directed, unweighted edges.
- Nodes have k_i and k_o incoming and outgoing edges, otherwise random.
- Network defined by joint in- and out-degree distribution: P_{k_i,k_o}
- Normalization: $\sum_{k_i=0}^{\infty} \sum_{k_o=0}^{\infty} P_{k_i,k_o} = 1$
 - Marginal in-degree and out-degree distributions:

$$P_{k_{\mathrm{i}}} = \sum_{k_{\mathrm{o}}=0}^{\infty} P_{k_{\mathrm{i}},k_{\mathrm{o}}} \text{ and } P_{k_{\mathrm{o}}} = \sum_{k_{\mathrm{i}}=0}^{\infty} P_{k_{\mathrm{i}},k}$$

Required balance:

$$\langle k_{\rm i}\rangle = \sum_{k_{\rm i}=0}^{\infty}\sum_{k_{\rm o}=0}^{\infty}k_{\rm i}P_{k_{\rm i},k_{\rm o}} = \sum_{k_{\rm i}=0}^{\infty}\sum_{k_{\rm o}=0}^{\infty}k_{\rm o}P_{k_{\rm i},k_{\rm o}} = \langle k_{\rm o}\rangle$$

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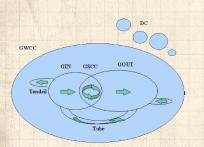
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Directed network structure:



From Boguñá and Serano.^[15]

GWCC = Giant Weakly Connected Component (directions removed);

GIN = Giant In-Component;

2

3

2

GOUT = Giant Out-Component;

GSCC = Giant Strongly Connected Component;

DC = Disconnected Components (finite). The PoCSverse Complex Networks 150 of 320

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When moving through a family of increasingly connected directed random networks, GWCC usually appears before GIN, GOUT, and GSCC which tend to appear together. ^[80, 15]

Observation:

- Directed and undirected random networks are separate families ...
- 🚳 ...and analyses are also disjoint.
- Need to examine a larger family of random networks with mixed directed and undirected edges.



Consider nodes with three types of edges:

- 1. $k_{\rm u}$ undirected edges,
- 2. k_i incoming directed edges,
- 3. k_{o} outgoing directed edges.

Define a node by generalized degree:

$$\vec{k} = [k_{\mathrm{u}} k_{\mathrm{i}} k_{\mathrm{o}}]^{\mathrm{T}}.$$

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Correlations:

🛞 Now add correlations (two point or Markovian) 🗆:

- 1. $P^{(u)}(\vec{k} | \vec{k}')$ = probability that an undirected edge leaving a degree \vec{k}' nodes arrives at a degree \vec{k} node.
- P⁽ⁱ⁾(k | k') = probability that an edge leaving a degree k' nodes arrives at a degree k node is an in-directed edge relative to the destination node.
 P^(o)(k | k') = probability that an edge leaving a degree k' nodes arrives at a degree k node is an out-directed edge relative to the destination node.

Now require more refined (detailed) balance.
 Conditional probabilities cannot be arbitrary.
 1. P^(u)(k | k') must be related to P^(u)(k' | k).

2. $P^{(0)}(\vec{k} | \vec{k}')$ and $P^{(i)}(\vec{k} | \vec{k}')$ must be connected.

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Correlations—Undirected edge balance:

- Randomly choose an edge, and randomly choose one end.
- Say we find a degree \vec{k} node at this end, and a degree \vec{k}' node at the other end.
- \clubsuit Define probability this happens as $P^{(u)}(\vec{k},\vec{k}')$.
 - Observe we must have $P^{(u)}(\vec{k}, \vec{k}') = P^{(u)}(\vec{k}', \vec{k})$.

 $\begin{array}{l} \bigotimes \\ \text{Conditional probability} \\ \text{connection:} \\ P^{(\mathsf{u})}(\vec{k},\vec{k}') &= P^{(\mathsf{u})}(\vec{k} \,|\, \vec{k}') \frac{k'_{\mathsf{u}} P(\vec{k}')}{\langle k'_{\mathsf{u}} \rangle} \end{array}$

 $P^{(\mathsf{u})}(\vec{k}',\vec{k}) = P^{(\mathsf{u})}(\vec{k}' \mid \vec{k}) \frac{k_{\mathsf{u}} P(\vec{k})}{\langle k_{\mathsf{u}} \rangle}.$

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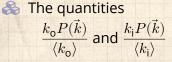
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Correlations—Directed edge balance:



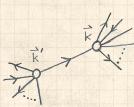
give the probabilities that in starting at a random end of a randomly selected edge, we begin at a degree \vec{k} node and then find ourselves travelling:

- 1. along an outgoing edge, or
- 2. against the direction of an incoming edge.

🚳 We therefore have

$$P^{(\mathsf{dir})}(\vec{k},\vec{k}') = P^{(\mathsf{i})}(\vec{k}\,|\,\vec{k}')\frac{k_{\mathsf{o}}'P(\vec{k}')}{\langle k_{\mathsf{o}}' \rangle} = P^{(\mathsf{o})}(\vec{k}'\,|\,\vec{k})\frac{k_{\mathsf{i}}P(\vec{k})}{\langle k_{\mathsf{i}} \rangle}$$

Note that $P^{(\text{dir})}(\vec{k}, \vec{k}')$ and $P^{(\text{dir})}(\vec{k}', \vec{k})$ are in general not related if $\vec{k} \neq \vec{k}'$.



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Summary of contagion conditions for uncorrelated networks:

 \mathfrak{R} I. Undirected, Uncorrelated— $f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{\mathsf{u}}} P^{(\mathsf{u})}(k_{\mathsf{u}} \,|\, *) \bullet (k_{\mathsf{u}} - 1) \bullet B_{k_{\mathsf{u}}, *}$$

 \mathfrak{R} II. Directed, Uncorrelated— $f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{\mathrm{i}}, k_{\mathrm{o}}} P^{(\mathrm{i})}(k_{\mathrm{i}}, k_{\mathrm{o}} \,|\, *) \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}}, *}$$

🚳 III. Mixed Directed and Undirected, Uncorrelated—

$$\begin{bmatrix} f^{(\mathrm{u})}(d+1) \\ f^{(\mathrm{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathrm{u})}(d) \\ f^{(\mathrm{o})}(d) \end{bmatrix}$$
$$\mathbf{R} = \sum_{\vec{k}} \begin{bmatrix} P^{(\mathrm{u})}(\vec{k} \mid *) \bullet (k_{\mathrm{u}} - 1) & P^{(\mathrm{i})}(\vec{k} \mid *) \bullet k_{\mathrm{u}} \\ P^{(\mathrm{u})}(\vec{k} \mid *) \bullet k_{\mathrm{o}} & P^{(\mathrm{i})}(\vec{k} \mid *) \bullet k_{\mathrm{o}} \end{bmatrix} \bullet B_{k_{\mathrm{u}}k_{\mathrm{i}}}$$

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Summary of contagion conditions for correlated networks:

$$R_{k_{\mathsf{u}}k_{\mathsf{u}}'} = P^{(\mathsf{u})}(k_{\mathsf{u}} \,|\, k_{\mathsf{u}}') \bullet (k_{\mathsf{u}} - 1) \bullet B_{k_{\mathsf{u}}k_{\mathsf{u}}'}$$

 $\textcircled{\begin{subarray}{c} \& \\ & \mathsf{Correlated}-f_{k_{\mathsf{i}}k_{\mathsf{o}}}(d+1) = \sum_{k_{\mathsf{i}}',k_{\mathsf{o}}'} R_{k_{\mathsf{i}}k_{\mathsf{o}}k_{\mathsf{i}}'k_{\mathsf{o}}'}f_{k_{\mathsf{i}}'k_{\mathsf{o}}'}(d) \end{array}$

$$R_{k_{\mathrm{i}}k_{\mathrm{o}}k_{\mathrm{i}}'k_{\mathrm{o}}'} = P^{(\mathrm{i})}(k_{\mathrm{i}},k_{\mathrm{o}}\,|\,k_{\mathrm{i}}',k_{\mathrm{o}}') \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}}k_{\mathrm{o}}k_{\mathrm{i}}'k_{\mathrm{o}}'}$$

🚳 VI. Mixed Directed and Undirected, Correlated—

$$\begin{bmatrix} f_{\vec{k}}^{(\mathrm{u})}(d+1) \\ f_{\vec{k}}^{(\mathrm{o})}(d+1) \end{bmatrix} = \sum_{k'} \mathbf{R}_{\vec{k}\vec{k}'} \begin{bmatrix} f_{\vec{k}'}^{(\mathrm{u})}(d) \\ f_{\vec{k}'}^{(\mathrm{o})}(d) \end{bmatrix}$$
$$\mathbf{R}_{\vec{k}\vec{k}'} = \begin{bmatrix} P^{(\mathrm{u})}(\vec{k} \mid \vec{k}') \bullet (k_{\mathrm{u}} - 1) & P^{(\mathrm{i})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{u}} \\ P^{(\mathrm{u})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{o}} & P^{(\mathrm{i})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{o}} \end{bmatrix} \bullet B_{\vec{k}\vec{k}'}$$

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Full generalization:

 $\vec{\alpha} = (\nu, \lambda)$

 $= (\nu', \lambda')$

$$f_{\vec{\alpha}}(d+1) = \sum_{\vec{\alpha}'} R_{\vec{\alpha}\vec{\alpha}'} f_{\vec{\alpha}'}(d)$$

 $R_{\vec{\alpha}\vec{\alpha}'}$ is the gain ratio matrix and has the form:

$$R_{\vec{\alpha}\vec{\alpha}'} = P_{\vec{\alpha}\vec{\alpha}'} \bullet k_{\vec{\alpha}\vec{\alpha}'} \bullet B_{\vec{\alpha}\vec{\alpha}'}$$

P_{\$\vec{a}\vec{a}\vec{a}'\$} = conditional probability that a type \$\lambda'\$ edge emanating from a type \$\nu'\$ node leads to a type \$\nu\$ node.

& k_{α̃α̃'} = potential number of newly infected edges of type λ emanating from nodes of type ν.
 & B_{α̃α̃'} = probability that a type ν node is eventually infected by a single infected type λ' link arriving from a neighboring node of type ν'.
 & Generalized contagion condition:

 $\max|\mu|:\mu\in\sigma\left(\mathbf{R}\right)>1$

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Some claims for social networks:

🙈 Social networks yes, but groups, groups, groups

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Some claims for social networks:

Social networks yes, but groups, groups, groups
 Sufficiently large social groups are:

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Some claims for social networks:

Social networks yes, but groups, groups, groups
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Social networks yes, but groups, groups, groups
 Sufficiently large social groups are:
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2. Pyramid Schemes,

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Social networks yes, but groups, groups, groups
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Homo narrativus: Storytellers, believers, spreaders.

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 - Stories \sim Characters + Time.

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- 1. Fandoms.
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- Homo narrativus: Storytellers, believers, spreaders.
- 🚳 Stories ~ Characters + Time.
- 🗞 Characters are shortcuts to stories.

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For novel diseases:

- 1. Can we predict the size of an epidemic?
- 2. How important is the reproduction number R_0 ?

R_0 approximately same for all of the following:

- 1918-19 "Spanish Flu" ~ 75,000,000 world-wide, 500,000 deaths in US.
- 1957-58 "Asian Flu" ~ 2,000,000 world-wide, 70,000 deaths in US.
- 1968-69 "Hong Kong Flu" ~ 1,000,000 world-wide, 34,000 deaths in US.
- 🗞 2003 "SARS Epidemic" ~ 800 deaths world-wide.

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Improving simple models

Idea for social networks: incorporate identity

Identity is formed from attributes such as:

- 🚳 Geographic location
- 🚳 Type of employment
- 🚳 Age
- 🚳 Recreational activities

Groups are crucial ...

- formed by people with at least one similar attribute
 - Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks.^[110]

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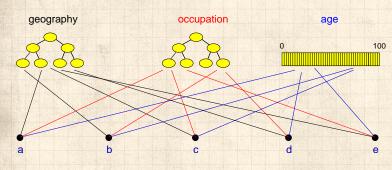
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Generalized context space



(Blau & Schwartz^[12], Simmel^[91], Breiger^[20])

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A toy agent-based model:



"Multiscale, resurgent epidemics in a hierarchcial metapopulation model" Watts et al., Proc. Natl. Acad. Sci., **102**, 11157–11162, 2005. [111]

Geography: allow people to move between contexts

- 🗞 Locally: standard SIR model with random mixing
- 🚳 discrete time simulation
- $\beta = infection probability$
- $rightarrow \gamma$ = recovery probability
- rightarrow P = probability of travel
- Solution Movement distance: $Pr(d) \propto exp(-d/\xi)$
- $\mathfrak{K} = \mathsf{typical travel distance}$

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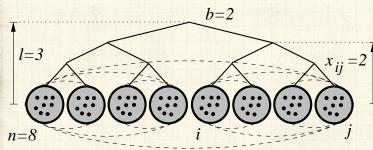
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A toy agent-based model

Schematic:



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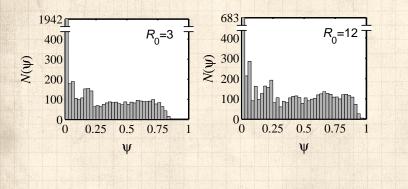
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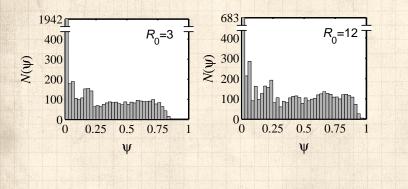
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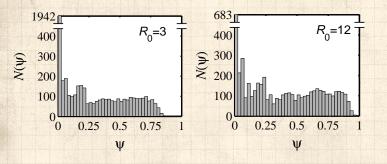
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 \mathfrak{S} Flat distributions are possible for certain ξ and P.

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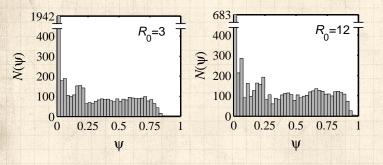
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Flat distributions are possible for certain ξ and P.
 Different R₀'s may produce similar distributions

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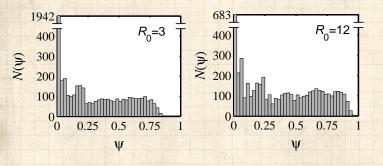
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Flat distributions are possible for certain ξ and P.
 Different R₀'s may produce similar distributions
 Same epidemic sizes may arise from different R₀'s

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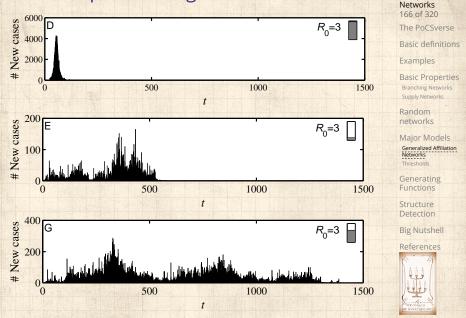
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Model output—resurgence



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Complex

Journal entry, 2020/02/21:

Twitter DMs to Sam Scarpino:

Solution Okay: The scientists studying pandemics need to be able to present some kind set of numbers that show how bad things are. The whole R_0 disaster has been waiting to happen because people have been ... lazily having fun with math models? Unconcerned about how to communicate vital scientific information? Stupid? I don't know. Maybe a radar plot visualization. I don't know.

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"When these three boundaries are crossed, we are in trouble" The PoCSverse Complex Networks 167 of 320

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- Solution Okay: The scientists studying pandemics need to be able to present some kind set of numbers that show how bad things are. The whole R_0 disaster has been waiting to happen because people have been ... lazily having fun with math models? Unconcerned about how to communicate vital scientific information? Stupid? I don't know. Maybe a radar plot visualization. I don't know.
- When these three boundaries are crossed, we are in trouble"
- Measles has an R_0 of 20. We should all have it. Of course, there's no f**king time scale for R_0 so we don't know when that happens.

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Understanding distributed social search

Milgram's social search experiment

THE MAN WHO Shocked the World

The Life and Legacy of Stanley Milgram



http://www.stanleymilgram.com

- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- 🚳 chain length \simeq 6.5.

Popular terms:

- The Small World Phenomenon;
- lix Degrees of Separation."

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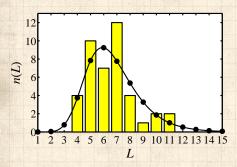
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The model—results

Milgram's Nebraska-Boston data:



Model parameters: $N = 10^8$, z = 300, g = 100, b = 10, $\alpha = 1, H = 2$;

$$\begin{array}{l} & \& \\ & \& \\ & L_{\mathsf{model}} \end{pmatrix} \simeq 6.5 \\ & & L_{\mathsf{data}} \simeq 6.5 \end{array}$$

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Social search—the Columbia experiment

local countries 60,000+ participants in 166 countries

- 🚳 18 targets in 13 countries including
 - a professor at an Ivy League university,
 - 文 an archival inspector in Estonia,
 - a technology consultant in India,
 - a policeman in Australia, and

a veterinarian in the Norwegian army.

🗞 24,000+ chains

We were lucky and contagious:

"Using E-Mail to Count Connections" 🕝, Sarah Milstein, New York Times, Circuits Section (December, 2001) The PoCSverse Complex Networks 171 of 320

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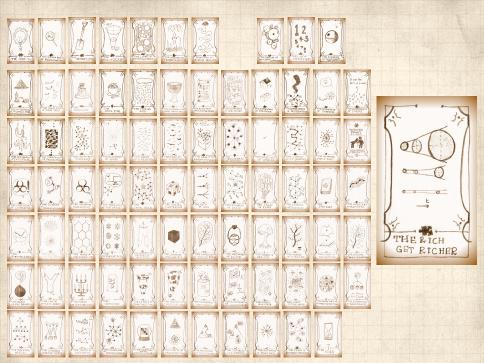
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Jonathan Harris's Wordcount:

A word frequency distribution explorer:

		Basic demnitions
WORDC	OUNT	Evenenles
		Examples
PREVIOUS WORD NE	XT WORD	Basic Properties
11		Branching Networks
		Supply Networks
		Random
1 2 3 4 5 5		networks
CURRENT WORD		Major Models
		Generalized Affiliation
FIND WORD: BY RANK: REQUESTED WORD: THE 86800 WORDS	IN ARCHIVE	Networks
RANK: 1 ABOUT V	VORDCOUNT	
		Generating
WORD	OUNT	Functions
		Structure
PREVIOUS WORD NE	XT WORD	Detection
		Big Nutshell
anitaharganaylaaturhanrannaha	Irol	
spitsbergeneylesturboproppaho		References
55059 55060 55061 55062	t is a	R. A
CURRENT WORD		
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FIND WORD: BY RANK: REQUESTED WORD: SPITSBERGEN 86600 WORDS RANK: 55059 ABOUT V	VORDCOUNT	THE CHURCH
10001 P		The WOWLEV-COL.

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Pacie definitions

The long tail of knowledge:



Take a scrolling voyage to the citational abyss, starting at the surface with the lonely, giant citaceans, moving down to the legion of strange, sometimes misplaced, unloved creatures, that dwell in Kahneman's Google Scholar page

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"Thing Explainer: Complicated Stuff in Simple Words " a, C by Randall Munroe (2015).^[70]

BOAT THAT GOES UNDER THE SEA

We've always had boats that go under the At first, we used those boats to shoot at Later, we found a new use for these boats sea, but in the last few hundred years, we've other boats, make holes in them, or stick keeping our city-burning machines hidden, learned to make ones that come back up. things to them that blew up.

safe, and ready to use if there's a war.

WORLD-ENDING BOAT

SLEEPING ROOMS

BREATHING STICK

SPECIAL SEA WORDS. HEAVY METAL POWER MACHINE.



EMPTY ROOMS -----

OTHER BOATS THAT GO UNDER THE SEA These are some other boats, drawn to show how big

MACHINES FOR BURNING CITIES.

MACHINES FOR SHOOTING BOATS

MIRROR LOOKERS

SOUND LOOKERS

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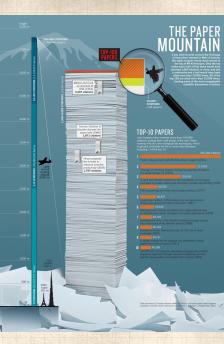
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Up goer five





Nature (2014): Most cited papers of all time

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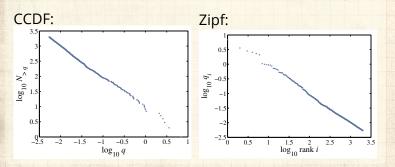
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Size distributions:

Brown Corpus (1,015,945 words):



The, of, and, to, a, ...= 'objects'
 'Size' = word frequency

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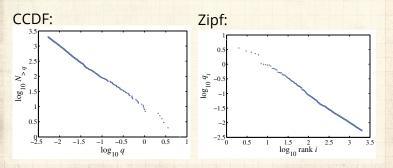
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Size distributions:

...





The, of, and, to, a, ...= 'objects'
 'Size' = word frequency
 Beep: (Important) CCDF and Zipf plots are related

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Pre-Zipf's law observations of Zipf's law

1910s: Word frequency examined re Stenography (a construction of brachygraphy or tachygraphy), Jean-Baptiste Estoup (a [36]. The PoCSverse Complex Networks 178 of 320

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1910s: Felix Auerbach pointed out the Zipfitude of city sizes in
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Species per Genus (offers first theoretical mechanism)

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1926: Lotka^[61]:

Scientific papers per author (Lotka's law)

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1949: Zipf's "Human Behaviour and the Principle of Least-Effort" is published. ^[120] The PoCSverse Complex Networks 179 of 320

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- 1953: Mandelbrot ^[62]: Optimality argument for Zipf's law; focus on language.

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- 1953: Mandelbrot ^[62]: Optimality argument for Zipf's law; focus on language.
- 1955: Herbert Simon ^[92, 120]: Zipf's law for word frequency, city size, income, publications, and species per genus.

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- 1965/1976: Derek de Solla Price ^[26, 83]: Network of Scientific Citations.

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- 1965/1976: Derek de Solla Price ^[26, 83]: Network of Scientific Citations.
- 1999: Barabasi and Albert^[8]: The World Wide Web, networks-at-large.

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Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at t = 1

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Random Competitive Replication (RCR):

- 1. Start with 1 elephant (or element) of a particular flavor at t = 1
- 2. At time t = 2, 3, 4, ..., add a new elephant in one of two ways:
 - With probability ρ, create a new elephant with a new flavor

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- 1. Start with 1 elephant (or element) of a particular flavor at t = 1
- 2. At time t = 2, 3, 4, ..., add a new elephant in one of two ways:
 - With probability ρ, create a new elephant with a new flavor

With probability $1 - \rho$, randomly choose from all existing elephants, and make a copy.

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Random Competitive Replication (RCR):

- 1. Start with 1 elephant (or element) of a particular flavor at t = 1
- 2. At time t = 2, 3, 4, ..., add a new elephant in one of two ways:
 - With probability ρ, create a new elephant with a new flavor

With probability $1 - \rho$, randomly choose from all existing elephants, and make a copy.

Elephants of the same flavor form a group

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 - With probability 1 ρ, randomly choose from all existing elephants, and make a copy.
 = Replication/Imitation

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Example: Words appearing in a language

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Example: Words appearing in a language

Consider words as they appear sequentially.
 With probability *ρ*, the next word has not previously appeared

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Example: Words appearing in a language

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Solution With probability $1 - \rho$, randomly choose one word from all words that have come before, and reuse this word

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Example: Words appearing in a language

- Consider words as they appear sequentially.
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Example: Words appearing in a language

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 With probability *ρ*, the next word has not previously appeared
 = Mutation/Innovation
- Solution With probability 1ρ , randomly choose one word from all words that have come before, and reuse this word
 - = Replication/Imitation

Note: This is a terrible way to write a novel.

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For example:



o 21 words used	0	21	words	used
-----------------	---	----	-------	------

- · next word is new with prob p
- next word is a Copy with prob 1- ρ prob: next word; $6/_{21}$ ook $4/_{21}$ the $3/_{21}$ and $2/_{21}$ penguin Y_{21} library

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$$\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}$$

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$$\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}$$

3 Observe $2 < \gamma < \infty$ for $0 < \rho < 1$.

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$$\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}$$

 $\gamma \simeq 2$

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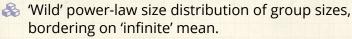
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$$\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}$$

$\gamma \simeq 2$

 $\gamma \simeq \infty$

Wild' power-law size distribution of group sizes, bordering on 'infinite' mean.
 For $\rho \simeq 1$ (high innovation rate):

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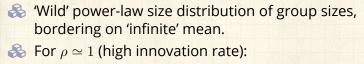
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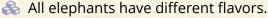
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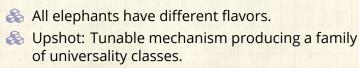


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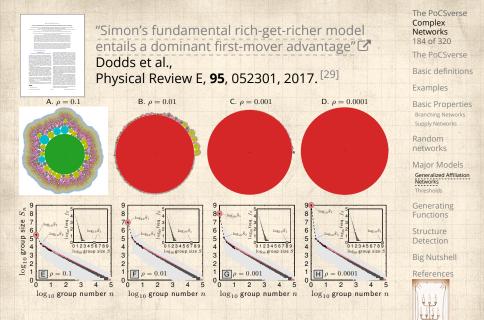
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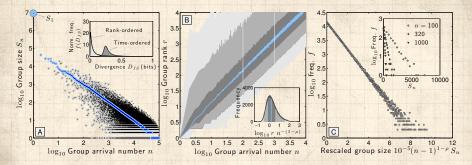
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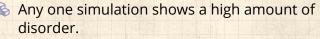




🚳 See visualization at paper's online app-endices 🗹

Arrival variability:

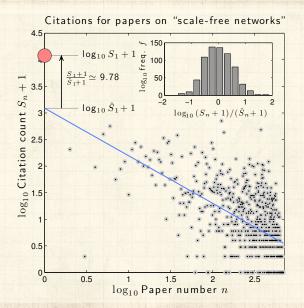




Two orders of magnitude variation in possible rank.

Rank ordering creates a smooth Zipf distribution.
 Size distribution for the *n*th arriving group show exponential decay.

Self-referential citation data:



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The Quickening C — Mandelbrot v. Simon: There Can Be Only One: C



Things there should be only one of: Theory, Highlander Films. The PoCSverse Complex Networks 187 of 320

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The Quickening C — Mandelbrot v. Simon: There Can Be Only One: C



Things there should be only one of: Theory, Highlander Films.

Feel free to play Queen's It's a Kind of Magic I' in your head (funding remains tight).

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We were born to be Princes of the Universe



vs.



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Mandelbrot vs. Simon:

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 - Simon (1960): "Some further notes on a class of skew distribution functions" ^[93]

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- Simon (1961): "Reply to 'final note' by Benoit Mandelbrot"^[95]

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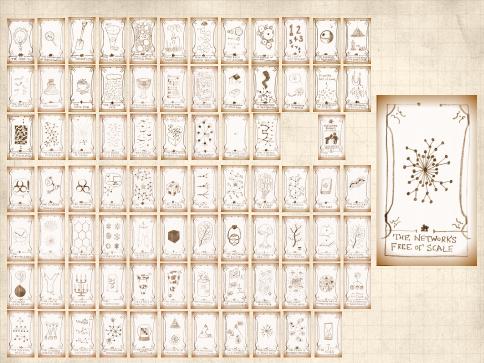
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Real networks with power-law degree distributions became known as scale-free networks.

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- Real networks with power-law degree distributions became known as scale-free networks.
- Scale-free refers specifically to the degree distribution having a power-law decay in its tail:

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- Real networks with power-law degree distributions became known as scale-free networks.
- Scale-free refers specifically to the degree distribution having a power-law decay in its tail:

 $P_k \sim k^{-\gamma}$ for 'large' k

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One of the seminal works in complex networks:



"Emergence of scaling in random networks" Barabási and Albert, Science, **286**, 509–511, 1999.^[8]

Times cited: ~ 43,853 🖸 (as of May 19, 2023)

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"Organization of Growing Random Networks" Krapivsky and Redner, Phys. Rev. E, **63**, 066123, 2001.^[57]

Fooling with the mechanism:

Krapivsky & Redner ^[57] explored the general attachment kernel: The PoCSverse Complex Networks 192 of 320

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Pr(attach to node *i*) $\propto A_k = k_i^{\nu}$

where A_k is the attachment kernel and $\nu > 0$.

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Krapivsky & Redner ^[57] explored the general attachment kernel:

 $\mathbf{Pr}(\text{attach to node } i) \propto A_k = k_i^{\nu}$

where A_k is the attachment kernel and $\nu > 0$. KR also looked at changing the details of the attachment kernel. The PoCSverse Complex Networks 192 of 320

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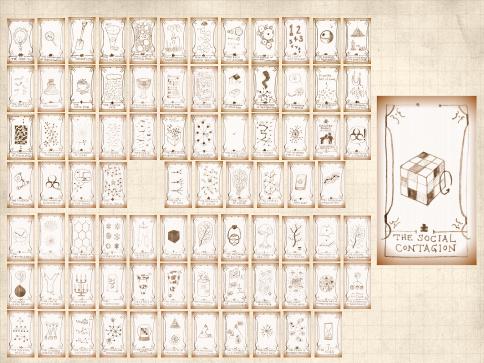
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'The rumor spread through the city like wildfire



"The Truth" **3** C by Terry Pratchett (2000). ^[82]

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'The rumor spread through the city like wildfire which had quite often spread through Ankh-Morpork



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The rumor spread through the city like wildfire which had quite often spread through Ankh-Morpork since its citizens had learned the words



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'The rumor spread through the city like wildfire which had quite often spread through Ankh-Morpork since its citizens had learned the words "fire insurance").'



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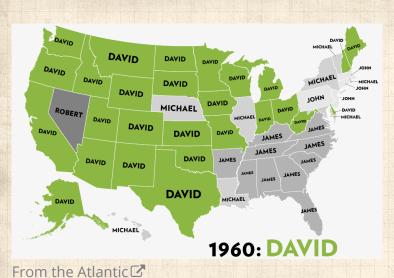
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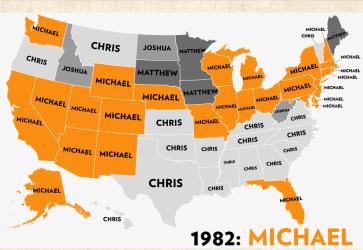
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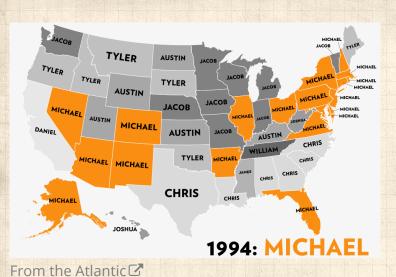
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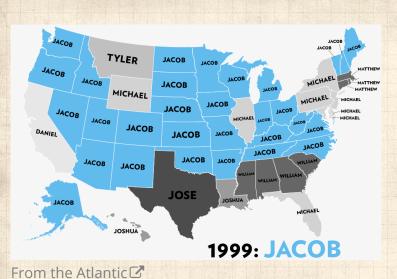
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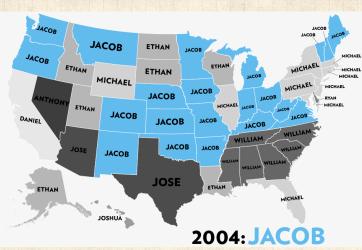
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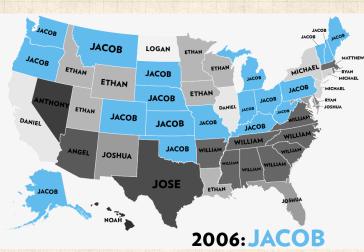
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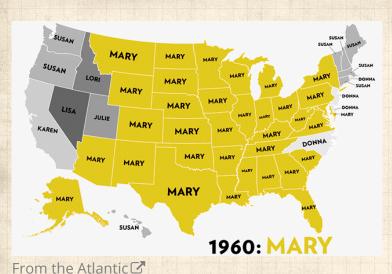
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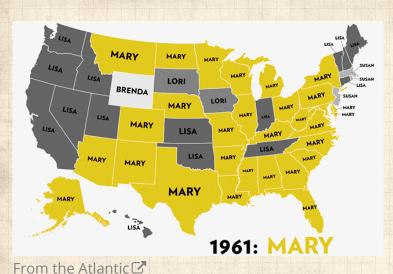
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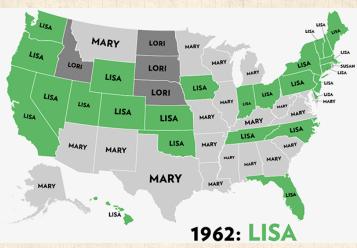
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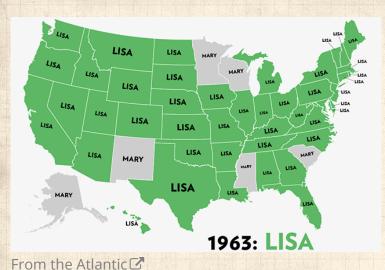
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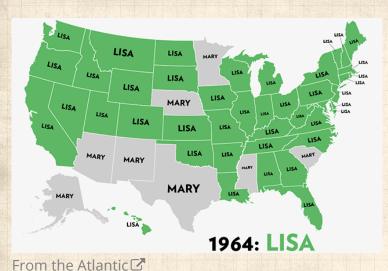
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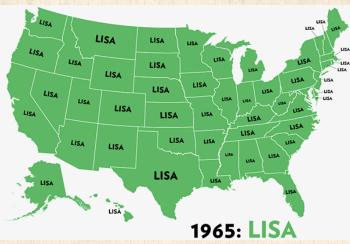
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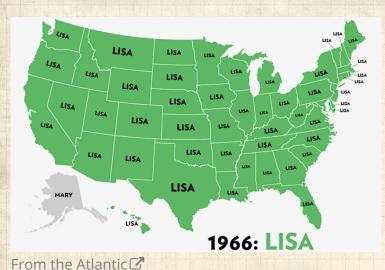
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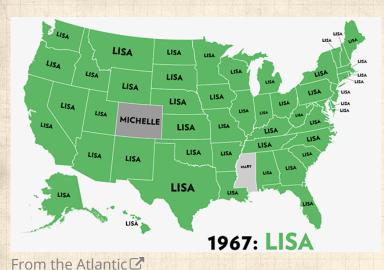
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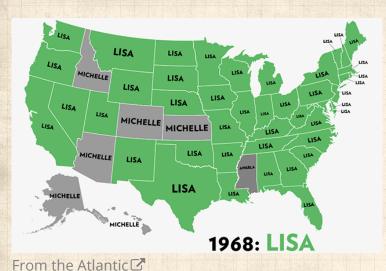
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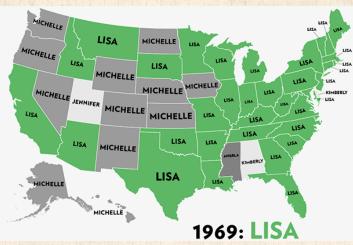
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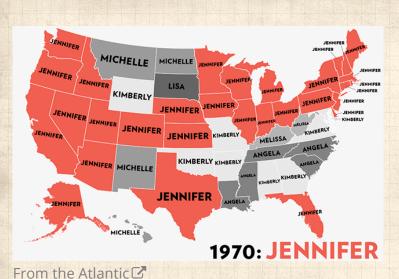
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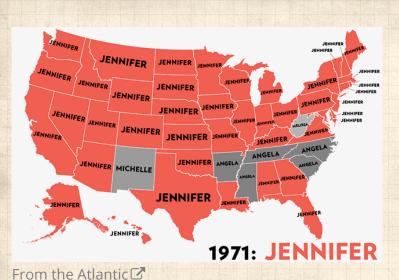
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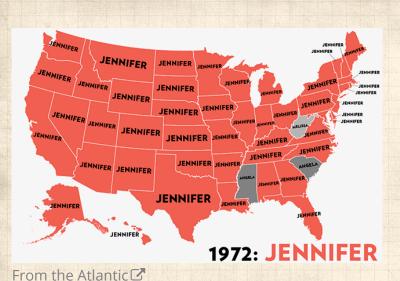
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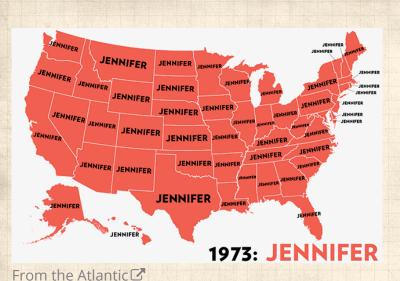
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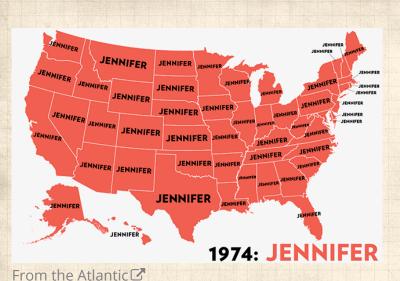
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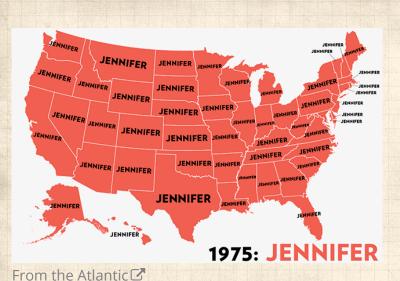
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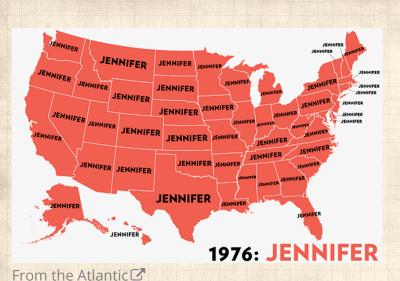
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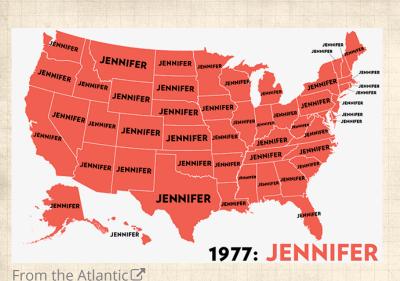
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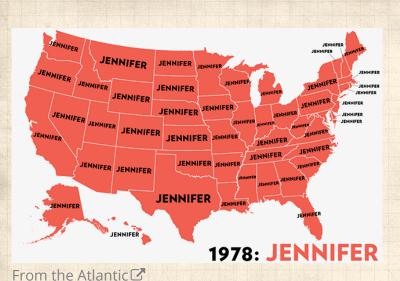
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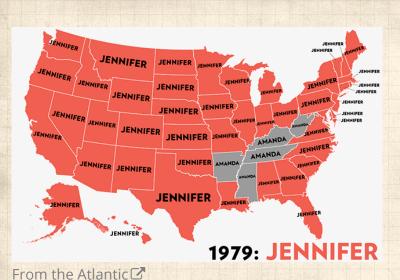
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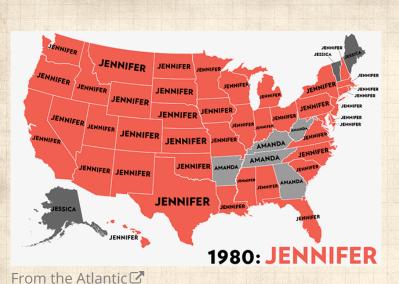
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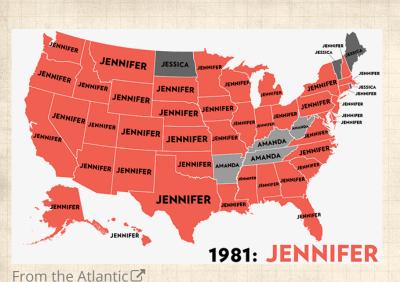
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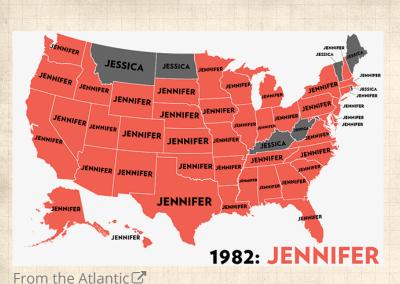
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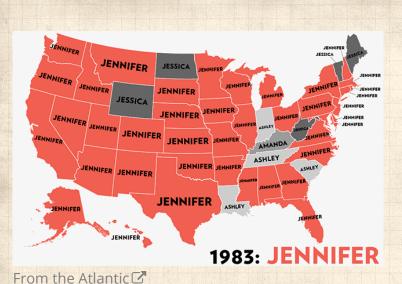
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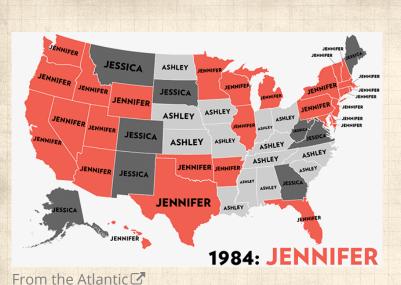
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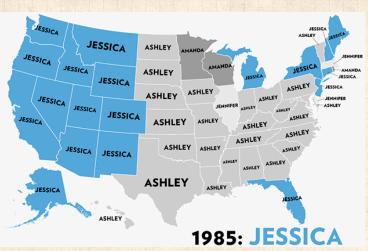
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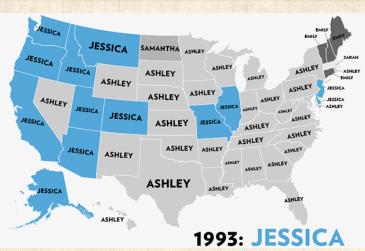
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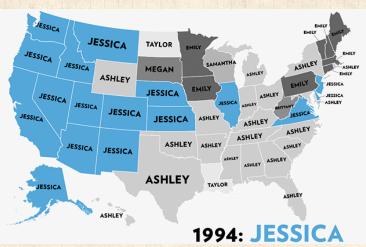
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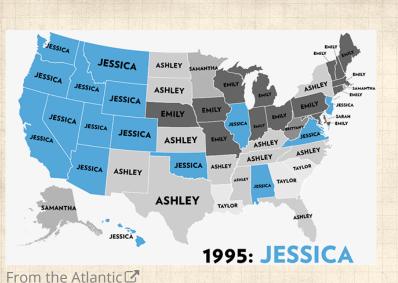
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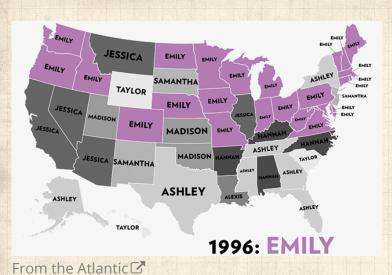
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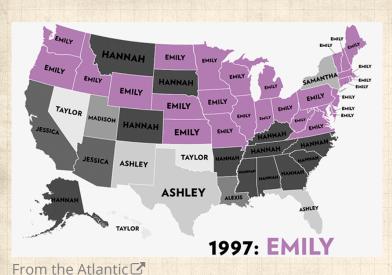
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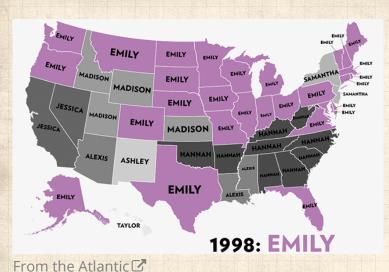
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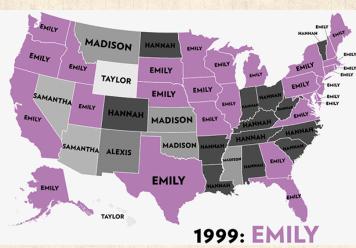
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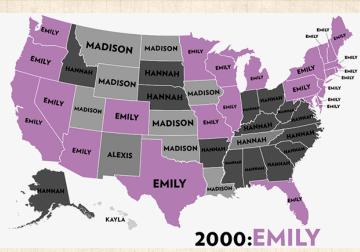
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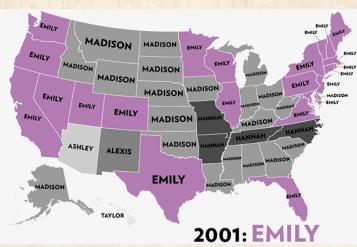
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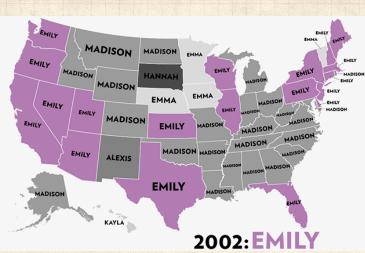
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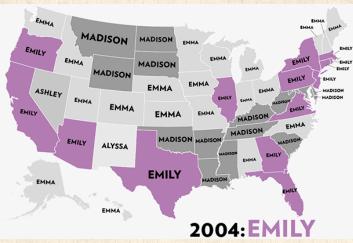
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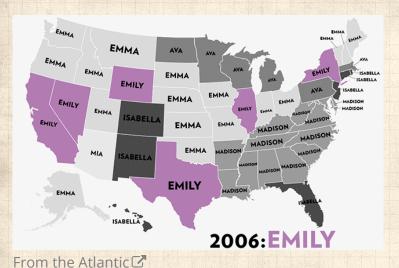
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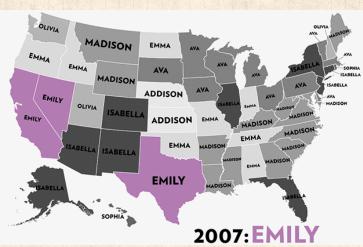
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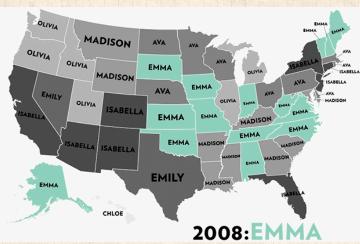
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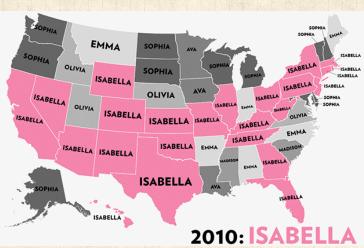
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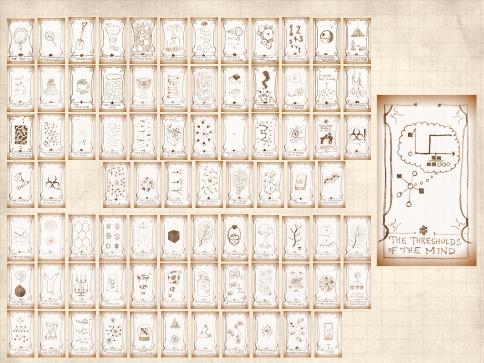
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Some important models:

Tipping models—Schelling (1971)^[85, 86, 87]

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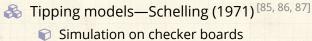
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Tipping models—Schelling (1971)^[85, 86, 87]

Simulation on checker boards
 Idea of thresholds

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Some important models:

Tipping models—Schelling (1971)^[85, 86, 87]

- Simulation on checker boards
- Idea of thresholds
- Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.)

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- Herding models—Bikhchandani, Hirschleifer, Welch (1992)^[10, 11]
 - Social learning theory, Informational cascades,...

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Thresholds

Basic idea: individuals adopt a behavior when a certain fraction of others have adopted

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Thresholds

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- Others' may be everyone in a population, an individual's close friends, any reference group.

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Social contagion models

Thresholds

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Some possible origins of thresholds:

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Some possible origins of thresholds:

Inherent, evolution-devised inclination to coordinate, to conform, to imitate.^[9]

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 - Economics: Network effects or network externalities
 - Externalities = Effects on others not directly involved in a transaction
 - Examples: telephones, fax machine, TikTok, operating systems
 - An individual's utility increases with the adoption level among peers and the population in general

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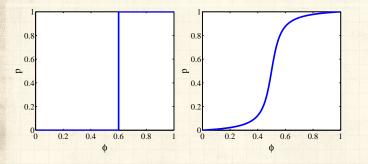
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Threshold models—response functions



Example threshold influence response functions: deterministic and stochastic The PoCSverse Complex Networks 202 of 320

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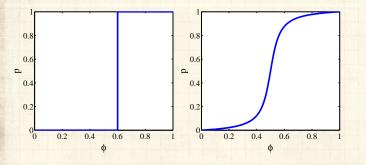
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Threshold models—response functions



Example threshold influence response functions: deterministic and stochastic

 $\Leftrightarrow \phi$ = fraction of contacts 'on' (e.g., rioting)

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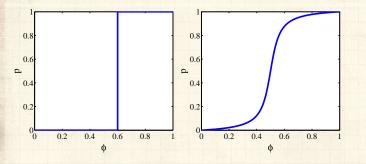
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Threshold models—response functions



Example threshold influence response functions: deterministic and stochastic

- $\Leftrightarrow \phi$ = fraction of contacts 'on' (e.g., rioting)
- 🚳 Two states: S and I.

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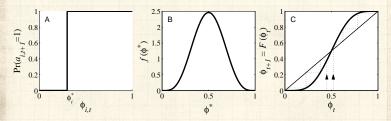
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Action based on perceived behavior of others:



🚳 Two states: S and I.

 $\Leftrightarrow \phi$ = fraction of contacts 'on' (e.g., rioting)

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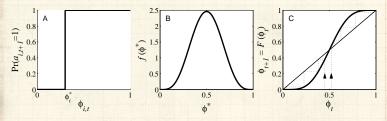
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Action based on perceived behavior of others:



🚳 Two states: S and I.

- $\Leftrightarrow \phi$ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)

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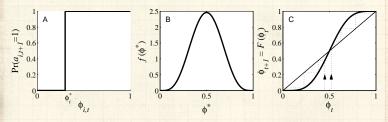
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Action based on perceived behavior of others:



- 🚳 Two states: S and I.
- $\Leftrightarrow \phi$ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- lis is a Critical mass model

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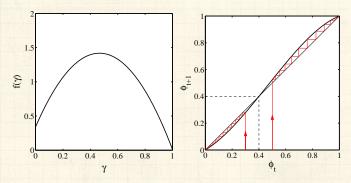
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Another example of critical mass model:



Solution Fragility of fixed point at $\phi = 0$. Critical slope = 1. The PoCSverse Complex Networks 204 of 320

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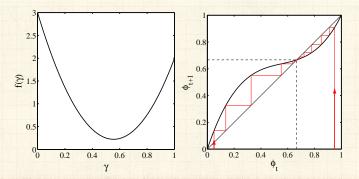
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Example of single stable state model:



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Implications for collective action theory:

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Implications for collective action theory: 1. Collective uniformity ⇒ individual uniformity

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Implications for collective action theory:

- 1. Collective uniformity \Rightarrow individual uniformity
- 2. Small individual changes \Rightarrow large global changes

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Implications for collective action theory:

- 1. Collective uniformity \Rightarrow individual uniformity
- 2. Small individual changes \Rightarrow large global changes
- The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.

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Implications for collective action theory:

- 1. Collective uniformity \Rightarrow individual uniformity
- 2. Small individual changes \Rightarrow large global changes
- The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
- 4. System stories live in left null space of our stories—we can't even see them.

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Implications for collective action theory:

- 1. Collective uniformity \Rightarrow individual uniformity
- 2. Small individual changes \Rightarrow large global changes
- The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
- 4. System stories live in left null space of our stories—we can't even see them.
- 5. But we happily impose simplistic, individual-centric stories—we can't help ourselves ☑.

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"A simple model of global cascades on random networks" D. J. Watts. Proc. Natl. Acad. Sci., 2002 ^[106]

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- "A simple model of global cascades on random networks"
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 - \bigcirc Mean field model ightarrow network model

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"A simple model of global cascades on random networks" D. J. Watts. Proc. Natl. Acad. Sci., 2002 ^[106]

✓ Mean field model → network model
 ✓ Individuals now have a limited view of the world

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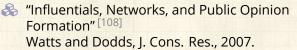
D. J. Watts. Proc. Natl. Acad. Sci., 2002^[106]

⑦ Mean field model → network model
 ⑦ Individuals now have a limited view of the world

Also consider:

"Seed size strongly affects cascades on random networks" [44] Gleeson and Cahalane, Phys. Rev. E, 2007.

"Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks"^[30] Dodds, Harris, and Payne, Phys. Rev. E, 2011



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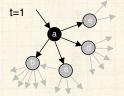
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Threshold model on a network



\clubsuit All nodes have threshold $\phi = 0.2$.

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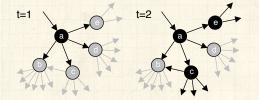
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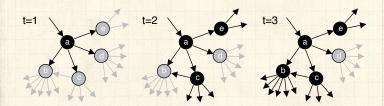
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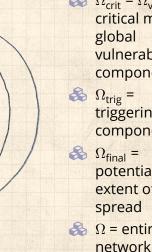
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Example random network structure:

inal



 $\Re \Omega_{\rm crit} = \Omega_{\rm vuln} =$ critical mass = vulnerable component triggering component potential extent of spread Ω = entire

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$$\Omega_{\text{crit}} \subset \Omega_{\text{trig}}; \ \Omega_{\text{crit}} \subset \Omega_{\text{final}}; \text{ and } \Omega_{\text{trig}}, \Omega_{\text{final}} \subset \Omega.$$

Back to following a link:

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Back to following a link:

A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.

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Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.

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Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- \lambda Normalization:

$$\sum_{k=0}^{\infty} k P_k = \langle k \rangle$$

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- Follows from there being k ways to connect to a node with degree k.
- 🚳 Normalization:

So

$$\sum_{k=0}^{\infty} k P_k = \langle k \rangle$$

 $P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$

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Next: Vulnerability of linked node

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Next: Vulnerability of linked node Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) \mathrm{d}\phi'_*$$

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Next: Vulnerability of linked node Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) \mathsf{d} \phi'_*$$

Solution If linked node is vulnerable, it produces k - 1 new outgoing active links

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Next: Vulnerability of linked node Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) \mathsf{d} \phi'_*$$

- Solution If linked node is vulnerable, it produces k 1 new outgoing active links
- If linked node is not vulnerable, it produces no active links.

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Putting things together:

Expected number of active edges produced by an active edge:

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Putting things together:

Expected number of active edges produced by an active edge:

+

$$R = \left[\sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} \right]$$

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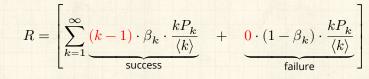
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Putting things together:

Expected number of active edges produced by an active edge:



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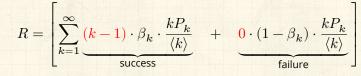
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Putting things together:

Expected number of active edges produced by an active edge:



$$=\sum_{k=1}^{\infty}(k-1)\cdot\beta_k\cdot\frac{kP_k}{\langle k\rangle}$$

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So... for random networks with fixed degree distributions, cacades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

 $\beta_k = \text{probability a degree } k \text{ node is vulnerable.}$ $P_k = \text{probability a node has degree } k.$

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Two special cases:

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Two special cases:

 \mathfrak{R} (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

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Two special cases:

 \mathfrak{R} (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

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$$1\cdot \sum_{k=1}^\infty (k-1)\cdot \frac{kP_k}{\langle k\rangle}>1.$$

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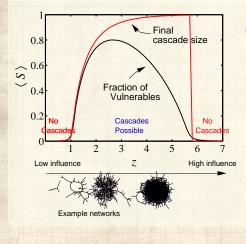
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Cascades on random networks



Cascades occur only if size of max vulnerable cluster > 0. The PoCSverse Complex Networks 215 of 320

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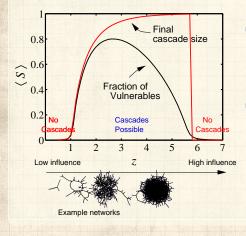
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Cascades on random networks



 Cascades occur only if size of max vulnerable cluster > 0.
 System may be 'robust-yetfragile'. The PoCSverse Complex Networks 215 of 320

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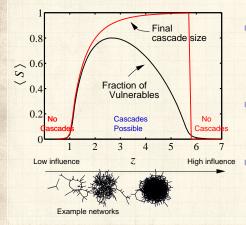
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Cascades on random networks



Cascades occur 2 only if size of max vulnerable cluster > 0. System may be 8 'robust-yetfragile'. 'Ignorance' 2 facilitates spreading.

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Expected size of spread

Pleasantness:

Taking off from a single seed story is about expansion away from a node.



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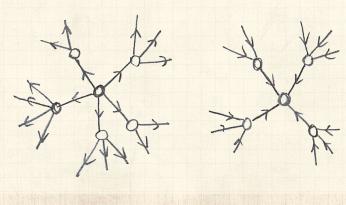
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Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.



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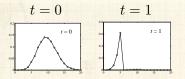
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 $P_{k,t}$ versus k

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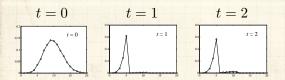
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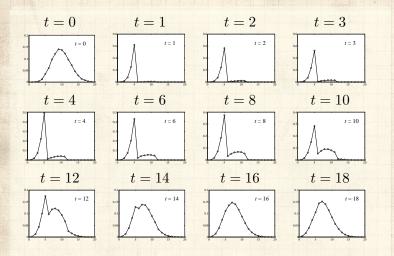
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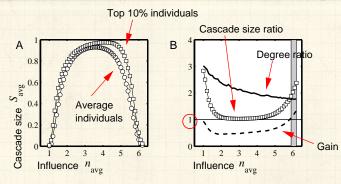
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 $P_{k,t} \operatorname{versus} k$

The multiplier effect:



Fairly uniform levels of individual influence.
 Multiplier effect is mostly below 1.

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"Threshold Models of Social Influence" Watts and Dodds, The Oxford Handbook of Analytical Sociology, **63**, 475–497, 2009. ^[109]

Assumption of sparse interactions is good

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"Threshold Models of Social Influence" Watts and Dodds, The Oxford Handbook of Analytical Sociology, **63**, 475–497, 2009. ^[109]

 Assumption of sparse interactions is good
 Degree distribution is (generally) key to a network's function The PoCSverse Complex Networks 219 of 320

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"Threshold Models of Social Influence" Watts and Dodds, The Oxford Handbook of Analytical Sociology, **63**, 475–497, 2009. ^[109]

Assumption of sparse interactions is good
 Degree distribution is (generally) key to a network's function

Still, random networks don't represent all networks The PoCSverse Complex Networks 219 of 320

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We have a second second in the second second

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 Assumption of sparse interactions is good
 Degree distribution is (generally) key to a network's function

- Still, random networks don't represent all networks
- 🚳 Major element missing: group structure

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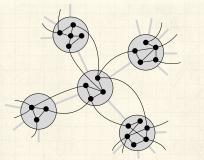
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Group structure—Ramified random networks



p = intergroup connection probability q = intragroup connection probability.

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Generalized affiliation model networks with triadic closure

Connect nodes with probability $\propto e^{-\alpha d}$ where α = homophily parameter and d = distance between nodes (height of lowest common ancestor) The PoCSverse Complex Networks 221 of 320

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Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto e^{-\alpha d}$ where α = homophily parameter and d = distance between nodes (beight of
 - *d* = distance between nodes (height of lowest common ancestor)
- $rac{1}{\tau_1}$ = intergroup probability of friend-of-friend connection

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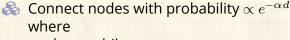
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Generalized affiliation model networks with triadic closure



 α = homophily parameter and

d = distance between nodes (height of lowest common ancestor)

- $rac{1}{\tau_1}$ = intergroup probability of friend-of-friend connection
- $rac{1}{2}$ = intragroup probability of friend-of-friend connection

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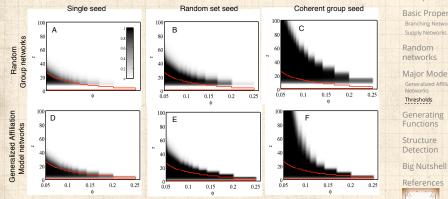
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Cascade windows for group-based networks



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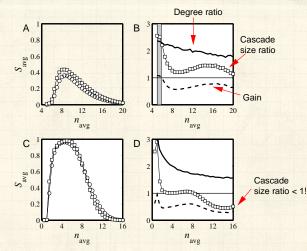
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Multiplier effect for group-based networks:



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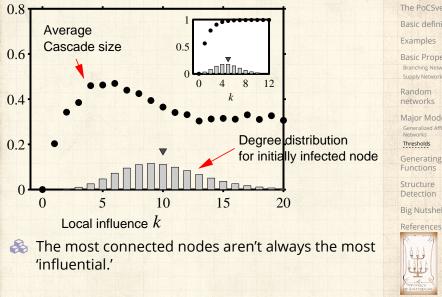
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🚳 Multiplier almost always below 1.

Assortativity in group-based networks



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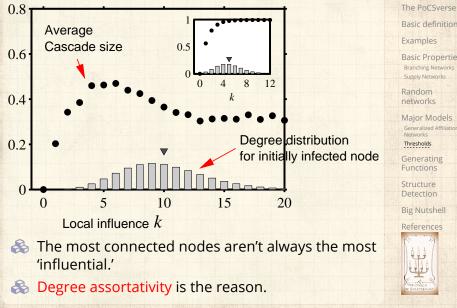
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Assortativity in group-based networks



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Social contagion

"Without followers, evil cannot spread." –Leonard Nimoy

Summary

linfluential vulnerables' are key to spread.

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"Without followers, evil cannot spread." –Leonard Nimoy

Summary

Influential vulnerables' are key to spread.
 Early adopters are mostly vulnerables.

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"Without followers, evil cannot spread." –Leonard Nimoy

Summary

- linfluential vulnerables' are key to spread.
- 🗞 Early adopters are mostly vulnerables.
- 🚳 Vulnerable nodes important but not necessary.



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"Without followers, evil cannot spread." –Leonard Nimoy

Summary

linfluential vulnerables' are key to spread.

- 🛞 Early adopters are mostly vulnerables.
- 🚳 Vulnerable nodes important but not necessary.
 - 🖇 Groups may greatly facilitate spread.

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"Without followers, evil cannot spread." –Leonard Nimoy

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- linfluential vulnerables' are key to spread.
- 🗞 Early adopters are mostly vulnerables.
- 🚳 Vulnerable nodes important but not necessary.
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- Seems that cascade condition is a global one.

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- 🚳 Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks

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- Most extreme/unexpected cascades occur in highly connected networks
- linfluentials' are posterior constructs.

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- 🚳 Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- linfluentials' are posterior constructs.
 - 🗞 Many potential influentials exist.

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Implications



Focus on the influential vulnerables.

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Implications



Focus on the influential vulnerables.

Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'

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Implications

- line and the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)

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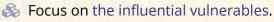
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Implications



- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.

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Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages; even so: buttons and hats).

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Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages; even so: buttons and hats).
- Entities can be novel or designed to combine with others, e.g. block another one.

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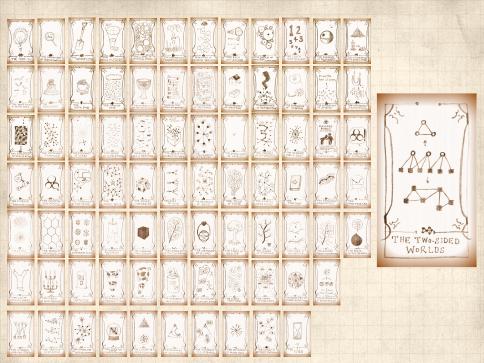
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"Flavor network and the principles of food pairing" Ahn et al., Nature Scientific Reports, **1**, 196, 2011.^[1]

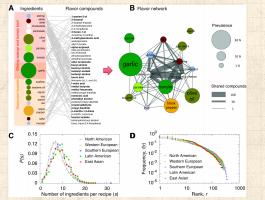


Figure 11 Have network (. 10) The impedents contained in two recipies (left column), together with the flavor compounds that are hown to be present in the impedience (regional). Each show compound is hinked to be impedient and notating is a hyperite track. Since compounds (shown in hisding) are already implicit impedients, (1)) We project the impedient approximation (histing a short interbasic, some compounds (shown in hisding) are already by multiple impedients, (1)) We project the impedient approximation (histing and the impedient approximation) are already of the impedient approximation (histing and the impedient approximation) are already of the impedient approximation (histing and histing The PoCSverse Complex Networks 228 of 320

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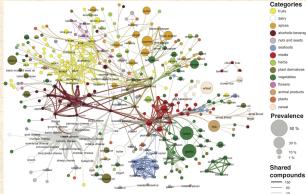
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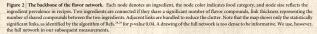
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"Flavor network and the principles of food pairing" Ahn et al., Nature Scientific Reports, 1, 196, 2011.^[1]





Categories alcoholic beverages nuts and seeds seafoods plant derivatives vegetables animal products Prevalence 50 % 30.%

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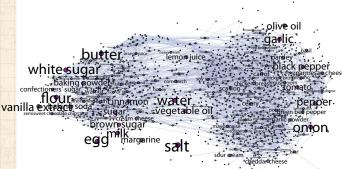
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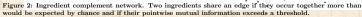
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"Recipe recommendation using ingredient networks" Teng, Lin, and Adamic, Proceedings of the 3rd Annual ACM Web Science Conference, **1**, 298–307, 2012.^[97]





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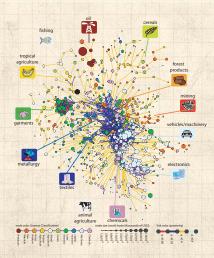
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"The Product Space Conditions the Development of Nations" Hidalgo et al., Science, **317**, 482–487, 2007.^[52]



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Networks and creativity:

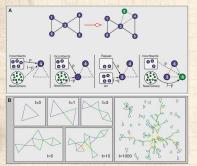


Fig. 2. Modeling the emergence of collaboration networks in creative enterprises. (A) Creation of a team with m - 3 agents. Consider, at time zero, a collaboration network comprising five agents, all incumbents (blue circles). Along with the incumbents, there is a large pool of newcomers (green circles) available to participate in new teams. Each agent in a team has a probability p of being drawn from the pool of incumbents and a probability 1 - p of being drawn from the pool of newcorners. For the second and subsequent agents selected from the incumbents' pool: (i) with probability q, the new agent is randomly selected from among the set of collaborators of a randomly selected incumbent already in the team: (ii) otherwise, he or she is selected at random among all incumbents in the network. For concreteness, let us assume that incumbent 4 is selected as the first agent in the new team (leftmost box). Let us also assume that the second agent is an incumbent, too (center-left box). In this example, the second agent is a past collaborator of agent 4, specifically agent 3 (center-right box). Lastly, the third agent is selected from the pool of newcomers: this agent becomes incumbent 6 (rightmost box). In these boxes and in the following panels and figures, blue lines indicate newcomernewcomer collaborations, green lines indicate newcomer-incumbent collaborations, vellow lines indicate new incumbent-incumbent collaborations, and red lines indicate repeat collaborations. (B) Time evolution of the network of collaborations according to the model for p = 0.5, q = 0.5, and m = 3.

Guimerà et al., Science 2005: ^[48] "Team Assembly Mechanisms Determine Collaboration Network Structure and Team Performance" **Broadway** musical industry Scientific collaboration in Social Psychology, Economics, Ecology, and Astronomy.

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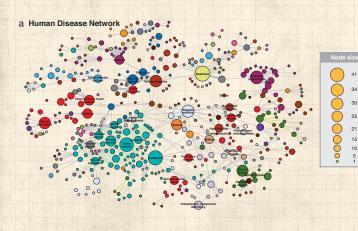
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"The human disease network" Goh et al., Proc. Natl. Acad. Sci., **104**, 8685–8690, 2007. ^[46]



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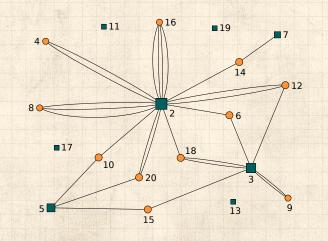
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"The complex architecture of primes and natural numbers" García-Pérez, Serrano, and Boguñá, https://arxiv.org/abs/1402.3612, 2014.^[39]



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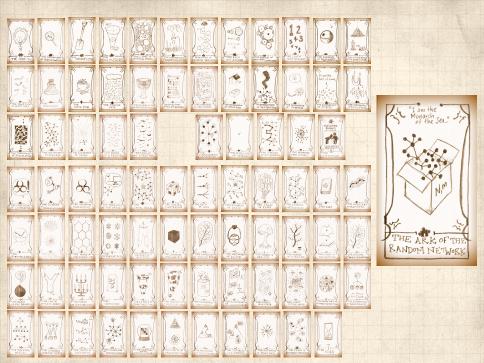
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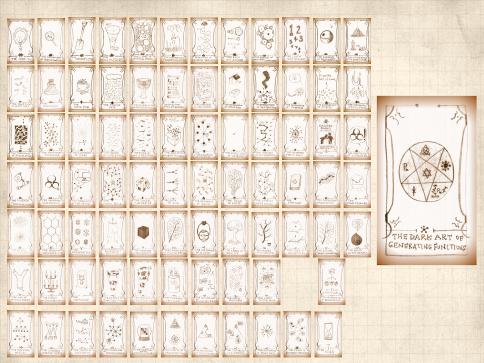
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Solution like the sequence a_0, a_1, a_2, \dots , associate each element with a distinct function or other mathematical object.

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- Solution like the sequence a_0, a_1, a_2, \dots , associate each element with a distinct function or other mathematical object.
- Well-chosen functions allow us to manipulate sequences and retrieve sequence elements.

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- Solution like the sequence a_0, a_1, a_2, \dots , associate each element with a distinct function or other mathematical object.
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Definition:

The generating function (g.f.) for a sequence $\{a_n\}$ is

$$F(x) = \sum_{n=0}^{\infty} a_n x^n$$

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Solution The generating function (g.f.) for a sequence $\{a_n\}$ is

$$F(x) = \sum_{n=0}^{\infty} a_n x^n$$

Roughly: transforms a vector in R^{∞} into a function defined on R^1 .

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- 3 Idea: Given a sequence a_0, a_1, a_2, \dots , associate each element with a distinct function or other mathematical object.
- 🚳 Well-chosen functions allow us to manipulate sequences and retrieve sequence elements.

Definition:

 \mathbb{R} The generating function (g.f.) for a sequence $\{a_n\}$ is

$$F(x) = \sum_{n=0}^{\infty} a_n x^n$$

 \mathfrak{S} Roughly: transforms a vector in R^{∞} into a function defined on R^1 .

🚳 Related to Fourier, Laplace, Mellin, ...

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Rolling dice and flipping coins:

 $\underset{k}{\circledast} p_{k}^{(\bigcirc)} = \mathbf{Pr}(\text{throwing a } k) = 1/6 \text{ where } k = 1, 2, \dots, 6.$

$$F^{(\textcircled{C})}(x) = \sum_{k=1}^{6} p_k^{(\textcircled{C})} x^k = \frac{1}{6} (x + x^2 + x^3 + x^4 + x^5 + x^6).$$

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$$p_0^{(coin)} = \mathbf{Pr}(head) = 1/2, p_1^{(coin)} = \mathbf{Pr}(tail) = 1/2.$$

$$F^{(\text{coin})}(x) = p_0^{(\text{coin})} x^0 + p_1^{(\text{coin})} x^1 = \frac{1}{2}(1+x).$$

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Rolling dice and flipping coins:

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A generating function for a probability distribution is called a Probability Generating Function (p.g.f.).

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Rolling dice and flipping coins:

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 A generating function for a probability distribution is called a Probability Generating Function (p.g.f.).
 We'll come back to these simple examples as we derive various delicious properties of generating functions. The PoCSverse **Basic definitions** Examples **Basic Properties** Branching Networks Supply Networks Random networks Major Models Generalized Affiliation Thresholds Generating Functions Structure **Big Nutshell** References

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🚳 Normalization:

F(1) = 1

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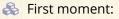
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🗞 Normalization:

F(1) = 1



 $\langle k\rangle = F'(1)$

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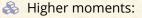


🚳 Normalization:

F(1) = 1

🚳 First moment:

 $\langle k\rangle = F'(1)$



$$\langle k^n \rangle = \left. \left(x \frac{\mathsf{d}}{\mathsf{d}x} \right)^n F(x) \right|_{x=1}$$

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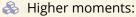
Useful pieces for probability distributions:

🚳 Normalization:

F(1) = 1

🚳 First moment:

 $\langle k\rangle = F'(1)$



$$\left| k^n \right\rangle = \left. \left(x \frac{\mathsf{d}}{\mathsf{d}x} \right)^n F(x) \right|_{x=1}$$

*k*th element of sequence (general):

$$P_k = \frac{1}{k!} \frac{\mathsf{d}^k}{\mathsf{d} x^k} F(x) \bigg|_{x=0}$$

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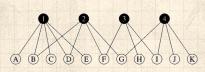
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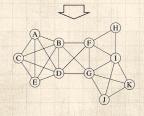


Random bipartite networks: We'll follow this rather well cited C paper:



"Random graphs with arbitrary degree distributions and their applications" Newman, Strogatz, and Watts, Phys. Rev. E, **64**, 026118, 2001.^[80]





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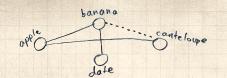
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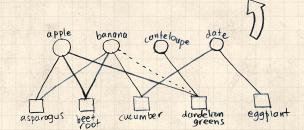
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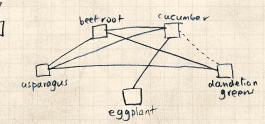
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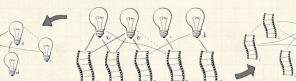
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Example of a bipartite affiliation network and the induced networks:



Center: A small story-trope bipartite graph. ^[28]
 Induced trope network and the induced story network are on the left and right.

The dashed edge in the bipartite affiliation network indicates an edge added to the system, resulting in the dashed edges being added to the two induced networks. The PoCSverse Complex Networks 242 of 320

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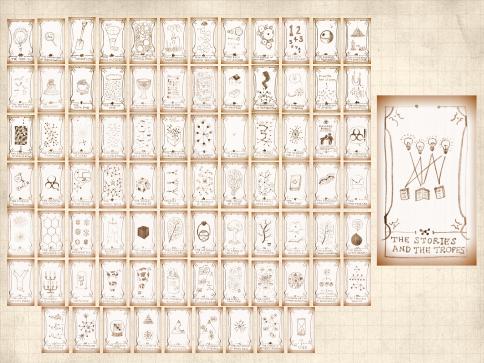
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An example of two inter-affiliated types:

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🚳 Stories contain tropes, tropes are in stories.

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Stories contain tropes, tropes are in stories.
 Consider a story-trope system with N_□ = # stories and N_Ω = # tropes.

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- 🚳 Stories contain tropes, tropes are in stories.
- Solution Consider a story-trope system with N_{\blacksquare} = # stories and N_{Q} = # tropes.
- $\mathfrak{B}_{\mathbf{H},\mathfrak{V}}$ = number of edges between \mathbf{H} and \mathfrak{V} .

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- 🚳 Stories contain tropes, tropes are in stories.
- Solution Consider a story-trope system with N_{\blacksquare} = # stories and N_{Q} = # tropes.
- *m*_{□,♀} = number of edges between □ and ♀.
 Let's have some underlying distributions for numbers of affiliations: *P*^(□)_k (a story has *k* tropes) and *P*^(♀)_k (a trope is in *k* stories).

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- 🚳 Stories contain tropes, tropes are in stories.
- Solution Consider a story-trope system with N_{\blacksquare} = # stories and N_{Q} = # tropes.
- $m_{\blacksquare, \Im} = \text{number of edges between } \blacksquare \text{ and } \Im.$
- Solution Let's have some underlying distributions for numbers of affiliations: $P_k^{(\blacksquare)}$ (a story has k tropes) and $P_k^{(Q)}$ (a trope is in k stories).
- \mathfrak{A} Average number of affiliations: $\langle k \rangle_{\mathbb{H}}$ and $\langle k \rangle_{\mathbb{Q}}$.

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- 🚳 Stories contain tropes, tropes are in stories.
- Solution Consider a story-trope system with N_{\blacksquare} = # stories and N_{Q} = # tropes.
- ♦ $m_{\blacksquare, Q}$ = number of edges between 🖽 and Q.
- Let's have some underlying distributions for numbers of affiliations: $P_k^{(\textcircled{B})}$ (a story has k tropes) and $P_k^{(\textcircled{Q})}$ (a trope is in k stories).
- Average number of affiliations: $\langle k \rangle_{\blacksquare}$ and $\langle k \rangle_{Q}$. $\langle k \rangle_{\blacksquare} = average number of tropes per story.$

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- 🚳 Stories contain tropes, tropes are in stories.
- Solution Consider a story-trope system with N_{\blacksquare} = # stories and N_{Q} = # tropes.
- ♦ $m_{\blacksquare, Q}$ = number of edges between 🖽 and Q.
- Let's have some underlying distributions for numbers of affiliations: $P_k^{(\textcircled{B})}$ (a story has k tropes) and $P_k^{(\textcircled{Q})}$ (a trope is in k stories).
- Average number of affiliations: $\langle k \rangle_{\blacksquare}$ and $\langle k \rangle_{Q}$. $\langle k \rangle_{\blacksquare} =$ average number of tropes per story.
 - $\langle k \rangle_{0}$ = average number of stories containing a given trope.

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- 🚳 Stories contain tropes, tropes are in stories.
- Solution Consider a story-trope system with N_{\blacksquare} = # stories and N_{Q} = # tropes.
- ♦ $m_{\blacksquare, \heartsuit}$ = number of edges between \blacksquare and \heartsuit .
- Solution Let's have some underlying distributions for numbers of affiliations: $P_k^{(\textcircled{B})}$ (a story has k tropes) and $P_k^{(\textcircled{Q})}$ (a trope is in k stories).
- Average number of affiliations: ⟨k⟩_□ and ⟨k⟩_♀.
 ⟨k⟩_□ = average number of tropes per story.
 ⟨k⟩_♀ = average number of stories containing a given trope.

 $Must have balance: N_{\blacksquare} \cdot \langle k \rangle_{\blacksquare} = m_{\blacksquare, \Im} = N_{\heartsuit} \cdot \langle k \rangle_{\heartsuit}.$

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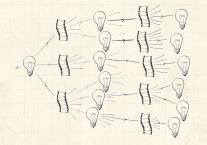
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Spreading through bipartite networks:



- View as bouncing back and forth between the two connected populations.^[28]
- Actual spread may be within only one population (ideas between between people) or through both (failures in physical and communication networks).
- The gain ratio for simple contagion on a bipartite random network = product of two gain ratios.

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Usual helpers for understanding network's structure:

Randomly select an edge connecting a \blacksquare to a \Im . Probability the \blacksquare contains k other tropes:

$$R_{k}^{(\textcircled{\hbox{\rm ID}})} = \frac{(k+1)P_{k+1}^{(\textcircled{\hbox{\rm ID}})}}{\sum_{j=0}^{N_{\fbox{\rm ID}}}(j+1)P_{j+1}^{(\textcircled{\hbox{\rm ID}})}} = \frac{(k+1)P_{k+1}^{(\textcircled{\hbox{\rm ID}})}}{\langle k \rangle_{\fbox{\rm ID}}}.$$

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Usual helpers for understanding network's structure:

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$$R_{k}^{(\textcircled{\hbox{\rm I}})} = \frac{(k+1)P_{k+1}^{(\textcircled{\hbox{\rm I}})}}{\sum_{j=0}^{N_{\textcircled{\hbox{\rm I}}}}(j+1)P_{j+1}^{(\textcircled{\hbox{\rm I}})}} = \frac{(k+1)P_{k+1}^{(\textcircled{\hbox{\rm I}})}}{\langle k\rangle_{\fbox{\hbox{\rm I}}}}.$$

 \mathfrak{S} Probability the \mathfrak{P} is in k other stories:

$$R_k^{(\mathbf{\widehat{Q}})} = \frac{(k+1)P_{k+1}^{(\mathbf{\widehat{Q}})}}{\sum_{j=0}^{N_\mathbf{\widehat{Q}}}(j+1)P_{j+1}^{(\mathbf{\widehat{Q}})}} = \frac{(k+1)P_{k+1}^{(\mathbf{\widehat{Q}})}}{\langle k\rangle_\mathbf{\widehat{Q}}}.$$

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Networks of \blacksquare and \heartsuit within bipartite structure: $\bigotimes P_{\text{ind},k}^{(\blacksquare)}$ = probability a random \blacksquare is connected to kstories by sharing at least one \heartsuit . The PoCSverse Complex Networks 247 of 320

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Networks of 🖽 and 😵 within bipartite structure:

- $P_{\text{ind},k}^{(\textcircled{H})} = \text{probability a random} \textcircled{H} \text{ is connected to } k$ stories by sharing at least one \Im .
- $P_{\text{ind},k}^{(Q)} = \text{probability a random } \text{ is connected to } k$ tropes by co-occurring in at least one **H**.

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Networks of 🖽 and 🖇 within bipartite structure:

- $P_{\text{ind},k}^{(\textcircled{\text{III}})} = \text{probability a random} \textcircled{\text{III}} \text{ is connected to } k$ stories by sharing at least one \Im .
- $P_{\text{ind},k}^{(\mathbf{Q})} = \text{probability a random } \mathbf{Q} \text{ is connected to } k$ tropes by co-occurring in at least one **H**.
- $R_{ind,k}^{(Q-\boxplus)} = \text{probability a random edge leads to a} \\ H \\ \text{which is connected to } k \text{ other stories by sharing at least one } Q.$

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Networks of 🖽 and 🖇 within bipartite structure:

- $P_{\text{ind},k}^{(\textcircled{\text{III}})} = \text{probability a random} \textcircled{\text{III}} \text{ is connected to } k$ stories by sharing at least one \heartsuit .
- $P_{\text{ind},k}^{(Q)} = \text{probability a random } Q \text{ is connected to } k$ tropes by co-occurring in at least one **H**.
- $R_{ind,k}^{(Q-\square)} = \text{probability a random edge leads to a} \\ \text{which is connected to } k \text{ other stories by sharing at least one } Q.$
- $R_{ind,k}^{(\blacksquare \mathbb{Q})} = \text{probability a random edge leads to a } \\ \text{which is connected to } k \text{ other tropes by} \\ \text{co-occurring in at least one } \blacksquare.$

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Networks of 🖽 and 🛿 within bipartite structure:

- $P_{\text{ind},k}^{(\textcircled{\text{III}})} = \text{probability a random} \textcircled{\text{III}} \text{ is connected to } k$ stories by sharing at least one \Im .
- $P_{\text{ind},k}^{(Q)} = \text{probability a random } Q \text{ is connected to } k$ tropes by co-occurring in at least one **H**.
- $\Re_{\text{ind},k}^{(\mathbb{Q}-\mathbb{H})} = \text{probability a random edge leads to a } \mathbb{H}$ which is connected to k other stories by sharing at least one \mathbb{Q} .
- $R_{ind,k}^{(\blacksquare \mathbb{Q})} = \text{probability a random edge leads to a } \\ \text{which is connected to } k \text{ other tropes by} \\ \text{co-occurring in at least one } \blacksquare.$
- 🚳 Goal: find these distributions 🛛.

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Networks of 🖽 and 🛿 within bipartite structure:

- $P_{\text{ind},k}^{(\textcircled{H})} = \text{probability a random} \textcircled{H} \text{ is connected to } k$ stories by sharing at least one \Im .
- $P_{\text{ind},k}^{(Q)} = \text{probability a random } Q \text{ is connected to } k$ tropes by co-occurring in at least one **H**.
- $\Re_{\text{ind},k}^{(\mathbb{Q}-\mathbb{H})} = \text{probability a random edge leads to a } \mathbb{H}$ which is connected to k other stories by sharing at least one \mathbb{Q} .
- $R_{ind,k}^{(\blacksquare \mathbb{Q})} = \text{probability a random edge leads to a } \\ \text{which is connected to } k \text{ other tropes by} \\ \text{co-occurring in at least one } \blacksquare.$
 - 🗞 Goal: find these distributions 🛛.
- Another goal: find the induced distribution of component sizes and a test for the presence or absence of a giant component.

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- $P_{\text{ind},k}^{(\textcircled{H})} = \text{probability a random} \textcircled{H} \text{ is connected to } k$ stories by sharing at least one \Im .
- $P_{\text{ind},k}^{(Q)} = \text{probability a random } Q \text{ is connected to } k$ tropes by co-occurring in at least one **H**.
- $\Re_{\text{ind},k}^{(\mathbb{Q}-\mathbb{H})} = \text{probability a random edge leads to a } \mathbb{H}$ which is connected to k other stories by sharing at least one \mathbb{Q} .
- $R_{ind,k}^{(\blacksquare \mathbb{Q})} = \text{probability a random edge leads to a } \\ \text{which is connected to } k \text{ other tropes by} \\ \text{co-occurring in at least one } \blacksquare.$
 - 🗞 Goal: find these distributions 🛛.
- Another goal: find the induced distribution of component sizes and a test for the presence or absence of a giant component.
 - Unrelated goal: be 10% happier/weep less.

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Always about the edges: when following a random edge toward a 🖽, what's the expected number of new edges leading to other stories via tropes?

- Always about the edges: when following a random edge toward a 🖽, what's the expected number of new edges leading to other stories via tropes?
- $\begin{aligned} & \& \end{aligned} \label{eq:kappa} \ensuremath{ \overset{\mbox{\footnotesize{\mbox{$\&$}$}}}{R_{\rm ind}}} = F_{R_{\rm ind}^{(\mbox{$$$$}\mbox{$$=$$}\mbox{$$]}}}(1) \mbox{ (and } \\ & F_{R_{\rm ind}^{(\mbox{$$$$}\mbox{$$=$$}\mbox{$$]}}}'(1) \mbox{ for the trope side of things). \end{aligned}$

- Always about the edges: when following a random edge toward a 🖽, what's the expected number of new edges leading to other stories via tropes?
- $\label{eq:constraint} \begin{array}{l} & \& \ensuremath{\mathbb{R}} \\ & \& \ensuremath{\mathbb{R}} \\ & & F'_{R_{\mathrm{ind}}^{(\mathbb{P}} \to \mathbb{Q})}(1) \mbox{ for the trope side of things).} \end{array}$

🚳 We compute with joy:

$$\langle k \rangle_{R,\boxplus,\mathrm{ind}} = \left. \frac{\mathrm{d}}{\mathrm{d}x} F_{R_{\mathrm{ind},k}^{(\mathrm{Q}-\boxplus)}}(x) \right|_{x=1} =$$

- Always about the edges: when following a random edge toward a , what's the expected number of new edges leading to other stories via tropes?
- $\label{eq:constraint} \begin{array}{l} & \& \ensuremath{\mathbb{R}} \\ & \& \ensuremath{\mathbb{R}} \\ & & F'_{R_{\mathrm{ind}}^{(\mathbb{Q}} \longrightarrow \mathbb{Q})}(1) \ensuremath{\text{for the trope side of things).}} \end{array} \\ & & F'_{R_{\mathrm{ind}}^{(\mathbb{Q}} \longrightarrow \mathbb{Q})}(1) \ensuremath{\text{for the trope side of things).}} \end{array}$
- 🚳 We compute with joy:

$$\langle k \rangle_{R,\boxplus,\mathrm{ind}} = \left. \frac{\mathrm{d}}{\mathrm{d}x} F_{R_{\mathrm{ind},k}^{(\mathrm{Q}-\boxplus)}}(x) \right|_{x=1} = \left. \frac{\mathrm{d}}{\mathrm{d}x} F_{R^{(\mathrm{E})}}\left(F_{R^{(\mathrm{Q})}}(x)\right) \right|_{x=1}$$

- Always about the edges: when following a random edge toward a
 , what's the expected number of new edges leading to other stories via tropes?
- $\label{eq:constraint} \begin{array}{l} & \& \ensuremath{\mathbb{R}} \\ & \& \ensuremath{\mathbb{R}} \\ & & F'_{R_{\mathrm{ind}}^{(\mathbb{P}-\mathbb{P})}}(1) \mbox{ for the trope side of things).} \end{array} \end{array}$

🚳 We compute with joy:

$$\langle k \rangle_{R,\boxplus,\mathrm{ind}} = \left. \frac{\mathrm{d}}{\mathrm{d}x} F_{R_{\mathrm{ind},k}^{(\mathrm{Q}-\boxplus)}}(x) \right|_{x=1} = \left. \frac{\mathrm{d}}{\mathrm{d}x} F_{R^{(\mathrm{E})}}\left(F_{R^{(\mathrm{Q})}}(x)\right) \right|_{x=1}$$

 $=F_{R^{(\mathrm{Q})}}'(1)F_{R^{(\mathrm{II})}}'\left(F_{R^{(\mathrm{Q})}}(1)\right)$

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 $=F'_{R^{(\underline{\mathbb{Q}})}}(1)F'_{R^{(\underline{\mathbb{H}})}}\left(F_{R^{(\underline{\mathbb{Q}})}}(1)\right)=F'_{R^{(\underline{\mathbb{Q}})}}(1)F'_{R^{(\underline{\mathbb{H}})}}(1)=\frac{F''_{P^{(\underline{\mathbb{Q}})}}(1)}{F'_{P^{(\underline{\mathbb{Q}})}}(1)}\frac{F''_{P^{(\underline{\mathbb{H}})}}(1)}{F'_{P^{(\underline{\mathbb{H}})}}(1)}$

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🗞 Note symmetry.

- Always about the edges: when following a random edge toward a , what's the expected number of new edges leading to other stories via tropes?
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- \$happiness++;

ln terms of the underlying distributions:

$$\langle k \rangle_{R,\boxplus,\mathrm{ind}} = \frac{\langle k(k-1) \rangle_{\boxplus}}{\langle k \rangle_{\boxplus}} \frac{\langle k(k-1) \rangle_{\textcircled{0}}}{\langle k \rangle_{\textcircled{0}}}$$

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$$\langle k \rangle_{R,\boxplus,\mathrm{ind}} = \frac{\langle k(k-1) \rangle_{\boxplus}}{\langle k \rangle_{\boxplus}} \frac{\langle k(k-1) \rangle_{\mathbb{Q}}}{\langle k \rangle_{\mathbb{Q}}}$$

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🚳 We have a giant component in both induced networks when

$$\langle k
angle_{R,\blacksquare,\mathsf{ind}} \equiv \langle k
angle_{R,\mathbf{Q},\mathsf{ind}} > 1$$

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We have a giant component in both induced networks when

 $\langle k \rangle_{R, \square, \text{ind}} \equiv \langle k \rangle_{R, \Omega, \text{ind}} > 1$

🚳 See this as the product of two gain ratios.

ln terms of the underlying distributions:

$$\langle k \rangle_{R,\boxplus,\mathrm{ind}} = \frac{\langle k(k-1) \rangle_{\textcircled{H}}}{\langle k \rangle_{\textcircled{H}}} \frac{\langle k(k-1) \rangle_{\textcircled{Q}}}{\langle k \rangle_{\textcircled{Q}}}$$

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We have a giant component in both induced networks when

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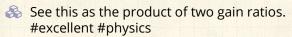
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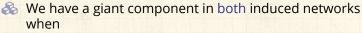
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 $\langle k \rangle_{B} \equiv ind \equiv \langle k \rangle_{B,Q,ind} > 1$

- See this as the product of two gain ratios. #excellent #physics
- lacktrian webs with this condition to make it webs with the second secon mathematically pleasant and pleasantly inscrutable:

$$\sum_{k=0}^{\infty}\sum_{k'=0}^{\infty}kk'(kk'-k-k')P_k^{(\textcircled{H})}P_{k'}^{(\textcircled{Q})}=0.$$

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Generating functions allow us to strangely calculate features of random networks. The PoCSverse Complex Networks 250 of 320

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Generating functions allow us to strangely calculate features of random networks.
 They're a bit scary and magical.

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Generating functions allow us to strangely calculate features of random networks.
 They're a bit scary and magical.
 Generating functions can be used to study contagion.

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Generating functions allow us to strangely calculate features of random networks.

- line a bit scary and magical.
- Generating functions can be used to study contagion.
- But: For essential results like possibility and probability of global spread, more direct, physics-bearing calculations are possible.

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- 🚳 Good real thing: Bipartite affiliation structures.

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Generating functions allow us to strangely calculate features of random networks.

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- But: For essential results like possibility and probability of global spread, more direct, physics-bearing calculations are possible.
- 🗞 Good real thing: Bipartite affiliation structures.
 - 🗞 Groups, groups, groups, ...

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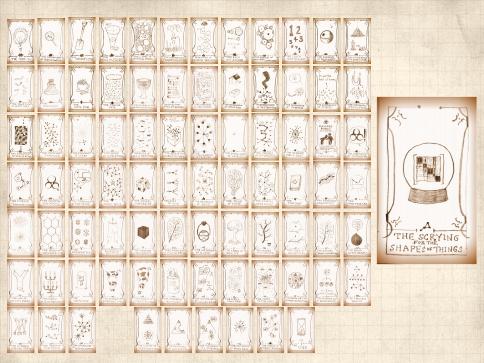
Major Models Generalized Affiliation Networks Thresholds

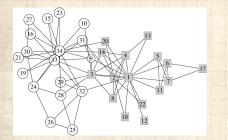
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▲ Zachary's karate club ^[119, 79]

\delta The issue:

how do we elucidate the internal structure of large networks across many scales? The PoCSverse Complex Networks 252 of 320

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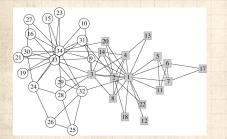
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Possible substructures: hierarchies, cliques, rings, ...

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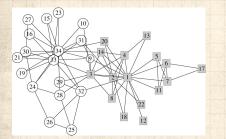
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 Possible substructures: hierarchies, cliques, rings, ...
 Plus: All combinations of substructures.

The issue:

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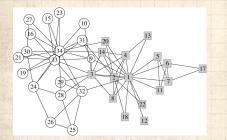
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 Possible substructures: hierarchies, cliques, rings, ...
 Plus: All combinations of substructures.
 Much focus on hierarchies (pyramids)

The issue:

how do we elucidate the internal structure of large networks across many scales? The PoCSverse Complex Networks 252 of 320

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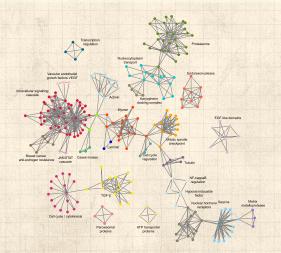
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"Community detection in graphs" Santo Fortunato, Physics Reports, **486**, 75–174, 2010. ^[38]



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Top down:

Idea: Identify global structure first and recursively uncover more detailed structure. The PoCSverse Complex Networks 254 of 320

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Top down:

- Idea: Identify global structure first and recursively uncover more detailed structure.
- Basic objective: find dominant components that have significantly more links within than without, as compared to randomized version.

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Top down:

- Idea: Identify global structure first and recursively uncover more detailed structure.
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 - "Community structure in social and biological networks" by Girvan and Newman (PNAS, 2002).^[42]

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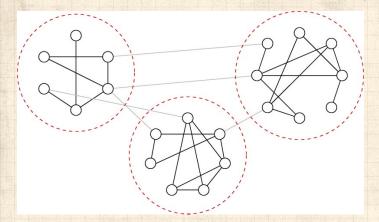
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Idea: Edges that connect communities have higher betweenness than edges within communities.

One class of structure-detection algorithms:

1. Compute edge betweenness for whole network.

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One class of structure-detection algorithms:

- 1. Compute edge betweenness for whole network.
- 2. Remove edge with highest betweenness.

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One class of structure-detection algorithms:

- 1. Compute edge betweenness for whole network.
- 2. Remove edge with highest betweenness.
- 3. Recompute edge betweenness

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One class of structure-detection algorithms:

- 1. Compute edge betweenness for whole network.
- 2. Remove edge with highest betweenness.
- 3. Recompute edge betweenness
- 4. Repeat steps 2 and 3 until all edges are removed.

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One class of structure-detection algorithms:

- 1. Compute edge betweenness for whole network.
- 2. Remove edge with highest betweenness.
- 3. Recompute edge betweenness
- 4. Repeat steps 2 and 3 until all edges are removed.
- 5 Record when components appear as a function of # edges removed.

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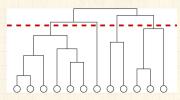
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One class of structure-detection algorithms:

- 1. Compute edge betweenness for whole network.
- 2. Remove edge with highest betweenness.
- 3. Recompute edge betweenness
- 4. Repeat steps 2 and 3 until all edges are removed.
- 5 Record when components appear as a function of # edges removed.
- 6 Generate dendogram revealing hierarchical structure.



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One class of structure-detection algorithms:

- 1. Compute edge betweenness for whole network.
- 2. Remove edge with highest betweenness.
- 3. Recompute edge betweenness
- 4. Repeat steps 2 and 3 until all edges are removed.
- 5 Record when components appear as a function of # edges removed.
- 6 Generate dendogram revealing hierarchical structure.

Red line indicates appearance of four (4) components at a certain level. The PoCSverse Complex Networks 256 of 320

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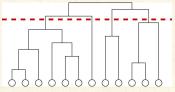
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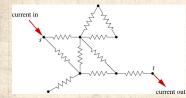
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Unit resistors on each edge.



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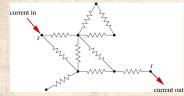
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Unit resistors on each edge.
 For every pair of nodes s (source) and t (sink), set up unit currents in at s and out at t.

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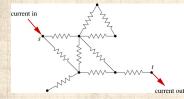
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Unit resistors on each edge.
 For every pair of nodes s (source) and t (sink), set up unit currents in at s and out at t.
 Measure absolute current along each edge l, |I_P st|.

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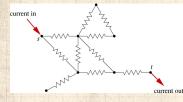
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Unit resistors on each edge. For every pair of nodes 4 s (source) and t (sink), set up unit currents in at s and out at t. Measure absolute current along each edge ℓ , $|I_{\ell,st}|$.

Sum $|I_{\ell,st}|$ over all pairs of nodes to obtain electronic betweenness for edge ℓ .

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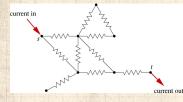
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Unit resistors on each edge. For every pair of nodes s (source) and t (sink), set up unit currents in at s and out at t. Measure absolute current along each edge ℓ , $|I_{\ell,st}|$.

Sum |I_{ℓ,st}| over all pairs of nodes to obtain electronic betweenness for edge ℓ.
 (Equivalent to random walk betweenness.)

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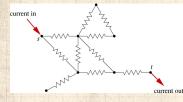
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Unit resistors on each edge. For every pair of nodes s (source) and t (sink), set up unit currents in at s and out at t. Measure absolute current along each edge ℓ , $|I_{\ell,st}|$.

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Sum |I_{ℓ,st}| over all pairs of nodes to obtain electronic betweenness for edge ℓ.
 (Equivalent to random walk betweenness.)
 Contributing electronic betweenness for edge between nodes *i* and *j*:

$$B_{ij,st}^{\text{elec}} = a_{ij} |V_{i,st} - V_{j,st}|.$$

lefine some arbitrary voltage reference.

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Define some arbitrary voltage reference.
 Kirchhoff's laws: current flowing out of node *i* must balance:

$$\sum_{j=1}^N \frac{1}{R_{ij}}(V_j-V_i) = \delta_{is}-\delta_{it}.$$

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Define some arbitrary voltage reference. 🚓 Kirchhoff's laws: current flowing out of node *i* must balance:

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Setween connected nodes, $R_{ij} = 1 = a_{ij} = 1/a_{ij}$.

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Some gentle jiggery-pokery on the left hand side: $\sum_{i} a_{ij} (V_i - V_j)$

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Write right hand side as $[I^{\text{ext}}]_{i,st} = \delta_{is} - \delta_{it}$, where I_{st}^{ext} holds external source and sink currents.

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Write right hand side as $[I^{\text{ext}}]_{i,st} = \delta_{is} - \delta_{it}$, where I_{st}^{ext} holds external source and sink currents. Matrixingly then:

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L = K – A is a beast of some utility—known as the Laplacian. The PoCSverse Complex Networks 259 of 320

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- L = K A is a beast of some utility—known as the Laplacian.
- Solve for voltage vector \vec{V} by **LU** decomposition (Gaussian elimination).

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Do not compute an inverse!

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- Do not compute an inverse!
 - Note: voltage offset is arbitrary so no unique solution.
- Presuming network has one component, null space of K A is one dimensional.
 - In fact, $\mathcal{N}(\mathbf{K} \mathbf{A}) = \{c\vec{1}, c \in R\}$ since $(\mathbf{K} \mathbf{A})\vec{1} = \vec{0}$.

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Random walk betweenness:

Asking too much: Need full knowledge of network to travel along shortest paths. The PoCSverse Complex Networks 260 of 320

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Random walk betweenness:

- Asking too much: Need full knowledge of network to travel along shortest paths.
- One of many alternatives: consider all random walks between pairs of nodes *i* and *j*.

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Random walk betweenness:

- Asking too much: Need full knowledge of network to travel along shortest paths.
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- Random walk betweenness of an edge = absolute difference in probability a random walk travels one way versus the other along the edge.

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Random walk betweenness:

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- One of many alternatives: consider all random walks between pairs of nodes *i* and *j*.
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- Record the number of times an edge is followed by a walk.
- 🚳 Consider all pairs of nodes.
- Random walk betweenness of an edge = absolute difference in probability a random walk travels one way versus the other along the edge.
 - Equivalent to electronic betweenness (see also diffusion).

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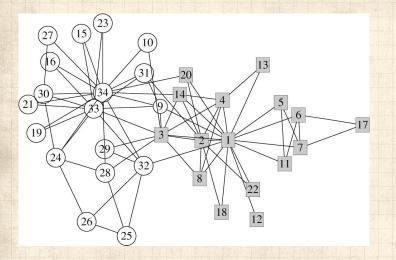
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Hierarchy by division



Factions in Zachary's karate club network. [119]

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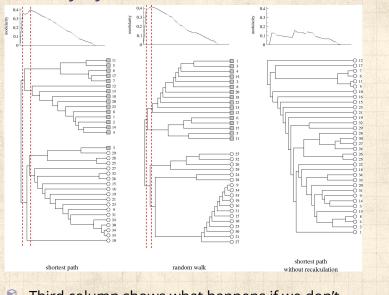
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Hierarchy by division



Third column shows what happens if we don't recompute betweenness after each edge removal.

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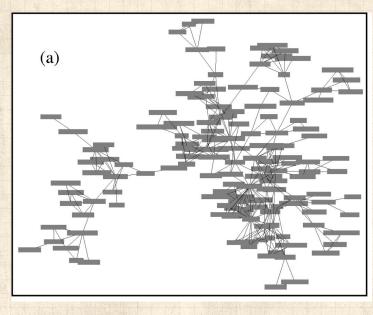
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Scientists working on networks (2004)



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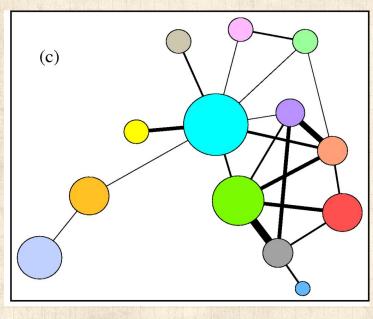
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Scientists working on networks (2004)



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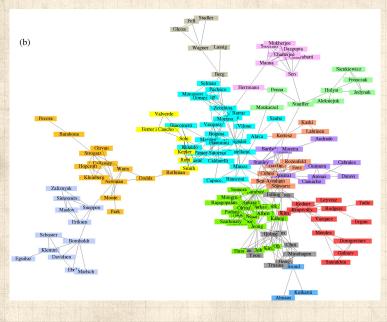
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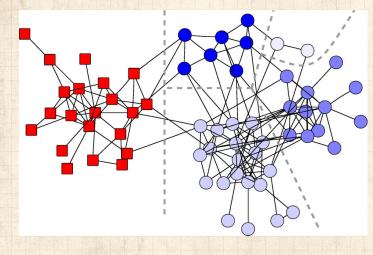
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Dolphins!



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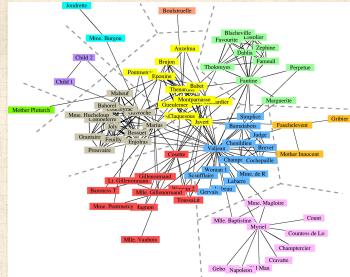
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Les Miserables



More network analyses for Les Miserables here and here . The PoCSverse Complex Networks 267 of 320

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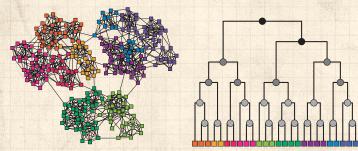
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Idea: Shades indicate probability that nodes in left and right subtrees of dendogram are connected. The PoCSverse Complex Networks 268 of 320

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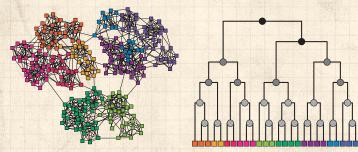
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 Idea: Shades indicate probability that nodes in left and right subtrees of dendogram are connected.
 Handle: Hierarchical random graph models. The PoCSverse Complex Networks 268 of 320

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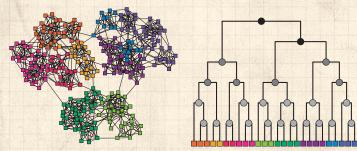
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 Handle: Hierarchical random graph models.
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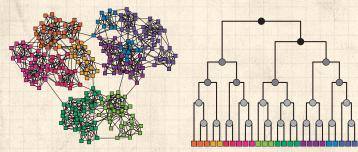
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- Idea: Shades indicate probability that nodes in left and right subtrees of dendogram are connected.
- 🚳 Handle: Hierarchical random graph models.
- Plan: Infer consensus dendogram for a given real network.
- Obtain probability that links are missing (big problem...).

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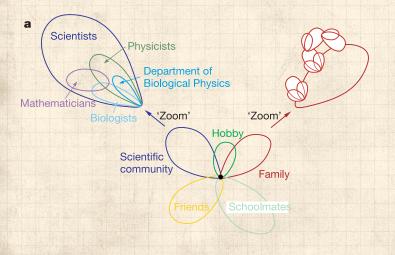
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"Uncovering the overlapping community structure of complex networks in nature and society" Palla et al., Nature, **435**, 814–818, 2005. ^[81]



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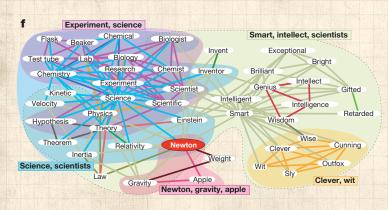
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"Link communities reveal multiscale complexity in networks" Ahn, Bagrow, and Lehmann, Nature, **466**, 761–764, 2010.^[2]



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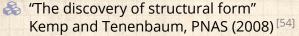
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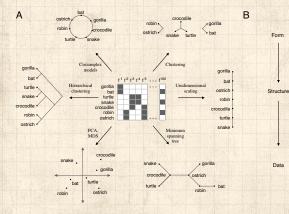
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General structure detection





Tree oorilla . bat turtle • snake • crocodile robin ostrich ·

Form

Data



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Example learned structures:



Brever White

Ginsburg

Blackmun Stevens Souter

Marshall

Brennan

С

O'Conno

Kennedy





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Biological features; Supreme Court votes; perceived color differences; face differences; & distances between cities.

Overview Key Points:

The field of complex networks came into existence in the late 1990s.

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To solve network problems: "Follow the edges."

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
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To solve network problems: "Follow the edges."

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Overview Key Points:

The field of complex networks came into existence in the late 1990s.

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 Hardened up much thinking about complex systems.

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- The field of complex networks came into existence in the late 1990s.
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- Specific focus on networks that are large-scale, sparse, natural or people-made, evolving and dynamic, and (crucially) measurable.

To solve network problems: "Follow the edges."

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- Specific focus on networks that are large-scale, sparse, natural or people-made, evolving and dynamic, and (crucially) measurable.
- 🚳 Three main (blurred) categories:
 - 1. Physical (e.g., river networks),
 - 2. Interactional (e.g., social networks),
 - 3. Abstract (e.g., thesauri).

To solve network problems: "Follow the edges."

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The map is not the territory.

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The map is not the territory.

Sometimes the map is not the territory because the territory does not exist.

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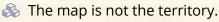
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- Sometimes the map is not the territory because the territory does not exist.
- "But it might one day!" yelled Captain Survivor Bias IV while holding up two pineapples to gauge the distance between waves.

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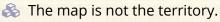
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line mapper is never the map.

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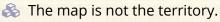
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- Sometimes the map is not the territory because the territory does not exist.
- "But it might one day!" yelled Captain Survivor Bias IV while holding up two pineapples to gauge the distance between waves.
- 🚳 And the mapper is never the map.
- (Scientific truths shouldn't be named after individuals.)

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Rather silly but great example of real science:

"How Cats Lap: Water Uptake by *Felis catus*" C Reis et al., *Science*, 2010.

A Study of Cat Lapping

Adult cats and dogs are unable to create suction in their mouths and must use their tongues to drink. A dog will scoop up liquid with the back of its tongue, but a cat will only touch the surface with the smooth tip of its tongue and pull a column of liquid into its mouth.











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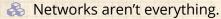
References



Source: Science

THE NEW YORK TIMES; IMAGES FROM VIDEO BY ROMAN STOCKER, SUNGHWAN JUNG, JEFFREY M. ARISTOFF AND PEDRO M. REIS

Amusing interview here



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Networks aren't everything. Famous models of networks aren't everything in networks.

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\delta Networks aren't everything.

- Famous models of networks aren't everything in networks.
- Even when networks are core to a system, the best level of analysis may involve some scale of grouping/averaging.

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\delta Networks aren't everything.

- Famous models of networks aren't everything in networks.
- Even when networks are core to a system, the best level of analysis may involve some scale of grouping/averaging.

Groups, groups, groups.

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\delta Networks aren't everything.

- Famous models of networks aren't everything in networks.
- Even when networks are core to a system, the best level of analysis may involve some scale of grouping/averaging.
- 🚳 Groups, groups, groups.
 - \diamond And pyramids (\sim hierarchies)

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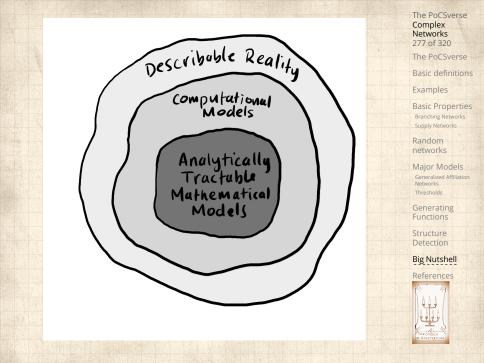
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Describuble Reality computational Models Analytically Tractuble Mathematical Models

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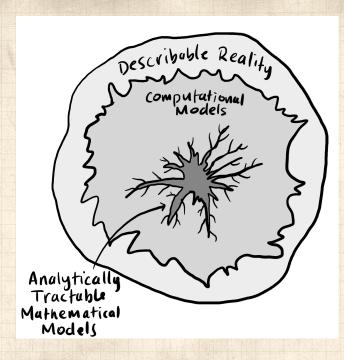
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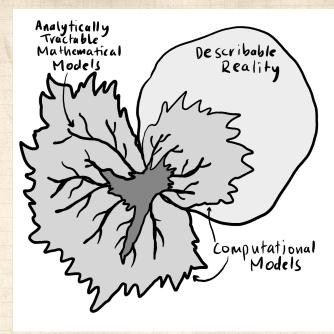
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Lord Kelvin (possibly):

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Lord Kelvin (possibly):
"To measure is to know."
"If you cannot measure it, you cannot improve it."

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Bonus: "X-rays will prove to be a hoax." The PoCSverse Complex Networks 281 of 320

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Bonus:

- 🚳 "X-rays will prove to be a hoax."
- 🚳 "There is nothing new to be discovered in physics now, All that remains is more and more precise measurement."



🚓 "Beards will always be cool."

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The Pyramid C knows what you did.

OTOT -

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The Pyramid C knows what you did.

O TOP

Mass surveillance by story.

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Modern basic science in three steps:

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Modern basic science in three steps:

1. Find interesting/meaningful/important phenomena, optionally involving spectacular amounts of data.

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Modern basic science in three steps:

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- 2. Describe what you see.

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Modern basic science in three steps:

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Modern basic science in three steps:

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If you succeed at 1–3:

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Modern basic science in three steps:

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- 2. Describe what you see.
- 3. Explain it.

If you succeed at 1–3: 4. Create.

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Modern basic science in three steps:

- 1. Find interesting/meaningful/important phenomena, optionally involving spectacular amounts of data.
- 2. Describe what you see.
- 3. Explain it.

If you succeed at 1–3:4. Create.5. Share.

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Modern basic science in three steps:

- 1. Find interesting/meaningful/important phenomena, optionally involving spectacular amounts of data.
- 2. Describe what you see.
- 3. Explain it.

If you succeed at 1–3: 4. Create. 5. Share.

Always:

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Modern basic science in three steps:

- 1. Find interesting/meaningful/important phenomena, optionally involving spectacular amounts of data.
- 2. Describe what you see.
- 3. Explain it.

If you succeed at 1–3:4. Create.5. Share.

Always: 6. Be good people. The PoCSverse Complex Networks 283 of 320

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